



$$2^{2^5} + 1 = 641 \cdot 6700417$$

$$\text{avg}(x, y) = (x \& y) + ((x \oplus y) \gg 1)$$

$$2^{2^6} + 1 = 274177 \cdot 67280421310721$$

$$x - y = x + \bar{y} + 1$$

$$\lfloor a \rfloor + \lfloor b \rfloor \leq \lfloor a + b \rfloor \leq \lfloor a \rfloor + \lfloor b \rfloor + 1 \quad \text{pop}(x) = -\sum_{i=0}^{31} (x \ll i) \text{rot}$$

George Boole
1815 - 1864

$$\lfloor \sqrt{11111111} \rfloor = 1111$$

$$(x \neq 0) = (x | \neg x) \gg 31$$

$$\text{mux}(x, y, m) = ((x \oplus y) \& m) \oplus y$$

$$A(n, d) = A(n-1, d-1), d \text{ even} \quad -\bar{x} = x + 1$$

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$\frac{1}{3} = 0.01010101\dots$

SECOND EDITION

$$1111^2 = 11100001$$

$$n = -2^{31}b_{31} + 2^{30}b_{30} + 2^{29}b_{29} + \dots + 2^0b_0$$

$$\lceil x \rceil = -\lfloor -x \rfloor \quad f(x, y, z) = g(x, y) \oplus zh(x, y)$$

$$\text{Num factors of 2 in } x = \log_2(x \& (-x)), x \neq 0$$

$$\text{rjust}(x) = x \gg (x \& -x), x \neq 0$$

$$p_n = 1 + \sum_{m=1}^n \left[\frac{1}{\sqrt{n}} \left[\sum_{k=1}^m \left[\cos^2 \pi \frac{(k-1)^2 + 1}{n} \right] \right] \right]^{1/n}$$

$$x \oplus y = (x | y) - (x \& y)$$

$$x + y = (x | y) + (x \& y)$$

HENRY S. WARREN, JR.

Hacker's Delight

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