Structural Steel Design

Fifth Edition

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ALWAYS LEARNING

PEARSON

STRUCTURAL STEEL DESIGN

FIFTH EDITION

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is for computer programs. You can see that it would be rather inconvenient to stop occasionally in the middle of a computer design to read K factors from the charts and input them to the computer. The equations, however, can easily be included in the programs, eliminating the necessity of using alignment charts.[3]

The alignment chart of Fig. 7.2(b) for frames with sidesway uninhibited always indicates that $K \geq 1.0$. In fact, calculated K factors of 2.0 to 3.0 are common, and even larger values are occasionally obtained. To many designers, such large factors seem completely unreasonable. If the designer derives seemingly high K factors, he or she should carefully review the numbers used to enter the chart (that is, the G values), as well as the basic assumptions made in preparing the charts. These assumptions are discussed in detail in Sections 7.4 and 7.5.

7.4 FRAMES NOT MEETING ALIGNMENT CHART ASSUMPTIONS AS TO JOINT ROTATIONS

In this section, a few comments are presented regarding frames whose joint rotations (and thus their beam stiffnesses) are not in agreement with the assumptions made for developing the charts.

It can be shown by structural analysis that the rotation at point B in the frame of Fig. 7.5 is twice as large as the rotation at B assumed in the construction of the nomographs. Therefore, beam BC in the figure is only one-half as stiff as the value assumed for the development of the alignment charts.

The Jackson and Moreland charts can be accurately used for situations in which the rotations are different from those assumed by making adjustments to the computed beam stiffnesses before the chart values are read. Relative stiffnesses for situations other than the one shown in Fig. 7.5 also can be determined by structural analysis. Table 7.1 presents correction factors to be multiplied by calculated beam stiffnesses, for situations where the beam end conditions are different from those assumed for the development of the charts.

Example 7-2 shows how the correction factors can be applied to a building frame where the rotations at the ends of some of the beams vary from the assumed conditions of the charts.

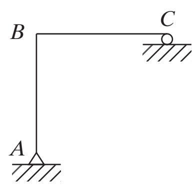

FIGURE 7.5

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[3]P. Dumonteil, "Simple Equations for Effective Length Factors," *Engineering Journal*, AISC, vol. 29, no. 3 (3rd Quarter, 1992), pp. 111–115.

TABLE 7.1 Multipliers for Rigidly Attached Members

Condition at Far End of Girder	Sidesway Prevented, Multiply by:	Sidesway Uninhibited, Multiply by:
Pinned	1.5	0.5
Fixed against rotation	2.0	0.67

Example 7-2

Determine K factors for each of the columns of the frame shown in Fig. 7.6. Here, W sections have been tentatively selected for each of the members of the frame and their I/L values determined and shown in the figure.

Solution. First, the G factors are computed for each joint in the frame. In this calculation, the I/L values for members FI and GJ are multiplied by the appropriate factors from Table 7.1.

1. For member FI, the I/L value is multiplied by 2.0, because its far end is fixed and there is no sidesway on that level.
2. For member, GJ, I/L is multiplied by 1.5, because its far end is pinned and there is no sidesway on that level.

$$G_A = 10 \text{ as described in Section 7.2, Pinned Column}$$

$$G_B = \frac{23.2 + 23.2}{70} = 0.663$$

$$G_C = \frac{23.2 + 20.47}{70} = 0.624$$

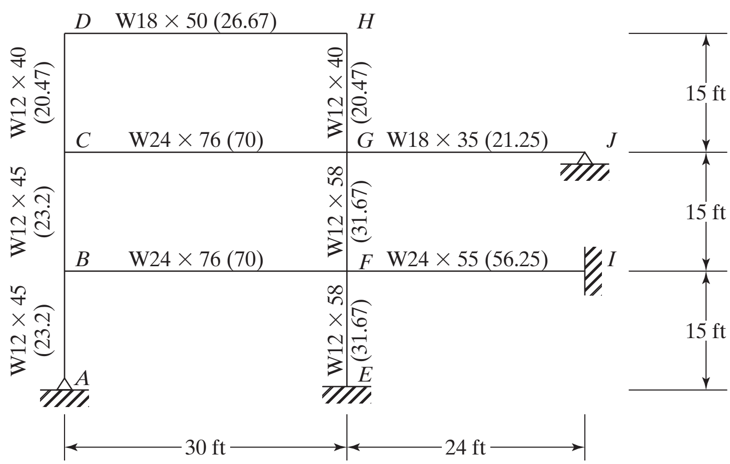

FIGURE 7.6
Steel shapes, including their I/L values.

$$G_D = \frac{20.47}{26.67} = 0.768$$

$$G_E = 1.0 \text{ as described in Section 7.2, Fixed Column}$$

$$G_F = \frac{31.67 + 31.67}{70 + (2.0)(56.25)} = 0.347$$

$$G_G = \frac{31.67 + 20.47}{70 + (1.5)(21.25)} = 0.512$$

$$G_H = \frac{20.47}{26.67} = 0.768$$

Finally, the K factors are selected from the appropriate alignment chart of Fig. 7.2.

Column	G Factors	Chart used	K Factors
AB	10 and 0.663	7.2 (a) no sidesway	0.83
BC	0.663 and 0.624	7.2 (a) no sidesway	0.72
CD	0.624 and 0.768	7.2 (b) has sidesway	1.23
EF	1.0 and 0.347	7.2 (a) no sidesway	0.71
FG	0.347 and 0.512	7.2 (a) no sidesway	0.67
GH	0.512 and 0.768	7.2 (b) has sidesway	1.21

Robins Air Force Base, GA. (Courtesy Britt, Peters and Associates.)

7.5 STIFFNESS-REDUCTION FACTORS

As previously mentioned, the alignment charts were developed according to a set of idealized conditions that are seldom, if ever, completely met in a real structure. Included among those conditions are the following: The column behavior is purely elastic, all columns buckle simultaneously, all members have constant cross sections, all joints are rigid, and so on.

If the actual conditions are different from these assumptions, unrealistically high K factors may be obtained from the charts, and overconservative designs may result. A large percentage of columns will appear in the inelastic range, but the alignment charts were prepared with the assumption of elastic failure. This situation, previously discussed in Chapter 5, is illustrated in Fig. 7.7. For such cases, the chart K values are too conservative and should be corrected as described in this section.

In the elastic range, the stiffness of a column is proportional to EI, where $E = 29{,}000$ ksi; in the inelastic range, its stiffness is more accurately proportional to $E_T I$, where E_T is a reduced or tangent modulus.

The buckling strength of columns in framed structures is shown in the alignment charts to be related to

$$G = \frac{\text{column stiffness}}{\text{girder stiffness}} = \frac{\Sigma(EI/L)\text{ columns}}{\Sigma(EI/L)\text{ girders}}$$

If the columns behave elastically, the modulus of elasticity will be canceled from the preceding expression for G. If the column behavior is inelastic, however, the column stiffness factor will be smaller and will equal $E_T I/L$. As a result, the G factor used to enter the alignment chart will be smaller, and the K factor selected from the chart will be smaller.

Though the alignment charts were developed for elastic column action, they may be used for an inelastic column situation if the G value is multiplied by a correction factor, τ_b. This reduction factor is specified in Section C2-3 of the AISC Specification.

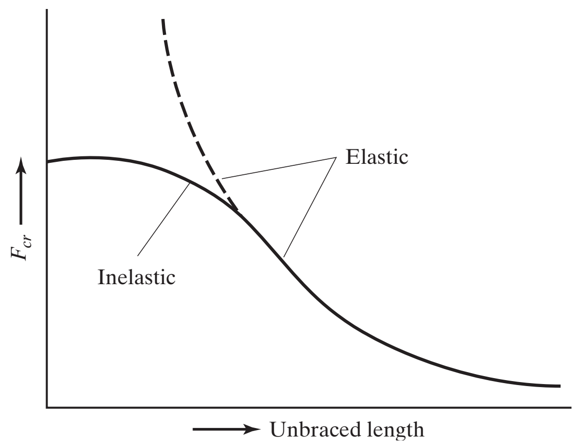

FIGURE 7.7

TABLE 7.2 Stiffness Reduction Factor, τ_b

ASD: $\frac{P_a}{A_g}$ / LRFD: $\frac{P_u}{A_g}$	F_y = 35 ksi ASD	35 LRFD	36 ASD	36 LRFD	42 ASD	42 LRFD	46 ASD	46 LRFD	50 ASD	50 LRFD
45	–	–	–	–	–	–	–	0.0851	–	0.360
44	–	–	–	–	–	–	–	0.166	–	0.422
43	–	–	–	–	–	–	–	0.244	–	0.482
42	–	–	–	–	–	–	–	0.318	–	0.538
41	–	–	–	–	–	0.0930	–	0.388	–	0.590
40	–	–	–	–	–	0.181	–	0.454	–	0.640
39	–	–	–	–	–	0.265	–	0.516	–	0.686
38	–	–	–	–	–	0.345	–	0.575	–	0.730
37	–	–	–	–	–	0.420	–	0.629	–	0.770
36	–	–	–	–	–	0.490	–	0.681	–	0.806
35	–	–	–	0.108	–	0.556	–	0.728	–	0.840
34	–	0.111	–	0.210	–	0.617	–	0.771	–	0.870
33	–	0.216	–	0.306	–	0.673	–	0.811	–	0.898
32	–	0.313	–	0.395	–	0.726	–	0.847	–	0.922
31	–	0.405	–	0.478	–	0.773	–	0.879	0.0317	0.942
30	–	0.490	–	0.556	–	0.816	–	0.907	0.154	0.960
29	–	0.568	–	0.627	–	0.855	–	0.932	0.267	0.974
28	–	0.640	–	0.691	–	0.889	0.102	0.953	0.373	0.986
27	–	0.705	–	0.750	–	0.918	0.229	0.970	0.470	0.994
26	–	0.764	–	0.802	0.0377	0.943	0.346	0.983	0.559	0.998
25	–	0.816	–	0.849	0.181	0.964	0.454	0.992	0.640	1.00
24	–	0.862	–	0.889	0.313	0.980	0.552	0.998	0.713	
23	–	0.901	–	0.923	0.434	0.991	0.640	1.00	0.777	
22	–	0.934	0.0869	0.951	0.543	0.998	0.719		0.834	
21	0.154	0.960	0.249	0.972	0.640	1.00	0.788		0.882	
20	0.313	0.980	0.395	0.988	0.726		0.847		0.922	
19	0.457	0.993	0.525	0.997	0.800		0.896		0.953	
18	0.583	0.999	0.640	1.00	0.862		0.936		0.977	
17	0.693	1.00	0.739		0.913		0.967		0.992	
16	0.786		0.822		0.952		0.987		0.999	
15	0.862		0.889		0.980		0.998		1.00	
14	0.922		0.940		0.996		1.00			
13	0.964		0.976		1.00					
12	0.991		0.996							
11	1.00		1.00							
10										
9										
8										
7										
6										
5										

– Indicates the stiffness reduction parameter is not applicable because the required strength exceeds the available strength for $KL/r = 0$.

Source: AISC Manual, Table 4-21, p. 4-321, 14th ed., 2011.

When $\alpha P_r/P_y$ is less than or equal to 0.5, then τ_b equals 1.0 per AISC Equation C2-2a. When $\alpha P_r/P_y$ is greater than 0.5, then $\tau_b = 4(\alpha P_r/P_y)[1 - (\alpha P_r/P_y)]$ per AISC Equation C2-2b. The factor, α, is taken as 1.0 for the LRFD method and 1.6 for the ASD design basis. P_r is the required axial compressive strength using LRFD or ASD load combinations, P_u or P_a respectively. P_y is the axial yield strength, F_y times the column gross area, A_g. Values of τ_b are shown for various P_u/A_g and P_a/A_g values in Table 7.2, which is Table 4-21 in the AISC Manual.

The τ_b factor is then used to reduce the column stiffness in the equation to calculate G, where $G_{(inelastic)} = \dfrac{\tau_b \sum(I_c/L_c)}{\sum(I_g/L_g)} = \tau_b G_{(elastic)}$. If the end of a column is pinned ($G = 10.0$) or fixed ($G = 1.0$), the value of G at that end should not be multiplied by a stiffness reduction factor.

Example 7-3 illustrates the steps used for the determination of the inelastic effective length factor for a column in a frame subject to sidesway. *In this example, note that the author has considered only in-plane behavior and only bending about the x axis*. As a result of inelastic behavior, the effective length factor is appreciably reduced.

Structures designed by inelastic analysis must meet the provisions of Appendix 1 of the AISC Specification.

Example 7-3

(a) Determine the effective length factor for column AB of the unbraced frame shown in Fig. 7.8, assuming that we have elastic behavior and that all of the other assumptions on which the alignment charts were developed are met. $P_D = 450$ k, $P_L = 700$ k, $F_y = 50$ ksi. Assume that column AB is a W12 × 170 and the columns above and below are as indicated on the figure.

(b) Repeat part (a) if inelastic column behavior is considered.

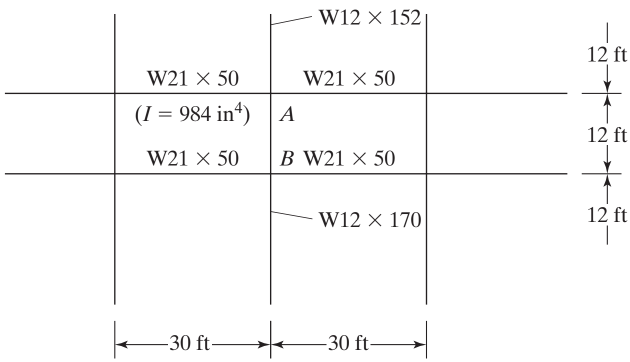

FIGURE 7.8

Solution

LRFD	ASD
$P_u = (1.2)(450) + (1.6)(700) = 1660$ k	$P_a = 450 + 700 = 1150$ k