MANAGERIAL DECISION MODELING

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Managerial Decision Modeling

As in the multiperiod production scheduling problem, we need to write balance constraints for each period (year). These constraints recognize the relationship between the investment decisions made in any given year and the investment decisions made in all prior years. Specifically, we need to ensure that the amount used for investment at the start of a given year is restricted to the amount maturing at the end of the previous year *less* any payments made for Susan's education that year. This relationship can be modeled as

$$\begin{pmatrix}\text{amount}\\ \text{invested at}\\ \text{start of}\\ \text{year } t\end{pmatrix} + \begin{pmatrix}\text{amount}\\ \text{paid for}\\ \text{education at}\\ \text{start of year } t\end{pmatrix} = \begin{pmatrix}\text{amound}\\ \text{maturing}\\ \text{at end}\\ \text{of year } (t-1)\end{pmatrix}$$

This equation is analogous to the inventory equations in the production scheduling problem.

At the start of year 2, the total amount maturing is $1.05A_1$ (investment in choice A in year 1 plus 5% interest). The constraint at the start of year 2 can therefore be written as

$$A_2 + B_2 + C_2 + D_2 = 1.05A_1 \qquad \text{(year 2 cash flow)}$$

These are the cash flow constraints.

Constraints at the start of years 3 through 6 are as follows and also include the amounts payable for Susan's education each year:

$$\begin{aligned}
A_3 + B_3 + C_3 + 20{,}000 &= 1.13B_1 + 1.05A_2 && \text{(year 3 cash flow)}\\
A_4 + B_4 + 22{,}000 &= 1.28C_1 + 1.13B_2 + 1.05A_3 && \text{(year 4 cash flow)}\\
A_5 + 24{,}000 &= 1.4D_1 + 1.28C_2 + 1.13B_3 + 1.05A_4 && \text{(year 5 cash flow)}\\
26{,}000 &= 1.4D_2 + 1.28C_3 + 1.13B_4 + 1.05A_5 && \text{(year 6 cash flow)}
\end{aligned}$$

These five constraints address the cash flow issues. However, they do not account for Larry's risk preference with regard to investments in choices C and D in any given year. To satisfy these requirements, we need to ensure that total investment in choices C and D in any year is no more than 20% of the total investment in *all* choices that year. In keeping track of these investments, it is important to also account for investments in *prior* years that may have still not matured. At the start of year 1, this constraint can be written as

$$C_1 + D_1 \le 0.2(A_1 + B_1 + C_1 + D_1) \qquad \text{(year 1 risk)}$$

These are the risk preference constraints.

In writing this constraint at the start of year 2, we must take into account the fact that investments B_1, C_1, and D_1 have still not matured. Therefore,

$$C_1 + D_1 + C_2 + D_2 \le 0.2(B_1 + C_1 + D_1 + A_2 + B_2 + C_2 + D_2) \qquad \text{(year 2 risk)}$$

Constraints at the start of years 3 through 5 are as follows. Note that there is no constraint necessary at the start of year 6 because there are no investments that year:

$$\begin{aligned}
C_1 + D_1 + C_2 + D_2 + C_3 &\le 0.2(C_1 + D_1 + B_2 + C_2 + D_2 + A_3 + B_3 + C_3) && \text{(year 3 risk)}\\
D_1 + C_2 + D_2 + C_3 &\le 0.2(D_1 + C_2 + D_2 + B_3 + C_3 + A_4 + B_4) && \text{(year 4 risk)}\\
D_2 + C_3 &\le 0.2(D_2 + C_3 + B_4 + A_5) && \text{(year 5 risk)}
\end{aligned}$$

Finally, we have the nonnegativity constraints:

All variables ≥ 0

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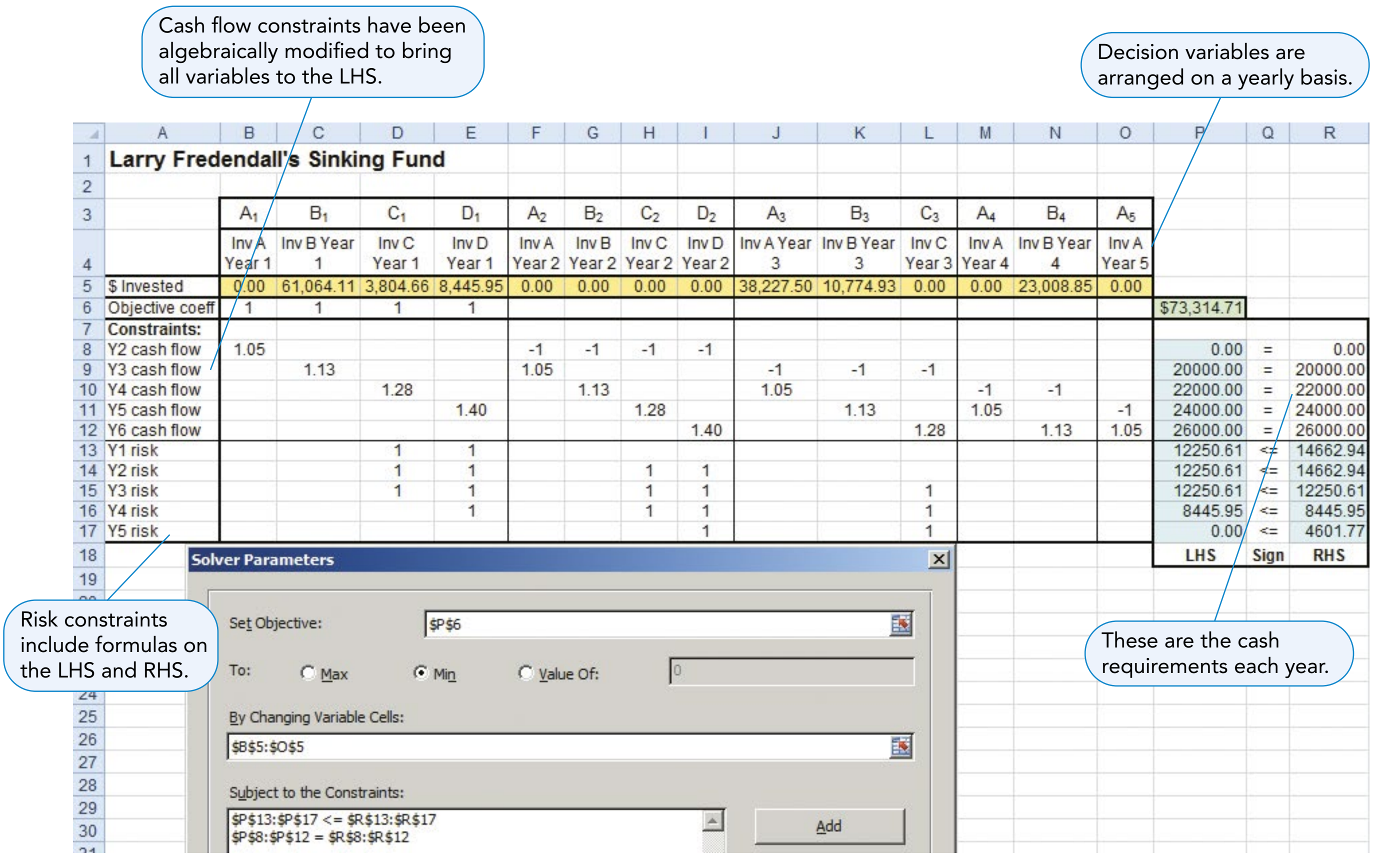
SOLVING THE PROBLEM AND INTERPRETING THE RESULTS Screenshot 3-12 shows the Excel layout and Solver entries for this model. As with the production scheduling problem, there are several alternate ways in which the Excel layout could be structured, depending on the preference and expertise of the analyst. In our implementation of this model, we have algebraically modified the cash flow constraints for each year so that all variables are on the LHS and the education cash outflows are on the RHS. We have, however, implemented the risk constraints as written in the formulation above. The modified cash flow constraints are as follows:

$$\begin{aligned}
1.05A_1 - A_2 - B_2 - C_2 - D_2 &= 0 && \text{(year 2 cash flow)}\\
1.13B_1 + 1.05A_2 - A_3 - B_3 - C_3 &= 20{,}000 && \text{(year 3 cash flow)}\\
1.28C_1 + 1.13B_2 + 1.05A_3 - A_4 - B_4 &= 22{,}000 && \text{(year 4 cash flow)}\\
1.4D_1 + 1.28C_2 + 1.13B_3 + 1.05A_4 - A_5 &= 24{,}000 && \text{(year 5 cash flow)}\\
1.4D_2 + 1.28C_3 + 1.13B_4 + 1.05A_5 &= 26{,}000 && \text{(year 6 cash flow)}
\end{aligned}$$

SCREENSHOT 3-12 Excel Layout and Solver Entries for Larry Fredendall's Sinking Fund



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	**Larry Fredendall's Sinking Fund**																	
2																		
3		A_1	B_1	C_1	D_1	A_2	B_2	C_2	D_2	A_3	B_3	C_3	A_4	B_4	A_5			
4		Inv A Year 1	Inv B Year 1	Inv C Year 1	Inv D Year 1	Inv A Year 2	Inv B Year 2	Inv C Year 2	Inv D Year 2	Inv A Year 3	Inv B Year 3	Inv C Year 3	Inv A Year 4	Inv B Year 4	Inv A Year 5			
5	$ Invested	0.00	61,064.11	3,804.66	8,445.95	0.00	0.00	0.00	0.00	38,227.50	10,774.93	0.00	0.00	23,008.85	0.00			
6	Objective coeff	1	1	1	1											$73,314.71		
7	**Constraints:**																	
8	Y2 cash flow	1.05				-1	-1	-1	-1							0.00	=	0.00
9	Y3 cash flow		1.13			1.05				-1	-1	-1				20000.00	=	20000.00
10	Y4 cash flow			1.28			1.13			1.05			-1	-1		22000.00	=	22000.00
11	Y5 cash flow				1.40			1.28			1.13		1.05		-1	24000.00	=	24000.00
12	Y6 cash flow								1.40			1.28		1.13	1.05	26000.00	=	26000.00
13	Y1 risk			1	1											12250.61	<=	14662.94
14	Y2 risk			1	1			1	1							12250.61	<=	14662.94
15	Y3 risk			1	1			1	1			1				12250.61	<=	12250.61
16	Y4 risk				1			1	1			1				8445.95	<=	8445.95
17	Y5 risk								1			1				0.00	<=	4601.77
18																**LHS**	**Sign**	**RHS**

The optimal solution requires Larry to invest a total of $73,314.71 at the start of year 1, putting $61,064.11 in choice B, $3,804.66 in choice C, and $8,445.95 in choice D. There is no money maturing for investment at the start of year 2. At the start of year 3, using the maturing amounts, Larry should pay off $20,000 for Susan's education, invest $38,227.50 in choice A, and invest $10,774.93 in choice B. At the start of year 4, Larry should use the maturing amounts to pay off $22,000 for Susan's education and invest $23,008.85 in choice B. The investments in place at that time will generate $24,000 at the start of year 5 and $26,000 at the start of year 6, meeting Larry's requirements in those years.

Summary

This chapter continues the discussion of LP models. To show ways of formulating and solving problems from a variety of disciplines, we examine applications from manufacturing, marketing, finance, employee scheduling, transportation, ingredient blending, and multiperiod planning. We also consider an example with a special type of objective function. For each example, we illustrate how to set up and solve all these models by using Excel's Solver add-in.

Solved Problem

Solved Problem 3-1

The Loughry Group has opened a new mall in Gainesville, Florida. Mark Loughry, the general manager of the mall, is trying to ensure that enough support staff are available to clean the mall before it opens each day. The mall operates seven days a week, and the cleaning staff work between 12:30 A.M. and 8:30 A.M. each night. Based on projected mall traffic data for the upcoming week, Mark estimates that the number of cleaning staff required each day will be as shown in Table 3.15.

TABLE 3.15
Cleaning Staff Requirement Data for Loughry Group's Mall

DAY OF WEEK	NUMBER OF STAFF REQUIRED
Monday	22
Tuesday	13
Wednesday	15
Thursday	20
Friday	18
Saturday	23
Sunday	27

TABLE 3.16
Schedule and Cost Data for Loughry Group's Mall

WORK SCHEDULE	WAGES PER WEEK
1. Saturday and Sunday off	\$350
2. Saturday and Tuesday off	\$375
3. Tuesday and Wednesday off	\$400
4. Monday and Thursday off	\$425
5. Tuesday and Friday off	\$425
6. Thursday and Friday off	\$400
7. Sunday and Thursday off	\$375
8. Sunday and Wednesday off	\$375

Mark can use the work schedules shown in Table 3.16 for the cleaning staff. The wages for each schedule are also shown in Table 3.16. In order to be perceived as being a fair employer, Mark wants to ensure that at least 75% of the workers have two consecutive days off and that at least 50% of the workers have at least one weekend day off. How should Mark schedule his cleaning staff in order to meet the mall's requirements?

Solution

FORMULATING AND SOLVING THE PROBLEM As noted in the previous labor staffing problem, the decision variables typically determine how many employees need to start their work at the different starting times permitted. In Mark's case, because there are eight possible work schedules, we have eight decision variables in the problem. Let

S_1 = number of employees who need to follow schedule 1 (Saturday and Sunday off)
S_2 = number of employees who need to follow schedule 2 (Saturday and Tuesday off)
S_3 = number of employees who need to follow schedule 3 (Tuesday and Wednesday off)
S_4 = number of employees who need to follow schedule 4 (Monday and Thursday off)
S_5 = number of employees who need to follow schedule 5 (Tuesday and Friday off)
S_6 = number of employees who need to follow schedule 6 (Thursday and Friday off)
S_7 = number of employees who need to follow schedule 7 (Sunday and Thursday off)
S_8 = number of employees who need to follow schedule 8 (Sunday and Wednesday off)

This is the objective function:

$$\text{Minimize total weekly wages} = \$350\,S_1 + \$375\,S_2 + \$400\,S_3 + \$425\,S_4 + \$425\,S_5 + \$400\,S_6 + \$375\,S_7 + \$375\,S_8$$

Subject to the constraints

$$S_1 + S_2 + S_3 + S_5 + S_6 + S_7 + S_8 \geq 22 \quad \text{(Monday requirement)}$$
$$S_1 + S_2 + S_4 + S_6 + S_7 + S_8 \geq 13 \quad \text{(Tuesday requirement)}$$
$$S_1 + S_2 + S_4 + S_5 + S_6 + S_7 \geq 15 \quad \text{(Wednesday requirement)}$$

$$S_1 + S_2 + S_3 + S_5 + S_8 \geq 20 \quad \text{(Thursday requirement)}$$
$$S_1 + S_2 + S_3 + S_4 + S_7 + S_8 \geq 18 \quad \text{(Friday requirement)}$$
$$S_3 + S_4 + S_5 + S_6 + S_7 + S_8 \geq 23 \quad \text{(Saturday requirement)}$$
$$S_2 + S_3 + S_4 + S_5 + S_6 \geq 27 \quad \text{(Sunday requirement)}$$

At least 75% of workers must have two consecutive days off each week, and at least 50% of workers must have at least one weekend day off each week. These constraints may be written, respectively, as

$$S_1 + S_3 + S_6 \geq 0.75\,(S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8)$$
$$S_1 + S_2 + S_7 + S_8 \geq 0.5\,(S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8)$$

Finally,

$$S_1, S_2, S_3, S_5, S_6, S_7, S_8 \geq 0$$

WWW

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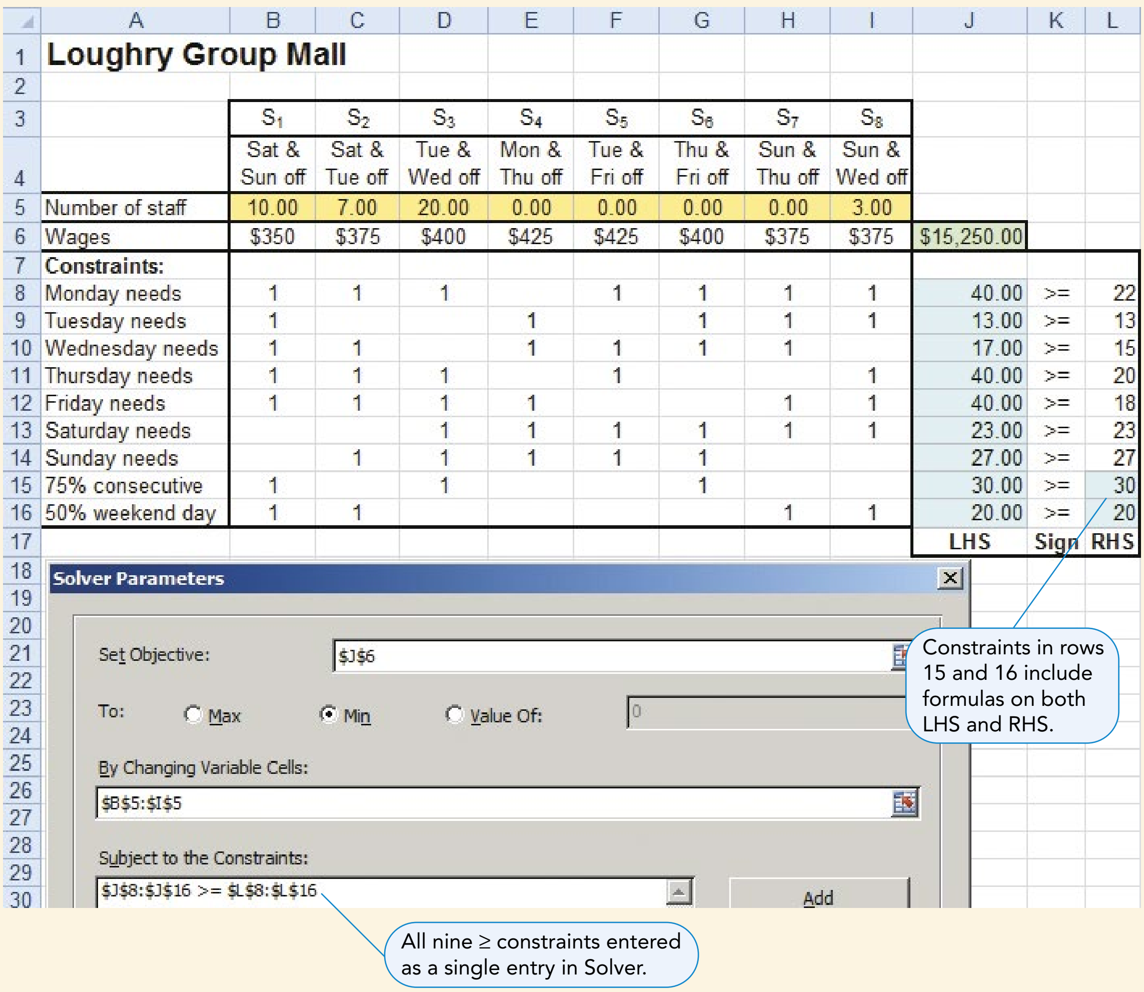
The Excel layout and Solver entries for this model are shown in Screenshot 3-13. For the last two constraints, the Excel layout includes formulas for both the LHS (cells J15:J16) and RHS (cells L15:L16) entries. While the formulas in cells J15:J16 use the usual SUMPRODUCT function, the formulas in cells L15:L16 are

Cell L15: =0.75*SUM($B$5:$I$5)
Cell L16: =0.50*SUM($B$5:$I$5)

SCREENSHOT 3-13 Excel Layout and Solver Entries for Loughry Group Mall



	A	B	C	D	E	F	G	H	I	J	K	L
1	Loughry Group Mall											
2												
3		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8			
4		Sat & Sun off	Sat & Tue off	Tue & Wed off	Mon & Thu off	Tue & Fri off	Thu & Fri off	Sun & Thu off	Sun & Wed off			
5	Number of staff	10.00	7.00	20.00	0.00	0.00	0.00	0.00	3.00			
6	Wages	$350	$375	$400	$425	$425	$400	$375	$375	$15,250.00		
7	Constraints:											
8	Monday needs	1	1	1		1	1	1	1	40.00	>=	22
9	Tuesday needs	1			1		1	1	1	13.00	>=	13
10	Wednesday needs	1	1		1	1	1	1		17.00	>=	15
11	Thursday needs	1	1	1		1			1	40.00	>=	20
12	Friday needs	1	1	1	1			1	1	40.00	>=	18
13	Saturday needs			1	1	1	1	1	1	23.00	>=	23
14	Sunday needs		1	1	1	1	1			27.00	>=	27
15	75% consecutive	1		1			1			30.00	>=	30
16	50% weekend day	1	1					1	1	20.00	>=	20
17										LHS	Sign	RHS

INTERPRETING THE RESULTS Screenshot 3-13 reveals that the optimal solution is to employ 10 people on schedule 1; 7 people on schedule 2; 20 people on schedule 3; and 3 people on schedule 8, for a total cost of $15,250 per week. As in the solution for the Hong Kong Bank problem (see Screenshot 3-6 on page 83), it turns out that this problem has alternate optimal solutions, too. For example, we can satisfy the staff requirements at the same cost by employing 10 people on schedule 1; 7 people on schedule 2; 3 people on schedule 3; 17 people on schedule 6; and 3 people on schedule 7.

The solution indicates that while exactly meeting staffing needs for Tuesday, Saturday, and Sunday, Mark is left with way more than he needs on Monday, Thursday, and Friday. He should perhaps consider using part-time help to alleviate this mismatch in his staffing needs.

Problems

3-1 A small backpack manufacturer carries four different models of backpacks, made of canvas, plastic, nylon, and leather. The bookstore, which will exclusively sell the backpacks, expects to be able to sell between 15 and 40 of each model. The store has agreed to pay $35.50 for each canvas backpack, $39.50 for each plastic backpack, $42.50 for each nylon backpack, and $69.50 for each leather backpack that can be delivered by the end of the following week.

One worker can work on either canvas or plastic, can complete a backpack in 1.5 hours, and will charge $7.00 per hour to do the work. This worker can work a maximum of 90 hours during the next week. Another worker can sew backpacks made of nylon fabric. He can complete a bag in 1.7 hours, will charge $8.00 per hour to work, and can work 42.5 hours in the next week. A third worker has the ability to sew leather. He each can complete a book bag in 1.9 hours, will charge $9.00 per hour to work, and can work 80 hours during the next week. The following table provides additional information about each backpack. What is the best combination of backpacks to provide the store to maximize the profit?

BACKPACK MODEL	MATERIAL REQUIRED (SQUARE YARDS)	MATERIAL AVAILABLE (SQUARE YARDS)	COST/ SQUARE YARD
Canvas	2.25	200	$4.50
Plastic	2.40	350	$4.25
Nylon	2.10	700	$7.65
Leather	2.60	550	$9.45

3-2 A contestant on the hit reality television show Top Bartender was asked to mix a variety of drinks, each consisting of 4 fluid ounces. No other ingredients were permitted. She was given the following quantities of liquor:

LIQUOR	QUANTITY
Bourbon	128 ounces
Brandy	128 ounces
Vodka	128 ounces
Dry Vermouth	32 ounces
Sweet Vermouth	32 ounces

The contestant is considering making the following four drinks:

- The New Yorker: 25% each of bourbon, brandy, vodka, and sweet vermouth
- The Garaboldi: 25% each of brandy and dry vermouth; 50% sweet vermouth
- The Kentuckian: 100% bourbon
- The Russian: 75% vodka and 25% dry vermouth

The contestant's objective is to make the largest number of drinks with the available liquor. What is the combination of drinks to meet her objective?

3-3 A manufacturer of travel pillows must determine the production plan for the next production cycle. He wishes to make at least 300 of each of the three models that his firm offers and no more than 1,200 of any one model. The specifics for each model are shown in the following table. How many pillows of each type should be manufactured in order to maximize total profit?

PILLOW MODEL	SELLING PRICE	CUTTING	SEWING	FINISHING	PACKING
Junior travel pillow	$5.75	0.10	0.05	0.18	0.20
Travel pillow	$6.95	0.15	0.12	0.24	0.20
Deluxe travel pillow	$7.50	0.20	0.18	0.20	0.20
Available hours		450	550	600	450
Cost per hour		$7.00	$9.00	$8.50	$7.25

Table for Problem 3-5

CABINET STYLE	CARPENTRY	PAINTING	FINISHING	PROFIT
Italian	3.00	1.50	0.75	$72
French	2.25	1.00	0.75	$65
Caribbean	2.50	1.25	0.85	$78
Available hours	1,360	700	430	

3-4 Students who are trying to raise funds have an agreement with a local pizza chain. The chain has agreed to sell them pizzas at a discount, which the students can then resell to families in the local community for a profit. It is expected that of the 500 families in the community, at most 70% will buy pizza. Based on a survey of their personal preferences, the students believe that they should order no more than 120 cheese pizzas, no more than 150 pepperoni pizzas, and no more than 100 vegetarian pizzas. They also want to make sure that at least 20% of the total pizzas are cheese and at least 50% of the pizzas are pepperoni. They make a profit of $1.45, $1.75, and $1.98, respectively, for each cheese, pepperoni, and vegetarian pizza they resell. How many pizzas of each type should they buy?

3-5 A furniture maker sells three different styles of cabinets, including an Italian model, a French Country model, and a Caribbean model. Each cabinet produced must go through three departments: carpentry, painting, and finishing. The table at the top of this page contains all relevant information concerning production times (hours per cabinet), production capacities for each operation per day, and profit ($ per unit). The owner has an obligation to deliver a minimum of 60 cabinets in each style to a furniture distributor. He would like to determine the product mix that maximizes his daily profit. Formulate the problem as an LP model and solve using Excel.

3-6 An electronics manufacturer has an option to produce six styles of cell phones. Each of these devices requires time, in minutes, on three types of electronic testing equipment, as shown in the table at the bottom of this page. The first two test devices are each available for 120 hours per week. Test device 3 requires more preventive maintenance and may be used only for 100 hours each week. The market for all six cell phones is vast, so the manufacturer believes that it can sell as many cell phones as it can manufacture. The table also summarizes the revenues and material costs for each type of phone.

In addition, variable labor costs are $15 per hour for test device 1, $12 per hour for test device 2, and $18 per hour for test device 3. Determine the product mix that would maximize profits. Formulate the problem as an LP model and solve it by using Excel.

3-7 A company produces three different types of wrenches: W111, W222, and W333. It has a firm order for 2,000 W111 wrenches, 3,750 W222 wrenches, and 1,700 W333 wrenches. Between now and the order delivery date, the company has only 16,500 fabrication hours and 1,600 inspection hours. The time that each wrench requires in each department is shown in the table at the bottom of this page. Also shown are the costs to manufacture the wrenches in-house and the costs to outsource them. For labeling considerations, the company wants to manufacture in-house at least 60% of each type of wrench that will be shipped. How many wrenches of each type should be made in-house and how many should be

Table for Problem 3-6

	SMARTPHONE	BLUEBERRY	MOPHONE	BOLDPHONE	LUXPHONE4G	TAP3G
Test device 1	7	3	12	6	18	17
Test device 2	2	5	3	2	15	17
Test device 3	5	1	3	2	9	2
Revenue per unit	$200	$120	$180	$200	$430	$260
Material cost per unit	$35	$25	$40	$45	$170	$60

Table for Problem 3-7

WRENCH	FABRICATION HOURS	INSPECTION HOURS	IN-HOUSE COST	OUTSOURCE COST
W111	2.50	0.25	$17.00	$20.40
W222	3.40	0.30	$19.00	$21.85
W333	3.80	0.45	$23.00	$25.76