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THE CLASSIC WORK  
EXTENDED AND REFINED

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# The Art of Computer Programming

VOLUME 4B

Combinatorial Algorithms

Part 2

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DONALD E. KNUTH

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# **THE ART OF COMPUTER PROGRAMMING**

# Art of Computer Programming, Volume 4B, The: Combinatorial Algorithms

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