

# Essential Mathematics

## for Economic Analysis

Knut Sydsæter, Peter Hammond,  
Arne Strøm & Andrés Carvajal

A decorative graphic consisting of a blue L-shaped element. It has a horizontal bar at the top and a vertical bar extending downwards from the left end of the horizontal bar.

ESSENTIAL MATHEMATICS FOR

# ECONOMIC ANALYSIS

# Essential Mathematics for Economic Analysis

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