**Introduction to Finite Elements in Engineering**

**Table of Contents**

Cover
PREFACE
ABOUT THE AUTHORS
Contents

1 FUNDAMENTAL CONCEPTS
   1.1 Introduction
   1.2 Historical Background
   1.3 Outline of Presentation
   1.4 Stresses and Equilibrium
   1.5 Boundary Conditions
   1.6 Strain-Displacement Relations
   1.7 Stress-Strain Relations
   Special Cases
   1.8 Temperature Effects
   1.9 Potential Energy and Equilibrium: The Rayleigh-Ritz Method
      Potential Energy, $\Phi$;
      Rayleigh-Ritz Method
   1.10 Galerkin's Method
   1.11 Saint Venant's Principle
   1.12 Von Mises Stress
   1.13 Principle of Superposition
Table of Contents

1.14 Computer Programs
1.15 Conclusion
Historical References
Problems

2 MATRIX ALGEBRA AND GAUSSIAN ELIMINATION

2.1 Matrix Algebra
   - Row and Column Vectors
   - Addition and Subtraction
   - Multiplication by a Scalar
   - Matrix Multiplication
   - Transposition
   - Differentiation and Integration
   - Square Matrix
   - Diagonal Matrix
   - Identity Matrix
   - Symmetric Matrix
   - Upper Triangular Matrix
   - Determinant of a Matrix
   - Matrix Inversion
   - Eigenvalues and Eigenvectors
   - Positive Definite Matrix
   - Cholesky Decomposition

2.2 Gaussian Elimination
   - General Algorithm for Gaussian Elimination
   - Symmetric Matrix
   - Symmetric Banded Matrices
   - Solution with Multiple Right Sides
   - Gaussian Elimination with Column Reduction
   - Skyline Solution
Table of Contents

Frontal Solution

2.3 Conjugate Gradient Method for Equation Solving
   Conjugate Gradient Algorithm
   Input Data/Output

Problems
   Program Listings

3 ONE-DIMENSIONAL PROBLEMS
   3.1 Introduction
   3.2 Finite Element Modeling
      Element Division
      Numbering Scheme
   3.3 Shape Functions and Local Coordinates
   3.4 The Potential-Energy Approach
      Element Stiffness Matrix
      Force Terms
   3.5 The Galerkin Approach
      Element Stiffness
      Force Terms
   3.6 Assembly of the Global Stiffness Matrix and Load Vector
   3.7 Properties of K
   3.8 The Finite Element Equations: Treatment of Boundary Conditions
      Types of Boundary Conditions
      Elimination Approach
      Penalty Approach
      Multipoint Constraints
   3.9 Quadratic Shape Functions
   3.10 Temperature Effects
# Table of Contents

3.11 Problem Modeling and Boundary Conditions
- Problem in Equilibrium
- Symmetry
- Two Elements with Same End Displacements
- Problem with a Closing Gap
- Input Data/Output

Problems
- Program Listing

4 TRUSSES
4.1 Introduction
4.2 Plane Trusses
- Local and Global Coordinate Systems
- Formulas for Calculating \( l \) and \( m \)
- Element Stiffness Matrix
- Stress Calculations
- Temperature Effects
4.3 Three-Dimensional Trusses
4.4 Assembly of Global Stiffness Matrix for the Banded and Skyline Solutions
- Assembly for Banded Solution
- Skyline Assembly
4.5 Problem Modeling and Boundary Conditions
- Inclined Support in Two Dimensions
- Inclined Support in Three Dimensions
- Line Constraint
- Plane Constraint
- Symmetry and Antisymmetry
- Input Data/Output

Problems
## Table of Contents

Program Listing

5 BEAMS AND FRAMES

5.1 Introduction
   Potential-Energy Approach
   Galerkin Approach

5.2 Finite Element Formulation
   Element Stiffness
   Direct Approach

5.3 Load Vector

5.4 Boundary Considerations

5.5 Shear Force and Bending Moment

5.6 Beams on Elastic Supports

5.7 Plane Frames

5.8 Three-Dimensional Frames

5.9 Problem Modeling and Boundary Conditions

5.10 Some Comments
   Input Data/Output

Problems

Program Listings

6 TWO-DIMENSIONAL PROBLEMS USING CONSTANT STRAIN TRIANGLES

6.1 Introduction

6.2 Finite Element Modeling

6.3 Constant Strain Triangle (CST)
   Isoparametric Representation
   Potential-Energy Approach
   Element Stiffness
   Force Terms
Table of Contents

Integration Formula on a Triangle
Galerkin Approach
Stress Calculations
Temperature Effects

6.4 Problem Modeling and Boundary Conditions
Some General Comments on Dividing into Elements

6.5 Patch Test and Convergence
Patch Test

6.6 Orthotropic Materials
Temperature Effects
Input Data/Output

Problems
Program Listing

7 AXISYMMETRIC SOLIDS SUBJECTED TO AXISYMMETRIC LOADING

7.1 Introduction

7.2 Axisymmetric Formulation

7.3 Finite Element Modeling: Triangular Element
Potential Energy Approach
Body Force Term
Rotating Flywheel
Surface Traction
Galerkin Approach
Stress Calculations
Temperature Effects

7.4 Problem Modeling and Boundary Conditions
Cylinder Subjected to Internal Pressure
Infinite Cylinder
Table of Contents

Press Fit on a Rigid Shaft
Press Fit on an Elastic Shaft
Belleville Spring
Thermal Stress Problem
Input Data/Output

Problems
Program Listing

8 TWO-DIMENSIONAL ISOPARAMETRIC ELEMENTS AND NUMERICAL INTEGRATION

8.1 Introduction

8.2 The Four-Node Quadrilateral
   Shape Functions
   Element Stiffness Matrix
   Element Force Vectors

8.3 Numerical Integration
   Two-Dimensional Integrals
   Stiffness Integration
   Stress Calculations

8.4 Higher Order Elements
   Nine-Node Quadrilateral
   Eight-Node Quadrilateral
   Six-Node Triangle
   Integration on a Triangle
   Symmetric Points
   Integration on a Triangle
   Degenerate Quadrilateral

8.5 Four-Node Quadrilateral for Axisymmetric Problems

8.6 Conjugate Gradient Implementation of the Quadrilateral Element

8.7 Concluding Remarks and Convergence

8.8 References for Convergence
9 THREE-DIMENSIONAL PROBLEMS IN STRESS ANALYSIS

9.1 Introduction
9.2 Finite Element Formulation
   Element Stiffness
   Force Terms
9.3 Stress Calculations
9.4 Mesh Preparation
9.5 Hexahedral Elements and Higher Order Elements
9.6 Problem Modeling
9.7 Frontal Method for Finite Element Matrices
   Connectivity and Prefront Routine
   Element Assembly and Consideration of Specified dof
   Elimination of Completed dof
   Backsubstitution
   Consideration of Multipoint Constraints
Input Data/Output
Problems
   Program Listings

10 SCALAR FIELD PROBLEMS

10.1 Introduction
10.2 Steady-State Heat Transfer
   One-Dimensional Heat Conduction
   One-Dimensional Heat Transfer in Thin Fins
   Two-Dimensional Steady-State Heat Conduction
   Two-Dimensional Fins
# Table of Contents

Preprocessing for Program HEAT2D

10.3 Torsion
   Triangular Element
   Galerkin Approach[^2]

10.4 Potential Flow, Seepage, Electric and Magnetic Fields, and Fluid Flow in Ducts
   Potential Flow
   Seepage
   Electrical and Magnetic Field Problems
   Fluid Flow in Ducts
   Acoustics
   Boundary Conditions
   One-Dimensional Acoustics
   One-Dimensional Axial Vibrations
   Two-Dimensional Acoustics

10.5 Conclusion
   Input Data/Output

Problems
   Program Listings

11 DYNAMIC CONSIDERATIONS

11.1 Introduction

11.2 Formulation
   Solid Body with Distributed Mass

11.3 Element Mass Matrices

11.4 Evaluation of Eigenvalues and Eigenvectors
   Properties of Eigenvectors
   Eigenvalue-Eigenvector Evaluation
   Inverse Iteration Method
Table of Contents

Generalized Jacobi Method
Tridiagonalization and Implicit Shift Approach
Bringing Generalized Problem to Standard Form
Tridiagonalization
Implicit Symmetric QR Step with Wilkinson Shift for Diagonalization\(^{(2)}\)

11.5 Interfacing with Previous Finite Element Programs and a Program for Determining Critical Speeds of Shafts

11.6 Guyan Reduction

11.7 Rigid Body Modes

11.8 Conclusion
  Input Data/Output

Problems
  Program Listings

12 PREPROCESSING AND POSTPROCESSING

12.1 Introduction

12.2 Mesh Generation
  Region and Block Representation
  Block Corner Nodes, Sides, and Subdivisions

12.3 Postprocessing
  Deformed Configuration and Mode Shape
  Contour Plotting
  Nodal Values from Known Constant Element Values for a Triangle
  Least-Squares Fit for a Four-Noded Quadrilateral

12.4 Conclusion
  Input Data/Output

Problems
  Program Listings

APPENDIX Proof of \(dA = \det Jd\)
Table of Contents

BIBLIOGRAPHY
ANSWERS TO SELECTED PROBLEMS
INDEX