

# CONCRETE MATHEMATICS

A FOUNDATION FOR COMPUTER SCIENCE

GRAHAM



KNUTH



PATASHNIK

SECOND EDITION



*This page intentionally left blank*

# Concrete Mathematics: A Foundation for Computer Science

## Table of Contents

Cover

Title Page

Copyright Page

Preface

A Note on Notation

Contents

1 Recurrent Problems

1.1 The Tower of Hanoi

1.2 Lines in the Plane

1.3 The Josephus Problem

Exercises

2 Sums

2.1 Notation

2.2 Sums and Recurrences

2.3 Manipulation of Sums

2.4 Multiple Sums

2.5 General Methods

2.6 Finite and Infinite Calculus

2.7 Infinite Sums

Exercises

3 Integer Functions

# Table of Contents

- 3.1 Floors and Ceilings
- 3.2 Floor/Ceiling Applications
- 3.3 Floor/Ceiling Recurrences
- 3.4  $\text{'mod'}$ : The Binary Operation
- 3.5 Floor/Ceiling Sums
- Exercises

## 4 Number Theory

- 4.1 Divisibility
- 4.2 Primes
- 4.3 Prime Examples
- 4.4 Factorial Factors
- 4.5 Relative Primality
- 4.6  $\text{'mod'}$ : The Congruence Relation
- 4.7 Independent Residues
- 4.8 Additional Applications
- 4.9 Phi and Mu
- Exercises

## 5 Binomial Coefficients

- 5.1 Basic Identities
- 5.2 Basic Practice
- 5.3 Tricks of the Trade
- 5.4 Generating Functions
- 5.5 Hypergeometric Functions
- 5.6 Hypergeometric Transformations
- 5.7 Partial Hypergeometric Sums
- 5.8 Mechanical Summation
- Exercises

# **Table of Contents**

## **6 Special Numbers**

- 6.1 Stirling Numbers
- 6.2 Eulerian Numbers
- 6.3 Harmonic Numbers
- 6.4 Harmonic Summation
- 6.5 Bernoulli Numbers
- 6.6 Fibonacci Numbers
- 6.7 Continuants
- Exercises

## **7 Generating Functions**

- 7.1 Domino Theory and Change
- 7.2 Basic Maneuvers
- 7.3 Solving Recurrences
- 7.4 Special Generating Functions
- 7.5 Convolutions
- 7.6 Exponential Generating Functions
- 7.7 Dirichlet Generating Functions
- Exercises

## **8 Discrete Probability**

- 8.1 Definitions
- 8.2 Mean and Variance
- 8.3 Probability Generating Functions
- 8.4 Flipping Coins
- 8.5 Hashing
- Exercises

## **9 Asymptotics**

# **Table of Contents**

9.1 A Hierarchy

9.2 O Notation

9.3 O Manipulation

9.4 Two Asymptotic Tricks

9.5 Euler's Summation Formula

9.6 Final Summations

Exercises

A: Answers to Exercises

B: Bibliography

C: Credits for Exercises

Index

List of Tables