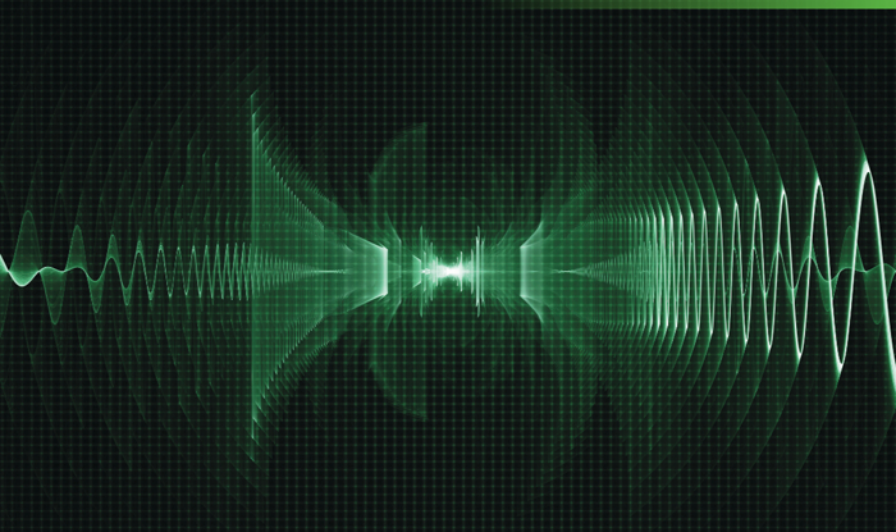


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# Mechanical Vibrations

Sixth Edition in SI Units

Singiresu S. Rao



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Sixth Edition in SI Units



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