GLOBAL EDITION

Precalculus

SEVENTH EDITION

Robert F. Blitzer

A Brief Guide to Getting the Most from This Book

1 Read the Book

Feature	Description	Benefit
Section-Opening Scenarios	Every section opens with a scenario presenting a unique application of algebra or trigonometry in your life outside the classroom.	Realizing that algebra and trigonometry are everywhere will help motivate your learning. **(See page 246.)**
EXAMPLE	Examples are clearly written and provide step-by-step solutions. No steps are omitted, and each step is thoroughly explained to the right of the mathematics.	The blue annotations will help you understand the solutions by providing the reason why every algebraic or trigonometric step is true. **(See page 129.)**
Applications Using Real-World Data	Interesting applications from nearly every discipline, supported by up-to-date real-world data, are included in every section.	Ever wondered how you'll use algebra and trigonometry? This feature will show you how they can solve real problems. **(See pages 239–241.)**
GREAT QUESTION!	Answers to students' questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.	By seeing common mistakes, you'll be able to avoid them. This feature should help you not to feel anxious or threatened when asking questions in class. **(See page 354.)**
BRIEF REVIEW	Brief Reviews cover skills you already learned but may have forgotten.	Having these refresher boxes easily accessible will help ease anxiety about skills you may have forgotten. **(See page 482.)**
BLITZER BONUS	These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra or trigonometry under study.	Yet even more proof that math is an interesting and dynamic discipline! **(See page 232.)**
Explanatory Voice Balloons	Voice balloons help to demystify algebra and trigonometry. They translate math into plain English, clarify problem-solving procedures, and present alternative ways of understanding.	Does math ever look foreign to you? This feature often translates math into everyday English. **(See page 468.)**
WHAT YOU'LL LEARN 1 Learning Objective	Every section begins with a list of objectives. Each objective is restated in the margin where the objective is covered.	The objectives focus your reading by emphasizing what is most important and where to find it. **(See page 646.)**
TECHNOLOGY	The screens displayed in the technology boxes show how graphing utilities verify and visualize algebraic or trigonometric results.	Even if you are not using a graphing utility in the course, this feature will help you understand different approaches to problem solving. **(See page 371.)**

C7. The Linear Factorization Theorem states that an nth-degree polynomial can be expressed as the product of a nonzero constant and ____ linear factors, where each linear factor has a leading coefficient of ____.

Use Descartes's Rule of Signs to determine whether each statement is true or false.

C8. A polynomial function with four sign changes must have four positive real zeros. ______

C9. A polynomial function with one sign change must have one positive real zero. ______

C10. A polynomial function with seven sign changes can have one, three, five, or seven positive real zeros. ______

2.5 EXERCISE SET

Practice Exercises

In Exercises 1–8, use the Rational Zero Theorem to list all possible rational zeros for each given function.

1. $f(x) = x^3 + x^2 - 4x - 4$

2. $f(x) = x^3 + 3x^2 - 6x - 8$

3. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$

4. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

5. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

6. $f(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$

7. $f(x) = x^5 - x^4 - 7x^3 + 7x^2 - 12x - 12$

8. $f(x) = 4x^5 - 8x^4 - x + 2$

In Exercises 9–16,

a. *List all possible rational zeros.*
b. *Use synthetic division to test the possible rational zeros and find an actual zero.*
c. *Use the quotient from part (b) to find the remaining zeros of the polynomial function.*

9. $f(x) = x^3 + x^2 - 4x - 4$

10. $f(x) = x^3 - 5x^2 - 9x + 45$

11. $f(x) = 2x^3 - 3x^2 - 11x + 6$

12. $f(x) = 7x^3 - 5x^2 - 63x + 45$

13. $f(x) = x^3 + 4x^2 - 3x - 6$

14. $f(x) = 2x^3 + x^2 - 3x + 1$

15. $f(x) = 2x^3 + 6x^2 + 5x + 2$

16. $f(x) = x^3 - 4x^2 + 8x - 5$

In Exercises 17–24,

a. *List all possible rational roots.*
b. *Use synthetic division to test the possible rational roots and find an actual root.*
c. *Use the quotient from part (b) to find the remaining roots and solve the equation.*

17. $x^3 - 2x^2 - 11x + 12 = 0$

18. $x^3 - 2x^2 - 7x - 4 = 0$

19. $x^3 - 10x - 12 = 0$

20. $x^3 - 5x^2 + 17x - 13 = 0$

21. $6x^3 + 25x^2 - 24x + 5 = 0$

22. $2x^3 - 5x^2 - 6x + 4 = 0$

23. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$

24. $x^4 - 2x^2 - 16x - 15 = 0$

In Exercises 25–32, find an nth-degree polynomial function with real coefficients satisfying the given conditions. If you are using a graphing utility, use it to graph the function and verify the real zeros and the given function value.

25. $n = 3$; 1 and $5i$ are zeros; $f(-1) = -104$

26. $n = 3$; 4 and $2i$ are zeros; $f(-1) = -50$

27. $n = 3$; -5 and $4 + 3i$ are zeros; $f(2) = 91$

28. $n = 3$; 6 and $-5 + 2i$ are zeros; $f(2) = -636$

29. $n = 4$; i and $3i$ are zeros; $f(-1) = 20$

30. $n = 4$; -2, $-\frac{1}{2}$, and i are zeros; $f(1) = 18$

31. $n = 4$; -2, 5, and $3 + 2i$ are zeros; $f(1) = -96$

32. $n = 4$; -4, $\frac{1}{3}$, and $2 + 3i$ are zeros; $f(1) = 100$

In Exercises 33–38, use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for each given function.

33. $f(x) = x^3 + 2x^2 + 5x + 4$

34. $f(x) = x^3 + 7x^2 + x + 7$

35. $f(x) = 5x^3 - 3x^2 + 3x - 1$

36. $f(x) = -2x^3 + x^2 - x + 7$

37. $f(x) = 2x^4 - 5x^3 - x^2 - 6x + 4$

38. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

In Exercises 39–52, find all zeros of the polynomial function or solve the given polynomial equation. Use the Rational Zero Theorem, Descartes's Rule of Signs, and possibly the graph of the polynomial function shown by a graphing utility as an aid in obtaining the first zero or the first root.

39. $f(x) = x^3 - 4x^2 - 7x + 10$

40. $f(x) = x^3 + 12x^2 + 21x + 10$

41. $2x^3 - x^2 - 9x - 4 = 0$

42. $3x^3 - 8x^2 - 8x + 8 = 0$

43. $f(x) = x^4 - 2x^3 + x^2 + 12x + 8$

44. $f(x) = x^4 - 4x^3 - x^2 + 14x + 10$

45. $x^4 - 3x^3 - 20x^2 - 24x - 8 = 0$

46. $x^4 - x^3 + 2x^2 - 4x - 8 = 0$

47. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$

48. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

49. $4x^4 - x^3 + 5x^2 - 2x - 6 = 0$

50. $3x^4 - 11x^3 - 3x^2 - 6x + 8 = 0$

51. $2x^5 + 7x^4 - 18x^2 - 8x + 8 = 0$

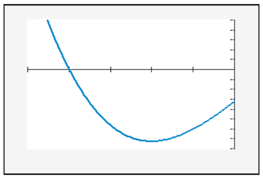
52. $4x^5 + 12x^4 - 41x^3 - 99x^2 + 10x + 24 = 0$

Practice PLUS

Exercises 53–60 show incomplete graphs of given polynomial functions.

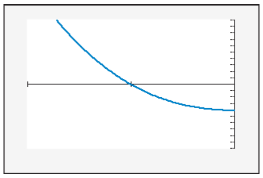
- ***a.*** *Find all the zeros of each function.*
- ***b.*** *Without using a graphing utility, draw a complete graph of the function.*

53. $f(x) = -x^3 + x^2 + 16x - 16$



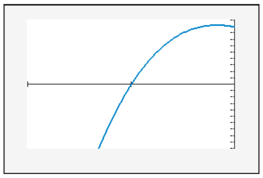
[−5, 0, 1] by [−40, 25, 5]

54. $f(x) = -x^3 + 3x^2 - 4$



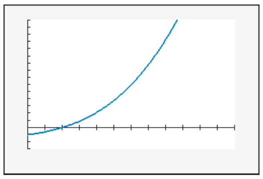
[−2, 0, 1] by [−10, 10, 1]

55. $f(x) = 4x^3 - 8x^2 - 3x + 9$



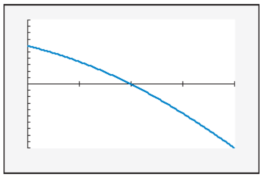
[−2, 0, 1] by [−10, 10, 1]

56. $f(x) = 3x^3 + 2x^2 + 2x - 1$



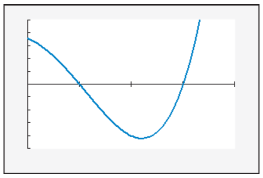
$[0, 2, \frac{1}{6}]$ by [−3, 15, 1]

57. $f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$



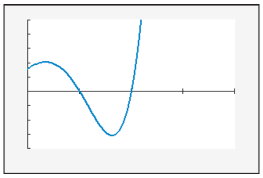
$[0, 1, \frac{1}{4}]$ by [−10, 10, 1]

58. $f(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 36$



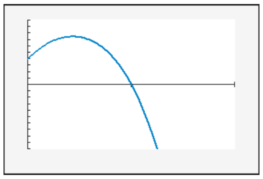
[0, 4, 1] by [−50, 50, 10]

59. $f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$



[0, 4, 1] by [−20, 25, 5]

60. $f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$



[0, 2, 1] by [−10, 10, 1]

Application Exercises

A popular model of carry-on luggage has a length that is 10 inches greater than its depth. Airline regulations require that the sum of the length, width, and depth cannot exceed 40 inches. These conditions, with the assumption that this sum is 40 inches, can be modeled by a function that gives the volume of the luggage, V, in cubic inches, in terms of its depth, x, in inches.

Volume = depth · length · width: 40 − (depth + length)

$$V(x) = x \cdot (x + 10) \cdot [40 - (x + x + 10)]$$

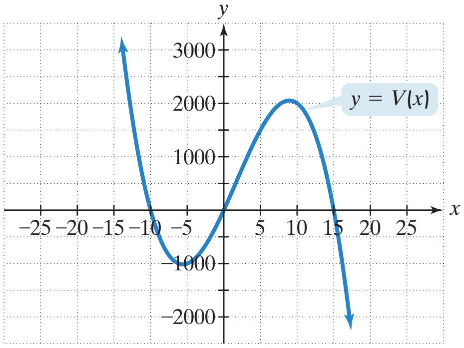
$$V(x) = x(x + 10)(30 - 2x)$$

Use function V to solve Exercises 61–62.

61. If the volume of the carry-on luggage is 2000 cubic inches, determine two possibilities for its depth. Where necessary, round to the nearest tenth of an inch.

62. If the volume of the carry-on luggage is 1500 cubic inches, determine two possibilities for its depth. Where necessary, round to the nearest tenth of an inch.

Use the graph of the function modeling the volume of the carry-on luggage to solve Exercises 63–64.



63. a. Identify your answers from Exercise 61 as points on the graph.

b. Use the graph to describe a realistic domain, x, for the volume function, where x represents the depth of the carry-on luggage.

64. a. Identify your answers from Exercise 62 as points on the graph.

b. Use the graph to describe a realistic domain, x, for the volume function, where x represents the depth of the carry-on luggage.

Explaining the Concepts

65. Describe how to find the possible rational zeros of a polynomial function.

66. How does the linear factorization of $f(x)$, that is,

$$f(x) = a_n(x - c_1)(x - c_2)\cdots(x - c_n),$$

show that a polynomial equation of degree n has n roots?

67. Describe how to use Descartes's Rule of Signs to determine the possible number of positive real zeros of a polynomial function.

68. Describe how to use Descartes's Rule of Signs to determine the possible number of negative roots of a polynomial equation.

69. Why must every polynomial equation with real coefficients of degree 3 have at least one real root?

70. Explain why the equation $x^4 + 6x^2 + 2 = 0$ has no rational roots.

71. Suppose $\frac{3}{4}$ is a root of a polynomial equation. What does this tell us about the leading coefficient and the constant term in the equation?

Technology Exercises

The equations in Exercises 72–75 have real roots that are rational. Use the Rational Zero Theorem to list all possible rational roots. Then graph the polynomial function in the given viewing rectangle to determine which possible rational roots are actual roots of the equation.

72. $2x^3 - 15x^2 + 22x + 15 = 0$; $[-1, 6, 1]$ by $[-50, 50, 10]$

73. $6x^3 - 19x^2 + 16x - 4 = 0$; $[0, 2, 1]$ by $[-3, 2, 1]$

74. $2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0$; $[-4, 3, 1]$ by $[-45, 45, 15]$

75. $4x^4 + 4x^3 + 7x^2 - x - 2 = 0$; $[-2, 2, 1]$ by $[-5, 5, 1]$

76. Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros of $f(x) = 3x^4 + 5x^2 + 2$. What does this mean in terms of the graph of f? Verify your result by using a graphing utility to graph f.

77. Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros of $f(x) = x^5 - x^4 + x^3 - x^2 + x - 8$. Verify your result by using a graphing utility to graph f.

78. Write equations for several polynomial functions of odd degree and graph each function. Is it possible for the graph to have no real zeros? Explain. Try doing the same thing for polynomial functions of even degree. Now is it possible to have no real zeros?

Use a graphing utility to obtain a complete graph for each polynomial function in Exercises 79–82. Then determine the number of real zeros and the number of imaginary zeros for each function.

79. $f(x) = x^3 - 6x - 9$

80. $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$

81. $f(x) = 3x^4 + 4x^3 - 7x^2 - 2x - 3$

82. $f(x) = x^6 - 64$

Critical Thinking Exercises

Make Sense? *In Exercises 83–86, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

83. I've noticed that $f(-x)$ is used to explore the number of negative real zeros of a polynomial function, as well as to determine whether a function is even, odd, or neither.

84. By using the quadratic formula, I do not need to bother with synthetic division when solving polynomial equations of degree 3 or higher.

85. I'm working with a fourth-degree polynomial function with integer coefficients and zeros at 1 and $3 + \sqrt{5}$. I'm certain that $3 + \sqrt{2}$ cannot also be a zero of this function.

86. I'm working with the polynomial function $f(x) = x^4 + 3x^2 + 2$ that has four possible rational zeros but no actual rational zeros.

In Exercises 87–90, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

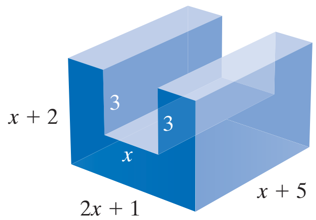
87. The equation $x^3 + 5x^2 + 6x + 1 = 0$ has one positive real root.

88. Descartes's Rule of Signs gives the exact number of positive and negative real roots for a polynomial equation.

89. Every polynomial equation of degree 3 with real coefficients has at least one real root.

90. Every polynomial equation of degree n has n distinct solutions.

91. If the volume of the solid shown in the figure is 208 cubic inches, find the value of x.



92. In this exercise, we lead you through the steps involved in the proof of the Rational Zero Theorem. Consider the polynomial equation

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x + a_0 = 0,$$

and let $\frac{p}{q}$ be a rational root reduced to lowest terms.

a. Substitute $\frac{p}{q}$ for x in the equation and show that the equation can be written as

$$a_np^n + a_{n-1}p^{n-1}q + a_{n-2}p^{n-2}q^2 + \cdots + a_1pq^{n-1} = -a_0q^n.$$

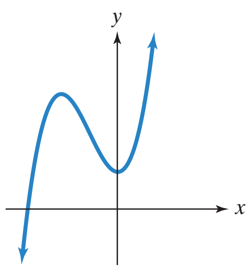
b. Why is p a factor of the left side of the equation?

c. Because p divides the left side, it must also divide the right side. However, because $\frac{p}{q}$ is reduced to lowest terms, p and q have no common factors other than -1 and 1. Because p does divide the right side and has no factors in common with q^n, what can you conclude?

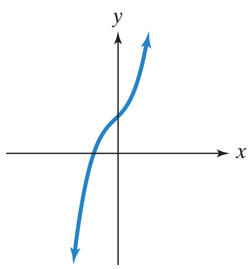
d. Rewrite the equation from part (a) with all terms containing q on the left and the term that does not have a factor of q on the right. Use an argument that parallels parts (b) and (c) to conclude that q is a factor of a_n.

In Exercises 93–96, the graph of a polynomial function is given. What is the smallest degree that each polynomial could have?

93.



94.



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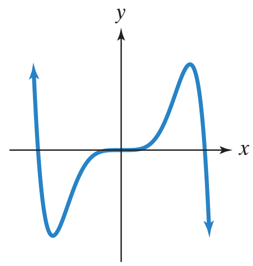
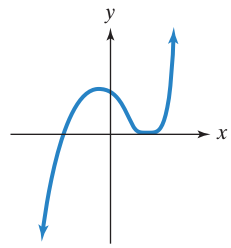


96.



97. Explain why a polynomial function of degree 20 cannot cross the x-axis exactly once.

Retaining the Concepts

98. Let $f(x) = 4 - x^2$ and $g(x) = x + 5$.

a. Find $(f \circ g)(x)$.

(Section 1.7, Example 5)

b. Find $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$, and simplify.

(Section 1.3, Example 8)

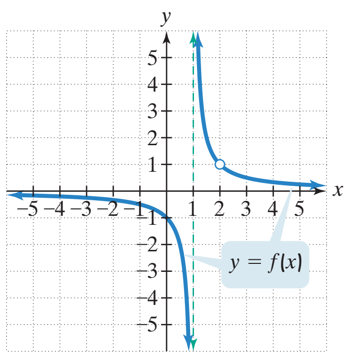
99. Write an equation in point-slope form and general form of the line passing through $(-5, 3)$ and perpendicular to the line whose equation is $x + 5y - 7 = 0$.

(Section 1.5, Example 2)

100. Find the average rate of change of $f(x) = \sqrt{x}$ from $x_1 = 4$ to $x_2 = 9$. (Section 1.5, Example 4)

Preview Exercises

Exercises 101–103 will help you prepare for the material covered in the next section. Use the graph of function f to solve each exercise.



101. For what values of x is the function undefined?

102. Write the equation of the vertical asymptote, or the vertical line that the graph of f approaches but does not touch.

103. Write the equation of the horizontal asymptote, or the horizontal line that the graph of f approaches but does not touch.

CHAPTER 2 Mid-Chapter Check Point

WHAT YOU KNOW: We performed operations with complex numbers and used the imaginary unit i $(i = \sqrt{-1},$ where $i^2 = -1)$ to represent solutions of quadratic equations with negative discriminants. Only real solutions correspond to x-intercepts. We graphed quadratic functions using vertices, intercepts, and additional points, as necessary. We learned that the vertex of $f(x) = a(x-h)^2 + k$ is (h, k) and the vertex of

$f(x) = ax^2 + bx + c$ is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. We used the vertex to solve problems that involved minimizing or maximizing quadratic functions. We learned a number of techniques for finding the zeros of a polynomial function f of degree 3 or higher or, equivalently, finding the roots, or solutions, of the equation $f(x) = 0$. For some functions, the zeros were found by factoring $f(x)$. For other functions, we listed possible rational zeros and used synthetic division and the Factor Theorem to determine the zeros. We saw that graphs cross the x-axis at zeros of odd multiplicity and touch the x-axis and turn around at zeros of even multiplicity. We learned to graph polynomial functions using zeros, the Leading Coefficient Test, intercepts, and symmetry. We checked graphs using the fact that a polynomial function of degree n has a graph with at most $n - 1$ turning points. After finding zeros of polynomial functions, we reversed directions by using the Linear Factorization Theorem to find functions with given zeros.

In Exercises 1–6, perform the indicated operations and write the result in standard form.

1. $(6 - 2i) - (7 - i)$

2. $3i(2 + i)$

3. $(1 + i)(4 - 3i)$

4. $\frac{1 + i}{1 - i}$

5. $\sqrt{-75} - \sqrt{-12}$

6. $(2 - \sqrt{-3})^2$

7. Solve and express solutions in standard form:

$$x(2x - 3) = -4.$$

In Exercises 8–11, graph the given quadratic function. Give each function's domain and range.

8. $f(x) = (x - 3)^2 - 4$

9. $f(x) = 5 - (x + 2)^2$

10. $f(x) = -x^2 - 4x + 5$

11. $f(x) = 3x^2 - 6x + 1$

In Exercises 12–20, find all zeros of each polynomial function. Then graph the function.

12. $f(x) = (x - 2)^2(x + 1)^3$

13. $f(x) = -(x - 2)^2(x + 1)^2$

14. $f(x) = x^3 - x^2 - 4x + 4$

15. $f(x) = x^4 - 5x^2 + 4$

16. $f(x) = -(x + 1)^6$

17. $f(x) = -6x^3 + 7x^2 - 1$

18. $f(x) = 2x^3 - 2x$

19. $f(x) = x^3 - 2x^2 + 26x$

20. $f(x) = -x^3 + 5x^2 - 5x - 3$

In Exercises 21–26, solve each polynomial equation.

21. $x^3 - 3x + 2 = 0$

22. $6x^3 - 11x^2 + 6x - 1 = 0$

23. $(2x + 1)(3x - 2)^3(2x - 7) = 0$

24. $2x^3 + 5x^2 - 200x - 500 = 0$

25. $x^4 - x^3 - 11x^2 = x + 12$

26. $2x^4 + x^3 - 17x^2 - 4x + 6 = 0$

27. A company manufactures and sells wireless speakers. The function

$$P(x) = -x^2 + 150x - 4425$$

models the company's daily profit, $P(x)$, when x speakers are manufactured and sold per day. How many speakers should be manufactured and sold per day to maximize the company's profit? What is the maximum daily profit?

28. Among all pairs of numbers whose sum is -18, find a pair whose product is as large as possible. What is the maximum product?

29. The base of a triangle measures 40 inches minus twice the measure of its height. For what measure of the height does the triangle have a maximum area? What is the maximum area?

In Exercises 30–31, divide, using synthetic division if possible.

30. $(6x^4 - 3x^3 - 11x^2 + 2x + 4) \div (3x^2 - 1)$

31. $(2x^4 - 13x^3 + 17x^2 + 18x - 24) \div (x - 4)$

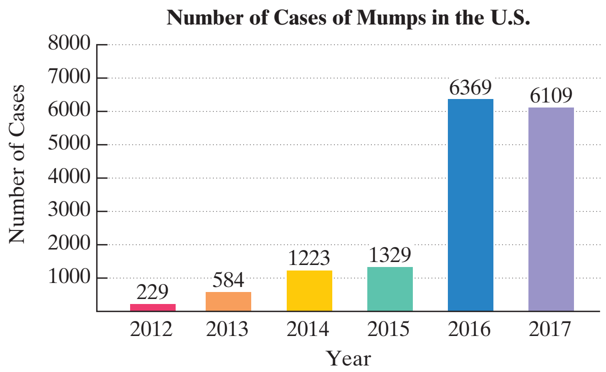
In Exercises 32–33, find an nth-degree polynomial function with real coefficients satisfying the given conditions.

32. $n = 3$; 1 and i are zeros; $f(-1) = 8$

33. $n = 4$; 2 (with multiplicity 2) and $3i$ are zeros; $f(0) = 36$

34. Does $f(x) = x^3 - x - 5$ have a real zero between 1 and 2?

35. Mumps is a vaccine-preventable virus with symptoms involving headaches, fever, loss of appetite, and painful swelling of the salivary glands under one or both ears. In 1960, before the vaccine, 42 Americans died from the virus. Although the disease has been vaccine-preventable since 1967, some people choose not to be vaccinated, as shown by the following bar graph.



Source: CDC

Here are two polynomial functions that model the data:

$$f(x) = 259x^2 + 42x + 158$$
$$g(x) = -107x^3 + 1059x^2 - 1420x + 478.$$

The functions model the number of cases of mumps, $f(x)$ or $g(x)$, x years after 2012.

a. Which function is a better model for the number of cases of mumps in 2017?

b. Use end behavior to explain why the function that you selected in part (a) is only appropriate for a limited time period.

SECTION 2.6 Rational Functions and Their Graphs

WHAT YOU'LL LEARN

1. Find the domains of rational functions.
2. Use arrow notation.
3. Identify vertical asymptotes.
4. Identify horizontal asymptotes.
5. Use transformations to graph rational functions.
6. Graph rational functions.
7. Identify slant asymptotes.
8. Solve applied problems involving rational functions.

The current generation of college students grew up playing interactive online games and many continue to play in school. Hundreds of colleges have formed organized gaming teams, many as campus clubs. Enter the Oculus Rift, revolutionizing the way people experience gaming. The Oculus Rift is a virtual reality headset that enables users to experience video games as immersive three-dimensional environments. Basically, it puts the gamer inside the game.

The cost of manufacturing Oculus Rift headsets can be modeled by rational functions. In this section, you will see that high production levels of the Oculus Rift can keep the price of this amazing invention low, perhaps making this the device that brings home virtual reality to reality.

1 Find the domains of rational functions.

Rational Functions

Rational functions are quotients of polynomial functions. This means that rational functions can be expressed as

$$f(x) = \frac{p(x)}{q(x)},$$

where p and q are polynomial functions and $q(x) \neq 0$. The **domain** of a rational function is the set of all real numbers except the x-values that make the denominator zero. For example, the domain of the rational function

$$f(x) = \frac{x^2 + 7x + 9}{x(x-2)(x+5)}$$

This is $p(x)$.

This is $q(x)$.

is the set of all real numbers except 0, 2, and -5.

EXAMPLE 1 Finding the Domain of a Rational Function

Find the domain of each rational function:

a. $f(x) = \dfrac{x^2 - 9}{x - 3}$ **b.** $g(x) = \dfrac{x}{x^2 - 9}$ **c.** $h(x) = \dfrac{x + 3}{x^2 + 9}$.

Solution Rational functions contain division. Because division by 0 is undefined, we must exclude from the domain of each function values of x that cause the polynomial function in the denominator to be 0.

a. The denominator of $f(x) = \dfrac{x^2 - 9}{x - 3}$ is 0 if $x = 3$. Thus, x cannot equal 3.

The domain of f consists of all real numbers except 3. We can express the domain in set-builder or interval notation:

$$\text{Domain of } f = \{x \mid x \neq 3\}$$
$$\text{Domain of } f = (-\infty, 3) \cup (3, \infty).$$