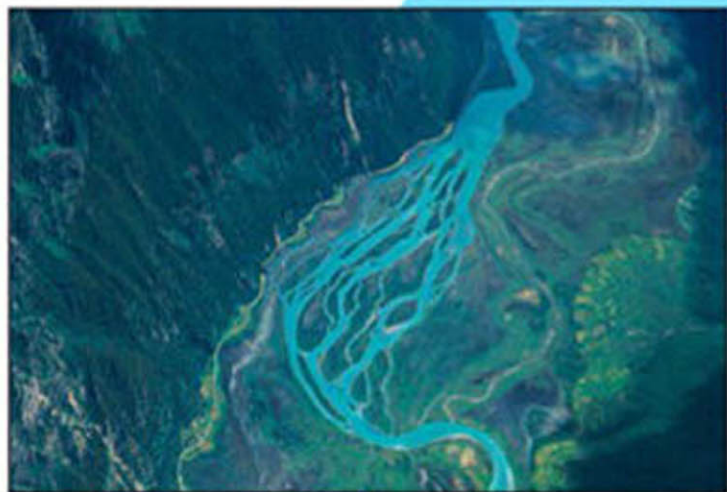




ADDISON-WESLEY PROFESSIONAL COMPUTING SERIES

Mathematical Foundations of Computer Networking

Srinivasan Keshav



Mathematical Foundations of Computer Networking

Mathematical Foundations of Computer Networking

Table of Contents

Contents

Preface

Chapter 1 Probability

1.1 Introduction

1.1.1 Outcomes

1.1.2 Events

1.1.3 Disjunctions and Conjunctions of Events

1.1.4 Axioms of Probability

1.1.5 Subjective and Objective Probability

1.2 Joint and Conditional Probability

1.2.1 Joint Probability

1.2.2 Conditional Probability

1.2.3 Bayess Rule

1.3 Random Variables

1.3.1 Distribution

1.3.2 Cumulative Density Function

1.3.3 Generating Values from an Arbitrary Distribution

1.3.4 Expectation of a Random Variable

1.3.5 Variance of a Random Variable

1.4 Moments and Moment Generating Functions

1.4.1 Moments

1.4.2 Moment Generating Functions

1.4.3 Properties of Moment Generating Functions

Table of Contents

1.5 Standard Discrete Distributions

- 1.5.1 Bernoulli Distribution
- 1.5.2 Binomial Distribution
- 1.5.3 Geometric Distribution
- 1.5.4 Poisson Distribution

1.6 Standard Continuous Distributions

- 1.6.1 Uniform Distribution
- 1.6.2 Gaussian, or Normal, Distribution
- 1.6.3 Exponential Distribution
- 1.6.4 Power-Law Distribution

1.7 Useful Theorems

- 1.7.1 Markov's Inequality
- 1.7.2 Chebyshev's Inequality
- 1.7.3 Chernoff Bound
- 1.7.4 Strong Law of Large Numbers
- 1.7.5 Central Limit Theorem

1.8 Jointly Distributed Random Variables

- 1.8.1 Bayesian Networks

1.9 Further Reading

1.10 Exercises

Chapter 2 Statistics

2.1 Sampling a Population

- 2.1.1 Types of Sampling
- 2.1.2 Scales
- 2.1.3 Outliers

2.2 Describing a Sample Parsimoniously

- 2.2.1 Tables
- 2.2.2 Bar Graphs, Histograms, and Cumulative Histograms
- 2.2.3 The Sample Mean

Table of Contents

- 2.2.4 The Sample Median
- 2.2.5 Measures of Variability
- 2.3 Inferring Population Parameters from Sample Parameters
- 2.4 Testing Hypotheses about Outcomes of Experiments
 - 2.4.1 Hypothesis Testing
 - 2.4.2 Errors in Hypothesis Testing
 - 2.4.3 Formulating a Hypothesis
 - 2.4.4 Comparing an Outcome with a Fixed Quantity
 - 2.4.5 Comparing Outcomes from Two Experiments
 - 2.4.6 Testing Hypotheses about Quantities Measured on Ordinal Scales
 - 2.4.7 Fitting a Distribution
 - 2.4.8 Power
- 2.5 Independence and Dependence: Regression and Correlation
 - 2.5.1 Independence
 - 2.5.2 Regression
 - 2.5.3 Correlation
- 2.6 Comparing Multiple Outcomes Simultaneously: Analysis of Variance
 - 2.6.1 One-Way Layout
 - 2.6.2 Multiway Layouts
- 2.7 Design of Experiments
- 2.8 Dealing with Large Data Sets
- 2.9 Common Mistakes in Statistical Analysis
 - 2.9.1 Defining Population
 - 2.9.2 Lack of Confidence Intervals in Comparing Results
 - 2.9.3 Not Stating the Null Hypothesis
 - 2.9.4 Too Small a Sample
 - 2.9.5 Too Large a Sample
 - 2.9.6 Not Controlling All Variables When Collecting Observations

Table of Contents

2.9.7 Converting Ordinal to Interval Scales

2.9.8 Ignoring Outliers

2.10 Further Reading

2.11 Exercises

Chapter 3 Linear Algebra

3.1 Vectors and Matrices

3.2 Vector and Matrix Algebra

3.2.1 Addition

3.2.2 Transpose

3.2.3 Multiplication

3.2.4 Square Matrices

3.2.5 Exponentiation

3.2.6 Matrix Exponential

3.3 Linear Combinations, Independence, Basis, and Dimension

3.3.1 Linear Combinations

3.3.2 Linear Independence

3.3.3 Vector Spaces, Basis, and Dimension

3.4 Using Matrix Algebra to Solve Linear Equations

3.4.1 Representation

3.4.2 Elementary Row Operations and Gaussian Elimination

3.4.3 Rank

3.4.4 Determinants

3.4.5 Cramers Theorem

3.4.6 The Inverse of a Matrix

3.5 Linear Transformations, Eigenvalues, and Eigenvectors

3.5.1 A Matrix as a Linear Transformation

3.5.2 The Eigenvalue of a Matrix

3.5.3 Computing the Eigenvalues of a Matrix

3.5.4 The Importance of Eigenvalues

Table of Contents

3.5.5 The Role of the Principal Eigenvalue

3.5.6 Finding Eigenvalues and Eigenvectors

3.5.7 Similarity and Diagonalization

3.6 Stochastic Matrices

3.6.1 Computing State Transitions by Using a Stochastic Matrix

3.6.2 Eigenvalues of a Stochastic Matrix

3.7 Exercises

Chapter 4 Optimization

4.1 System Modeling and Optimization

4.2 Introduction to Optimization

4.3 Optimizing Linear Systems

4.3.1 Network Flow

4.4 Integer Linear Programming

4.4.1 Total Unimodularity

4.4.2 Weighted Bipartite Matching

4.5 Dynamic Programming

4.6 Nonlinear Constrained Optimization

4.6.1 Lagrangian Techniques

4.6.2 Karush-Kuhn-Tucker Conditions for Nonlinear Optimization

4.7 Heuristic Nonlinear Optimization

4.7.1 Hill Climbing

4.7.2 Genetic Algorithms

4.8 Exercises

Chapter 5 Signals, Systems, and Transforms

5.1 Background

5.1.1 Sinusoids

5.1.2 Complex Numbers

5.1.3 Eulers Formula

Table of Contents

5.1.4 Discrete-Time Convolution and the Impulse Function

5.1.5 Continuous-Time Convolution and the Dirac Delta Function

5.2 Signals

5.2.1 The Complex Exponential Signal

5.3 Systems

5.4 Analysis of a Linear Time-Invariant System

5.4.1 The Effect of an LTI System on a Complex Exponential Input

5.4.2 The Output of an LTI System with a Zero Input

5.4.3 The Output of an LTI System for an Arbitrary Input

5.4.4 Stability of an LTI System

5.5 Transforms

5.6 The Fourier Series

5.7 The Fourier Transform and Its Properties

5.8 The Laplace Transform

5.8.1 Poles, Zeroes, and the Region of Convergence

5.8.2 Properties of the Laplace Transform

5.9 The Discrete Fourier Transform and Fast Fourier Transform

5.9.1 The Impulse Train

5.9.2 The Discrete-Time Fourier Transform

5.9.3 Aliasing

5.9.4 The Discrete-Time-and-Frequency Fourier Transform

5.9.5 The Fast Fourier Transform

5.10 The Z Transform

5.10.1 Relationship between Z and Laplace Transforms

5.10.2 Properties of the Z Transform

5.11 Further Reading

5.12 Exercises

Chapter 6 Stochastic Processes and Queueing Theory

Table of Contents

6.1 Overview

6.1.1 A General Queueing System

6.1.2 Little's Theorem

6.2 Stochastic Processes

6.2.1 Discrete and Continuous Stochastic Processes

6.2.2 Markov Processes

6.2.3 Homogeneity, State-Transition Diagrams, and the
Chapman-Kolmogorov Equations

6.2.4 Irreducibility

6.2.5 Recurrence

6.2.6 Periodicity

6.2.7 Ergodicity

6.2.8 A Fundamental Theorem

6.2.9 Stationary (Equilibrium) Probability of a Markov Chain

6.2.10 A Second Fundamental Theorem

6.2.11 Mean Residence Time in a State

6.3 Continuous-Time Markov Chains

6.3.1 Markov Property for Continuous-Time Stochastic Processes

6.3.2 Residence Time in a Continuous-Time Markov Chain

6.3.3 Stationary Probability Distribution for a Continuous-Time Markov Chain

6.4 Birth-Death Processes

6.4.1 Time-Evolution of a Birth-Death Process

6.4.2 Stationary Probability Distribution of a Birth-Death Process

6.4.3 Finding the Transition-Rate Matrix

6.4.4 A Pure-Birth (Poisson) Process

6.4.5 Stationary Probability Distribution for a Birth-Death Process

6.5 The M/M/1 Queue

6.6 Two Variations on the M/M/1 Queue

6.6.1 The M/M/∞ Queue: A Responsive Server

6.6.2 M/M/1/K: Bounded Buffers

Table of Contents

6.7 Other Queueing Systems

6.7.1 M/D/1: Deterministic Service Times

6.7.2 G/G/1

6.7.3 Networks of Queues

6.8 Further Reading

6.9 Exercises

Chapter 7 Game Theory

7.1 Concepts and Terminology

7.1.1 Preferences and Preference Ordering

7.1.2 Terminology

7.1.3 Strategies

7.1.4 Game Representation

7.1.5 Response and Best Response

7.1.6 Dominant and Dominated Strategy

7.1.7 Bayesian Games

7.1.8 Repeated Games

7.2 Solving a Game

7.2.1 Solution Concept and Equilibrium

7.2.2 Dominant-Strategy Equilibria

7.2.3 Iterated Removal of Dominated Strategies

7.2.4 Maximin Equilibrium

7.2.5 Nash Equilibria

7.2.6 Correlated Equilibria

7.2.7 Other Solution Concepts

7.3 Mechanism Design

7.3.1 Practical Mechanisms

7.3.2 Three Negative Results

7.3.3 Examples of Mechanism Design

7.3.4 Formalization

Table of Contents

7.3.5 Desirable Properties of a Mechanism

7.3.6 Revelation Principle

7.3.7 Vickrey-Clarke-Groves Mechanism

7.3.8 Problems with VCG Mechanisms

7.4 Limitations of Game Theory

7.5 Further Reading

7.6 Exercises

Chapter 8 Elements of Control Theory

8.1 Overview of a Controlled System

8.2 Modeling a System

8.2.1 Modeling Approach

8.2.2 Mathematical Representation

8.3 A First-Order System

8.4 A Second-Order System

8.4.1 Case 1 (Undamped System): $\zeta = 0$

8.4.2 Case 2 (Underdamped System): $0 < \zeta < 1$

8.4.3 Case 3 (Critically Damped System): $\zeta = 1$

8.4.4 Case 4 (Overdamped System): $\zeta > 1$

8.5 Basics of Feedback Control

8.5.1 System Goal

8.5.2 Constraints

8.6 PID Control

8.6.1 Proportional-Mode Control

8.6.2 Integral-Mode Control

8.6.3 Derivative-Mode Control

8.6.4 Combining Modes

8.7 Advanced Control Concepts

8.7.1 Cascade Control

8.7.2 Control Delay

Table of Contents

8.8 Stability

8.8.1 BIBO Stability Analysis of a Linear Time-Invariant System

8.8.2 Zero-Input Stability Analysis of a SISO Linear Time-Invariant System

8.8.3 Placing System Roots

8.8.4 Lyapunov Stability

8.9 State SpaceBased Modeling and Control

8.9.1 State SpaceBased Analysis

8.9.2 Observability and Controllability

8.9.3 Controller Design

8.10 Digital Control

8.11 Partial Fraction Expansion

8.11.1 Distinct Roots

8.11.2 Complex Conjugate Roots

8.11.3 Repeated Roots

8.12 Further Reading

8.13 Exercises

Chapter 9 Information Theory

9.1 Introduction

9.2 A Mathematical Model for Communication

9.3 From Messages to Symbols

9.4 Source Coding

9.5 The Capacity of a Communication Channel

9.5.1 Modeling a Message Source

9.5.2 The Capacity of a Noiseless Channel

9.5.3 A Noisy Channel

9.6 The Gaussian Channel

9.6.1 Modeling a Continuous Message Source

9.6.2 A Gaussian Channel

Table of Contents

9.6.3 The Capacity of a Gaussian Channel

9.7 Further Reading

9.8 Exercises

Solutions to Exercises

Index