GLOBAL EDITION

Quantitative Analysis for Management

FOURTEENTH EDITION

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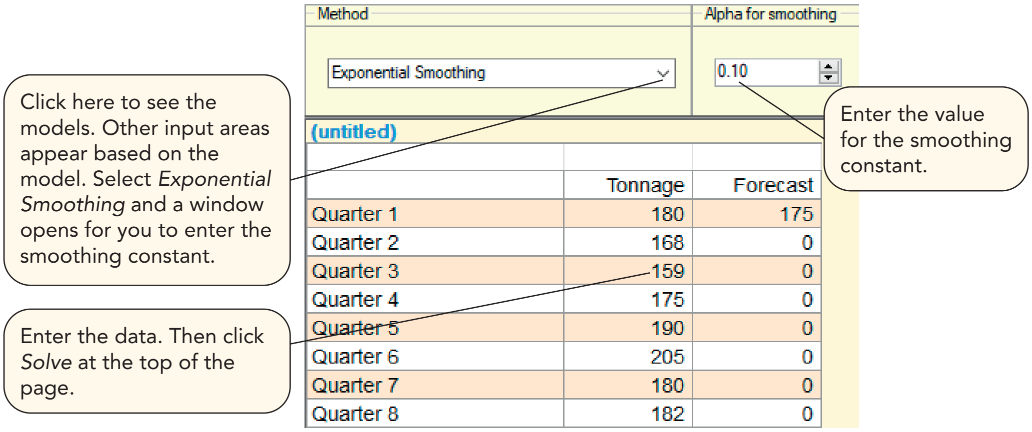
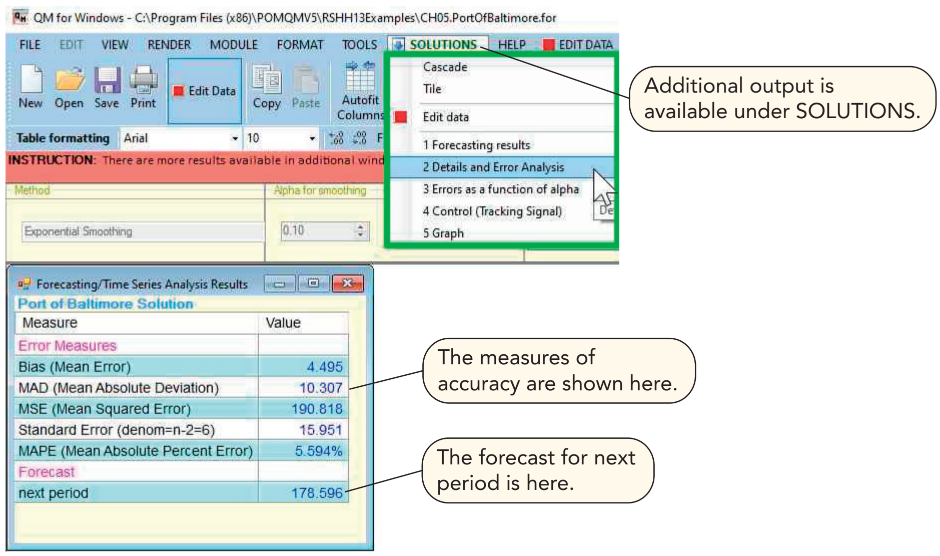
PROGRAM 5.2C

Selecting the Model and Entering Data for the Port of Baltimore Example in QM for Windows



PROGRAM 5.2D

Output for the Port of Baltimore Example in QM for Windows



5.5 Forecasting Models—Trend and Random Variations

If trend is present in a time series, the forecasting model must account for it and cannot simply average past values. Two very common techniques will be presented here. The first is exponential smoothing with trend, and the second is trend projection, or simply a trend line.

Exponential Smoothing with Trend

An extension of the exponential smoothing model that will explicitly adjust for trend is called the exponential smoothing with trend model. The idea is to develop an exponential smoothing forecast and then adjust this for trend. Two smoothing constants, α and β, are used in this model, and both of these values must be between 0 and 1. The level of the forecast is adjusted by multiplying the first smoothing constant, α, by the most recent forecast error and adding it to the previous forecast. The trend is adjusted by multiplying the second smoothing constant, β, by the most recent error or excess amount in the trend. A higher value gives more weight to recent observations and thus responds more quickly to changes in the patterns.

Two smoothing constants are used.

As with simple exponential smoothing, the first time a forecast is developed, a previous forecast (F_t) must be given or estimated. If none is available, often the initial forecast is assumed to be perfect. In addition, a previous trend (T_t) must be given or estimated. This is often estimated by using other past data, if available; by using subjective means; or by calculating the increase (or decrease) observed during the first few time periods of the data available. Without such an estimate available, the trend is sometimes assumed to be 0 initially, although this may lead to poor forecasts if the trend is large and β is small. Once these initial conditions

Estimate or assume initial values for F_t and T_t.

have been set, the exponential smoothing forecast including trend (FIT_t) is developed using three steps:

Step 1. Compute the smoothed forecast (F_{t+1}) for time period $t + 1$ using the equation

$$\text{Smoothed forecast} = \text{Previous forecast including trend} + \alpha(\text{Last error})$$
$$F_{t+1} = FIT_t + \alpha(Y_t - FIT_t) \tag{5-12}$$

Step 2. Update the trend (T_{t+1}) using the equation

$$\text{Smoothed trend} = \text{Previous trend} + \beta(\text{Error or excess in trend})$$
$$T_{t+1} = T_t + \beta(F_{t+1} - FIT_t) \tag{5-13}$$

Step 3. Calculate the trend-adjusted exponential smoothing forecast (FIT_{t+1}) using the equation

$$\text{Forecast including trend}(FIT_{t+1}) = \text{Smoothed forecast}(F_{t+1}) + \text{Smoothed trend}(T_{t+1})$$
$$FIT_{t+1} = F_{t+1} + T_{t+1} \tag{5-14}$$

where

T_t = smoothed trend for time period t
F_t = smoothed forecast for time period t
FIT_t = forecast including trend for time period t
α = smoothing constant for forecasts
β = smoothing constant for trend

Consider the case of Midwestern Manufacturing Company, which has a demand for electrical generators over the period 2007 to 2013 as shown in Table 5.6. To use the trend-adjusted exponential smoothing method, first set initial conditions (previous values for F and T) and choose α and β. Assuming that F_1 is perfect and T_1 is 0 and picking 0.3 and 0.4 for the smoothing constants, we have

$$F_1 = 74 \quad T_1 = 0 \quad \alpha = 0.3 \quad \beta = 0.4$$

This results in

$$FIT_1 = F_1 + T_1 = 74 + 0 = 74$$

Following the three steps to get the forecast for 2008 (time period 2), we have

Step 1. Compute F_{t+1} using the equation

$$F_{t+1} = FIT_t + \alpha(Y_t - FIT_t)$$
$$F_2 = FIT_1 + 0.3(Y_1 - FIT_1) = 74 + 0.3(74 - 74) = 74$$

Step 2. Update the trend (T_{t+1}) using the equation

$$T_{t+1} = T_t + \beta(F_{t+1} - FIT_t)$$
$$T_2 = T_1 + 0.4(F_2 - FIT_1) = 0 + 0.4(74 - 74) = 0$$

TABLE 5.6 Midwestern Manufacturing's Demand

YEAR	ELECTRICAL GENERATORS SOLD
2007	74
2008	79
2009	80
2010	90
2011	105
2012	142
2013	122

TABLE 5.7 Midwestern Manufacturing Exponential Smoothing with Trend Forecasts

TIME (T)	DEMAND (Y_T)	$F_{t+1} = FIT_t + 0.3(Y_t - FIT_t)$	$T_{t+1} = T_t + 0.4(F_{t+1} - FIT_t)$	$FIT_{t+1} = F_{t+1} + T_{t+1}$
1	74	74	0	74
2	79	$74 = 74 + 0.3(74 - 74)$	$0 = 0 + 0.4(74 - 74)$	$74 = 74 + 0$
3	80	$75.5 = 74 + 0.3(79 - 74)$	$0.6 = 0 + 0.4(75.5 - 74)$	$76.1 = 75.5 + 0.6$
4	90	77.270 $= 76.1 + 0.3(80 - 76.1)$	1.068 $= 0.6 + 0.4(77.27 - 76.1)$	$78.338 = 77.270 + 1.068$
5	105	81.837 $= 78.338 + 0.3(90 - 78.338)$	2.468 $= 1.068 + 0.4(81.837 - 78.338)$	$84.305 = 81.837 + 2.468$
6	142	90.514 $= 84.305 + 0.3(105 - 84.305)$	4.952 $= 2.468 + 0.4(90.514 - 84.305)$	$95.466 = 90.514 + 4.952$
7	122	109.426 $= 95.466 + 0.3(142 - 95.466)$	10.536 $= 4.952 + 0.4(109.426 - 95.466)$	$119.962 = 109.426 + 10.536$
8		120.573 $= 119.962 + 0.3(122 - 119.962)$	10.780 $= 10.536 + 0.4(120.573 - 119.962)$	$131.353 = 120.573 + 10.780$

Step 3. Calculate the trend-adjusted exponential smoothing forecast (FIT_{t+1}) using the equation

$$FIT_2 = F_2 + T_2 = 74 + 0 = 74$$

For 2009 (time period 3), we have

Step 1.

$$F_3 = FIT_2 + 0.3(Y_2 - FIT_2) = 74 + 0.3(79 - 74) = 75.5$$

Step 2.

$$T_3 = T_2 + 0.4(F_3 - FIT_2) = 0 + 0.4(75.5 - 74) = 0.6$$

Step 3.

$$FIT_3 = F_3 + T_3 = 75.5 + 0.6 = 76.1$$

The other results are shown in Table 5.7. The forecast for 2014 would be about 131.35.

To have Excel QM perform the calculations in Excel, from the Excel QM ribbon, select the alphabetical list of techniques and choose *Forecasting* and then *Trend-Adjusted Exponential Smoothing*. After specifying the number of past observations, enter the data and the values for and β, as shown in Program 5.3.

PROGRAM 5.3
Output from Excel QM in Excel for the Trend-Adjusted Exponential Smoothing Example

	A	B	C	D	E	F	G	H	I	J
1	Midwestern Manufacturing									
2										
3	Forecasting		Trend adjusted exponential smoothing							
4										
5										
6										
7	Alpha	0.3								
8	Beta	0.4								
9	Data			Forecasts and Error Analysis						
10	Period	Demand		Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t	Error	Absolute	Squared	Abs Pct Err
11	Period 1	74		74		74	0	0	0	00.00%
12	Period 2	79		74	0	74	5	5	25	06.33%
13	Period 3	80		75.5	0.6	76.1	3.9	3.9	15.21	04.88%
14	Period 4	90		77.27	1.068	78.338	11.662	11.662	136.00224	12.96%
15	Period 5	105		81.8366	2.46744	84.30404	20.69596	20.69596	428.32276	19.71%
16	Period 6	142		90.512828	4.9509552	95.463783	46.536217	46.53622	2165.6195	32.77%
17	Period 7	122		109.42465	10.535301	119.95995	2.0400505	2.040051	4.1618062	0.0167217
18		Next period		**120.57196**	**10.780107**	**131.35207**				
19				Total			89.834227	89.83423	2774.3163	78.32%
20				**Average**			**12.833461**	**12.83346**	**396.3309**	**11.19%**
21							**Bias**	**MAD**	**MSE**	**MAPE**
22								**SE**	**23.555536**	

The forecast for next period is here.

Trend Projections

A trend line is a regression equation with time as the independent variable.

Another method for forecasting time series with trend is called **trend projection**. This technique fits a trend line to a series of historical data points and then projects the line into the future for medium- to long-range forecasts. Several mathematical trend equations can be developed (e.g., exponential and quadratic), but in this section we look at linear (straight-line) trends only. A trend line is simply a linear regression equation in which the independent variable (X) is the time period. The first time period will be time period 1. The second time period will be time period 2, and so forth. The last time period will be time period n. The form of this is

$$\hat{Y} = b_0 + b_1X$$

where

$$\begin{aligned}
\hat{Y} &= \text{predicted value}\\
b_0 &= \text{intercept}\\
b_1 &= \text{slope of the line}\\
X &= \text{time period}\,(\text{i.e.,} X = 1, 2, 3 \ldots, n)
\end{aligned}$$

The **least-squares** regression method may be applied to find the coefficients that minimize the sum of the squared errors, thereby also minimizing the mean squared error (MSE). Chapter 4 provides a detailed explanation of least-squares regression and the formulas to calculate the coefficients by hand. In this section, we use computer software to perform the calculations.

Let us consider the case of Midwestern Manufacturing's demand for generators that was presented in Table 5.6. A trend line can be used to predict demand (Y) based on the time period using a regression equation. For the first time period, which was 2007, we let $X = 1$. For 2008, we let $X = 2$, and so forth. Using computer software to develop a regression equation as we did in Chapter 4, we get the following equation:

$$\hat{Y} = 56.71 + 10.54X$$

To project demand in 2014, we first denote the year 2014 in our new coding system as $X = 8$:

$$\begin{aligned}
(\text{Sales in 2014}) &= 56.71 + 10.54(8)\\
&= 141.03, \text{ or } 141 \text{ generators}
\end{aligned}$$

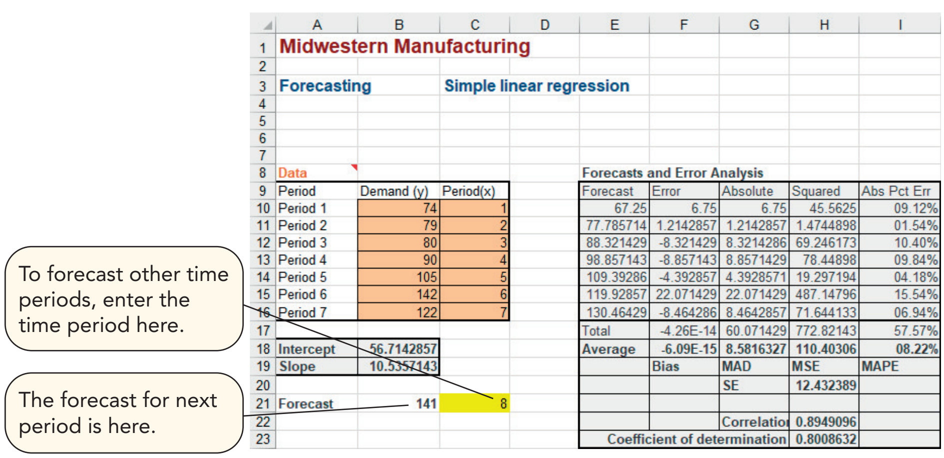
We can estimate demand for 2015 by inserting $X = 9$ in the same equation:

$$\begin{aligned}
(\text{Sales in 2015}) &= 56.71 + 10.54(9)\\
&= 151.57, \text{ or } 152 \text{ generators}
\end{aligned}$$

Program 5.4 provides output from Excel QM in Excel. To run this model, from the Excel QM ribbon, select *Alphabetical* to see the techniques. Then select *Forecasting* and *Regression / Trend Analysis*. When the input window opens, enter the number of past observations (7),

PROGRAM 5.4

Output from Excel QM in Excel for Trend Line Example



	A	B	C	D	E	F	G	H	I
1	Midwestern Manufacturing								
2									
3	Forecasting		Simple linear regression						
4									
5									
6									
7									
8	Data				Forecasts and Error Analysis				
9	Period	Demand (y)	Period(x)		Forecast	Error	Absolute	Squared	Abs Pct Err
10	Period 1	74	1		67.25	6.75	6.75	45.5625	09.12%
11	Period 2	79	2		77.785714	1.2142857	1.2142857	1.4744898	01.54%
12	Period 3	80	3		88.321429	-8.321429	8.3214286	69.246173	10.40%
13	Period 4	90	4		98.857143	-8.857143	8.8571429	78.44898	09.84%
14	Period 5	105	5		109.39286	-4.392857	4.3928571	19.297194	04.18%
15	Period 6	142	6		119.92857	22.071429	22.071429	487.14796	15.54%
16	Period 7	122	7		130.46429	-8.464286	8.4642857	71.644133	06.94%
17					Total	-4.26E-14	60.071429	772.82143	57.57%
18	Intercept	56.7142857			Average	-6.09E-15	8.5816327	110.40306	08.22%
19	Slope	10.5357143				Bias	MAD	MSE	MAPE
20							SE	12.432389	
21	Forecast	141	8						
22							Correlatio	0.8949096	
23					Coefficient of determination			0.8008632	

PROGRAM 5.5
Output from QM for Windows for Trend Line Example

Forecasting/Time Series Analysis Results

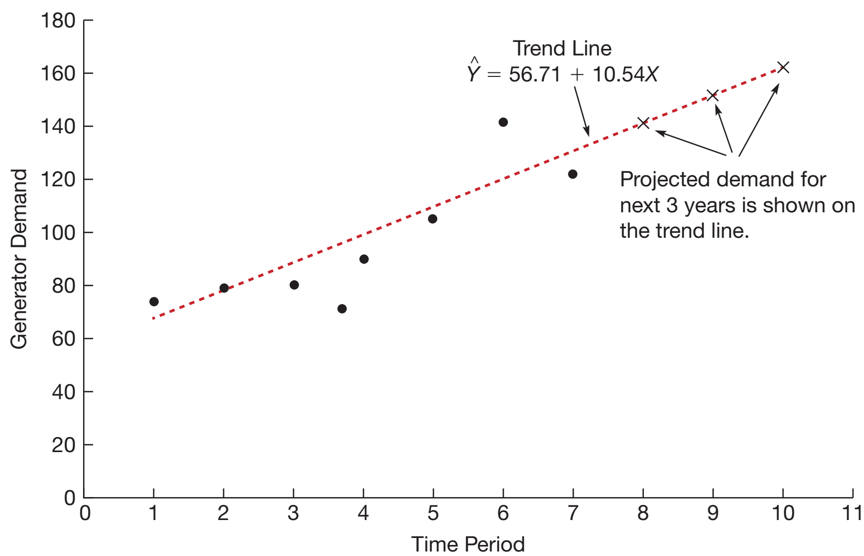
Midwestern Manufacturing Company Solution

Measure	Value	Future Period	Forecast
Error Measures		8	141
Bias (Mean Error)	0	9	151.536
MAD (Mean Absolute Deviation)	8.582	10	162.071
MSE (Mean Squared Error)	110.403	11	172.607
Standard Error (denom=n-2=5)	12.432	12	183.143
MAPE (Mean Absolute Percent Error)	8.224%	13	193.679
Regression line		14	204.214
Generators Sold = 56.714		15	214.75
+ 10.536 * Time(x)		16	225.286
Statistics		17	235.822
Correlation coefficient	.895	18	246.357
Coefficient of determination (r^2)	.801	19	256.893
		20	267.429
		21	277.964

Forecasts for future time periods are shown here.

The trend line is shown over two lines.

FIGURE 5.4
Generator Demand and Projections for Next 3 Years Based on Trend Line



and the spreadsheet is initialized, allowing you to input the seven Y values and the seven X values, as shown in Program 5.4.

This problem could also be solved using QM for Windows. To do so, select the *Forecasting* module, and then enter a new problem by selecting *New – Time Series Analysis*. When the next window opens, enter the number of observations (7) and press *OK*. Enter the values (Y) for the seven past observations when the input window opens. It is not necessary to enter the values for X (1, 2, 3, . . . , 7) because QM for Windows will automatically use these numbers. Then click *Solve* to see the results shown in Program 5.5.

Figure 5.4 provides a scatter diagram and a trend line for these data. The projected demand in each of the next 3 years is also shown on the trend line.

5.6 Adjusting for Seasonal Variations

Time-series forecasting such as that in the example of Midwestern Manufacturing involves looking at the *trend* of data over a series of time observations. Sometimes, however, recurring variations at certain seasons of the year make a *seasonal* adjustment in the trend line forecast necessary. Demand for coal and fuel oil, for example, usually peaks during cold winter months. Demand for golf clubs or suntan lotion may be highest in summer. Analyzing data in monthly or quarterly terms usually makes it easy to spot seasonal patterns. A seasonal index is often used in multiplicative time-series forecasting models to make an adjustment in the forecast when a seasonal component exists. An alternative is to use an additive model such as a regression model, which will be introduced in a later section.

Seasonal Indices

A **seasonal index** indicates how a particular season (e.g., month or quarter) compares with an average season. An index of 1 for a season would indicate that the season is average. If the index is higher than 1, the values of the time series in that season tend to be higher than average. If the index is lower than 1, the values of the time series in that season tend to be lower than average.

Seasonal indices are used with multiplicative forecasting models in two ways. In the first method, each observation in a time series is divided by the appropriate seasonal index to remove the impact of seasonality. The resulting values are called **deseasonalized data**. Using these deseasonalized values, forecasts for future values can be developed using a variety of forecasting techniques. Once the forecasts of future deseasonalized values have been developed, they are multiplied by the seasonal indices to develop the final forecasts, which now include variations due to seasonality. The second method, which is based on a centered-moving-average approach, is more difficult but it must be used when a trend is present.

Calculating Seasonal Indices with No Trend

An average season has an index of 1.

When no trend is present, the index can be found by dividing the average value for a particular season by the average of all the data. Thus, an index of 1 means the season is average. For example, if the average sales in January were 120 and the average sales in all months were 200, the seasonal index for January would be $120/200 = 0.60$, so January is below average. The next example illustrates how to compute seasonal indices from historical data and to use these indices in forecasting future values.

Monthly sales of one brand of telephone answering machine at Eichler Supplies are shown in Table 5.8 for the two most recent years. The average demand in each month is computed, and these values are divided by the overall average (94) to find the seasonal index for each month. We then use the seasonal indices from Table 5.8 to adjust future forecasts. For example, suppose we expected the third year's annual demand for answering machines to be 1,200 units, which is

TABLE 5.8
Answering Machine Sales and Seasonal Indices

MONTH	SALES DEMAND YEAR 1	SALES DEMAND YEAR 2	AVERAGE 2-YEAR DEMAND	MONTHLY DEMAND[a]	AVERAGE SEASONAL INDEX[b]
January	80	100	90	94	0.957
February	85	75	80	94	0.851
March	80	90	85	94	0.904
April	110	90	100	94	1.064
May	115	131	123	94	1.309
June	120	110	115	94	1.223
July	100	110	105	94	1.117
August	110	90	100	94	1.064
September	85	95	90	94	0.957
October	75	85	80	94	0.851
November	85	75	80	94	0.851
December	80	80	80	94	0.851
		Total average demand =	1,128		

[a] Average monthly demand $= \dfrac{1{,}128}{12 \text{ months}} = 94$

[b] Seasonal index $= \dfrac{\text{Average 2-year demand}}{\text{Average monthly demand}}$