

GLOBAL
EDITION



Mathematical Reasoning

for Elementary Teachers

SEVENTH EDITION

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Mathematical Reasoning

FOR ELEMENTARY TEACHERS

This book presents the mathematical knowledge needed for teaching, with an emphasis on why future teachers are learning the content, as well as when and how they will use it in the classroom. The Seventh Edition teaches the *content in context* to prepare today's students for tomorrow's classroom.



The **Common Core State Standards for Mathematics** include **8 Standards for Mathematical Practice (SMP)**, which have been integrated throughout this text. It's important for future teachers to know what will be expected of them when they are in the classroom, and these SMP references ensure that future teachers be both familiar and comfortable with these mathematical practices. Instances where an SMP applies are called out with an icon and highlighted text.

Continuing from the Common Core, the following eight Standards for Mathematical Practice are designed to teach students to

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice elaborate and reinforce the importance of Pólya's four principles of problems solving. In particular, special attention is given to the fourth principle, to

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quantities and their operation including differing units, such as cm, cm², cm³, Fahrenheit versus Celsius temperature, and so on. Computations with different units can cause a real change in a problem. Unfortunately, you will see an example of a disaster in the paragraph immediately after SMP 2.

"Mathematically proficient students make sense of quantities and their relationships in problem situations . . . Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them. . . ."

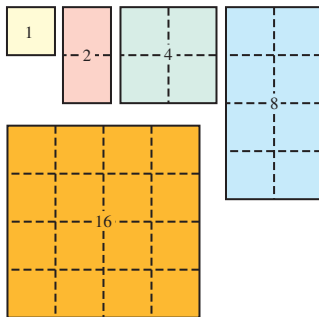
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COOPERATIVE INVESTIGATION

Numbers from Rectangles

Material Needed

1. One rectangle of each of these shapes for each student:



2. One record sheet like this for each student:

	16	8	4	2	1
0	0	0	0	0	0
1	0	0	0	0	1
18	1	0	0	1	0
19	1	0	0	1	1

	16	8	4	2	1
20	1	0	1	0	0
21					
38					
39					

Directions

- Step 1. Use the rectangles to determine whether or not there are representations of each of the numbers 0, 1, 2, . . . , 39 as a sum of the numbers 1, 2, 4, 8, or 16, with each of the latter group of numbers used at most once.
- Step 2. For each representation determined in step 1, record the numbers (rectangles) used by placing a 0 or a 1 in the appropriate columns of the record sheet. The rows for 0, 1, 18, 19, and 20 have been done for you.
 - (a) Do all the numbers from 0 through 39 have such a representation?
 - (b) What additional numbers could be represented if you had a 32 rectangle?
 - (c) Describe any interesting patterns you see on your record sheet.

COOPERATIVE INVESTIGATIONS begin each chapter, offering content-related games and puzzles that motivate the chapter. These can be easily adapted for use in the elementary classroom.

Mathematical Reasoning for Elementary Teachers, Global Edition

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