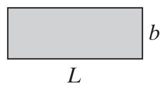
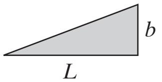
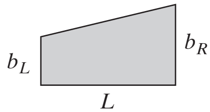
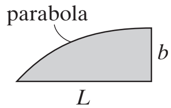
GLOBAL EDITION

# Structural Analysis

ELEVENTH EDITION IN SI UNITS

R. C. Hibbeler

## Table for Evaluating $\int_0^L ab\,dx$

| $\int_0^L ab\,dx$ |  |  |  |  |
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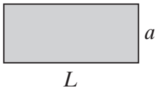
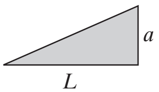
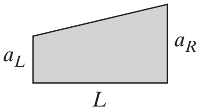
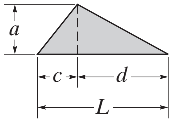
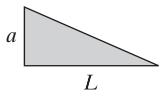
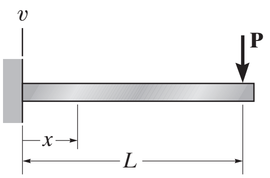
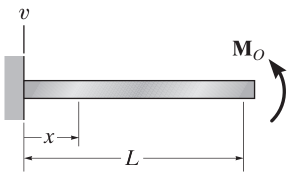
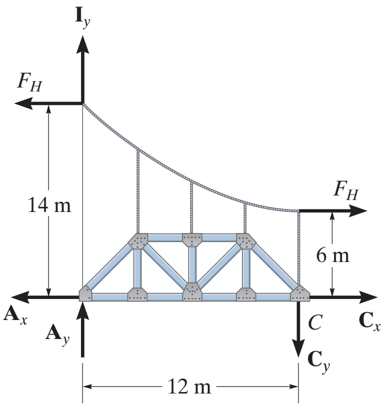
|  | $abL$ | $\frac{1}{2}abL$ | $\frac{1}{2}a(b_L + b_R)L$ | $\frac{2}{3}abL$ |
|  | $\frac{1}{2}abL$ | $\frac{1}{3}abL$ | $\frac{1}{6}a(b_L + 2b_R)L$ | $\frac{5}{12}abL$ |
|  | $\frac{1}{2}b(a_L + a_R)L$ | $\frac{1}{6}b(a_L + 2a_R)L$ | $\frac{1}{6}[2(a_Lb_L + a_Rb_R) + a_Lb_R + a_Rb_L]L$ (application also applies if $a_L < 0$ or $b_R < 0$) | $\frac{1}{12}[b(3a_L + 5a_R)]L$ |
|  | $\frac{1}{2}abL$ | $\frac{1}{6}ab(L + c)$ | $\frac{1}{6}a[b_L(L + d) + b_R(L + c)]$ | $\frac{1}{12}ab\left(3 + \frac{3c}{L} - \frac{c^2}{L^2}\right)L$ |
|  | $\frac{1}{2}abL$ | $\frac{1}{6}abL$ | $\frac{1}{6}a(2b_L + b_R)L$ | $\frac{1}{4}abL$ |

## Beam Deflections and Slopes

| Loading | $v$ + ↑ | $\theta$ + ↺ | |
|---|---|---|---|
|  | $v_{max} = -\frac{PL^3}{3EI}$ at $x = L$ | $\theta_{max} = -\frac{PL^2}{2EI}$ at $x = L$ | $v = \frac{P}{6EI}(x^3 - 3Lx^2)$ |
|  | $v_{max} = \frac{M_OL^2}{2EI}$ at $x = L$ | $\theta_{max} = \frac{M_OL}{EI}$ at $x = L$ | $v = \frac{M_O}{2EI}x^2$ |



(c)

If only half the suspended structure is considered, Fig. 5–6*c*, then summing moments about the pin at *C*, we have

$$\circlearrowright + \Sigma M_C = 0; \quad F_H(14 \text{ m}) - F_H(6 \text{ m}) - I_y(12 \text{ m}) - A_y(12 \text{ m}) = 0$$

$$I_y + A_y = 0.667F_H$$

From these two equations,

$$18.75 = 0.667F_H$$

$$F_H = 28.125 \text{ kN}$$

To obtain the maximum tension in the cable, we will use Eq. 5–11, but first it is necessary to determine the value of an assumed uniform distributed loading $w_0$ from Eq. 5–8:

$$w_0 = \frac{2F_H h}{L^2} = \frac{2(28.125 \text{ kN})(8 \text{ m})}{(12 \text{ m})^2} = 3.125 \text{ kN/m}$$

Thus, using Eq. 5–11, we have

$$T_{max} = w_0 L\sqrt{1 + (L/2h)^2}$$

$$= 3.125 \text{ kN/m}(12 \text{ m}) \sqrt{1 + (12 \text{ m}/2(8 \text{ m}))^2}$$

$$= 46.9 \text{ kN}$$ *Ans.*

*Note*: If the two pin-connected trusses are replaced by a single truss, $\Sigma M_C = 0$ is not applicable, and the cable-truss system becomes statically indeterminate to the first degree.

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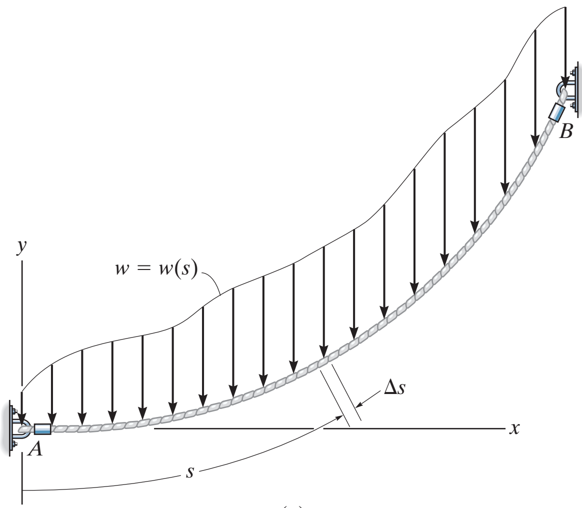



(a)

**Fig. 5–7**



## 5.4 CABLE SUBJECTED TO ITS OWN WEIGHT

When the weight of a cable becomes important in the force analysis, the loading function along the cable will be a function of the arc length $s$ rather than the projected length $x$. To analyze this problem, we will consider a generalized loading function $w = w(s)$ acting along the cable, as shown in Fig. 5–7*a*. The free-body diagram for a small segment $\Delta s$ of the cable is shown in Fig. 5–7*b*. Applying the equilibrium equations to the force system on this diagram, one obtains relationships identical to those given by Eqs. 5–1 through 5–3, but with $s$ replacing $x$ in Eqs. 5–1 and 5–2. Therefore, we can show that

$$T\cos\theta = F_H$$

$$T\sin\theta = \int w(s)\,ds \tag{5–12}$$

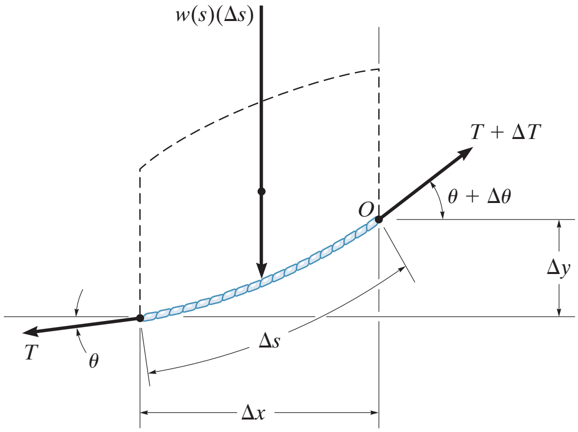
$$\frac{dy}{dx} = \frac{1}{F_H}\int w(s)\,ds \tag{5–13}$$

To perform a direct integration of Eq. 5–13, it is necessary to replace $dy/dx$ by $ds/dx$. Since

$$ds = \sqrt{dx^2 + dy^2}$$

then

$$\frac{dy}{dx} = \sqrt{\left(\frac{ds}{dx}\right)^2 - 1}$$




(b)

Therefore,

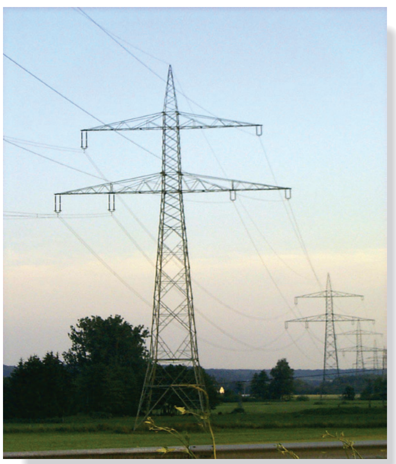
$$\frac{ds}{dx} = \left[1 + \frac{1}{F_H^2}\left(\int w(s)\,ds\right)^2\right]^{1/2}$$

Separating the variables and integrating we obtain

$$x = \int \frac{ds}{\left[1 + \frac{1}{F_H^2}\left(\int w(s)\,ds\right)^2\right]^{1/2}} \qquad (5\text{–}14)$$

The two constants of integration, say $C_1$ and $C_2$, are found using the boundary conditions for the curve.





Electrical transmission towers must be designed to support the weight of the suspended power lines. The weight and length of the cables can be determined since they each form a catenary curve.

## EXAMPLE 5.4

Determine the deflection curve, the length, and the maximum tension in the uniform cable shown in Fig. 5–8. The cable has a weight per unit length of $w_0 = 5\ \text{N/m}$.

### SOLUTION

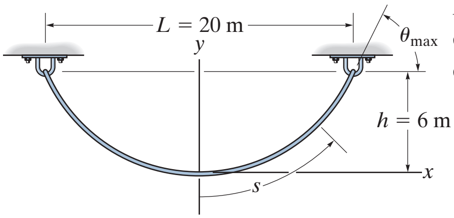



**Fig. 5–8**

For reasons of symmetry, the origin of coordinates is located at the center of the cable. The deflection curve is expressed as $y = f(x)$. We can determine it by first applying Eq. 5–14, where $w(s) = w_0$.

$$x = \int \frac{ds}{\left[1 + (1/F_H^2)\left(\int w_0\ ds\right)^2\right]^{1/2}}$$

Integrating the term under the integral sign in the denominator, we have

$$x = \int \frac{ds}{[1 + (1/F_H^2)(w_0 s + C_1)^2]^{1/2}}$$

Substituting $u = (1/F_H)(w_0 s + C_1)$ so that $du = (w_0/F_H)\ ds$, a second integration yields

$$x = \frac{F_H}{w_0}(\sinh^{-1} u + C_2)$$

or

$$x = \frac{F_H}{w_0}\left\{\sinh^{-1}\left[\frac{1}{F_H}(w_0 s + C_1)\right] + C_2\right\} \qquad (1)$$

To evaluate the constants note that, from Eq. 5–13,

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\ ds \quad \text{or} \quad \frac{dy}{dx} = \frac{1}{F_H}(w_0 s + C_1)$$

Since $dy/dx = 0$ at $s = 0$, then $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{w_0 s}{F_H} \qquad (2)$$

The constant $C_2$ may be evaluated by using the condition $s = 0$ at $x = 0$ in Eq. 1, in which case $C_2 = 0$. To obtain the deflection curve, solve for $s$ in Eq. 1, which yields

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right) \qquad (3)$$

Now substitute into Eq. 2, in which case

$$\frac{dy}{dx} = \sinh\left(\frac{w_0}{F_H}x\right)$$

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Hence,

$$y = \frac{F_H}{w_0}\cosh\left(\frac{w_0}{F_H}x\right) + C_3$$

If the boundary condition $y = 0$ at $x = 0$ is applied, the constant $C_3 = -F_H/w_0$, and therefore the deflection curve becomes

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right] \tag{4}$$

This equation defines the shape of a ***catenary curve***. The constant $F_H$ is obtained by using the boundary condition that $y = h$ at $x = L/2$, in which case

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0 L}{2F_H}\right) - 1\right] \tag{5}$$

Since $w_0 = 5\text{ N/m}$, $h = 6\text{ m}$, and $L = 20\text{ m}$, Eqs. 4 and 5 become

$$y = \frac{F_H}{5\text{ N/m}}\left[\cosh\left(\frac{5\text{ N/m}}{F_H}x\right) - 1\right] \tag{6}$$

$$6\text{ m} = \frac{F_H}{5\text{ N/m}}\left[\cosh\left(\frac{50\text{ N}}{F_H}\right) - 1\right] \tag{7}$$

Equation 7 can be solved for $F_H$ by using a trial-and-error procedure. The result is

$$F_H = 45.9\text{ N}$$

and therefore the deflection curve, Eq. 6, becomes

$$y = 9.19[\cosh(0.109x) - 1]\text{ m}$$ *Ans.*

Using Eq. 3, with $x = 10\text{ m}$, the half-length of the cable is

$$\frac{\mathscr{L}}{2} = \frac{45.9\text{ N}}{5\text{ N/m}}\sinh\left[\frac{5\text{ N/m}}{45.9\text{ N}}(10\text{ m})\right] = 12.1\text{ m}$$

Hence,

$$\mathscr{L} = 24.2\text{ m}$$ *Ans.*

Since $T = F_H/\cos\theta$, the maximum tension occurs when the absolute value of $\theta$ is maximum, i.e., at $s = \mathscr{L}/2 = 12.1\text{ m}$. Using Eq. 2 yields

$$\left.\frac{dy}{dx}\right|_{s=12.1\text{ m}} = \tan\theta_{max} = \frac{5\text{ N/m}(12.1\text{ m})}{45.9\text{ N}} = 1.32$$

$$\theta_{max} = 52.8^\circ$$

And so,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{45.9\text{ N}}{\cos 52.8^\circ} = 75.9\text{ N}$$ *Ans.*

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## 5.5 ARCHES

Like cables, arches can be used to reduce the bending moments in long-span structures. Essentially, an arch acts as an inverted cable, so it transmits its load mainly in compression although, because of its rigidity, it must also resist some bending and shear depending upon how it is loaded and shaped. If the arch has a ***parabolic shape*** and it is subjected to a *uniform* distributed vertical load, then from the analysis of cables it follows that *only compressive forces* will be resisted by the arch. Under these conditions the arch shape is called a ***funicular arch*** because no bending moment or shear force occurs within it. (See Example 5.5.)

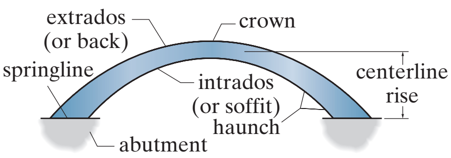
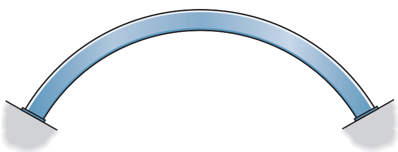



Fig. 5–9

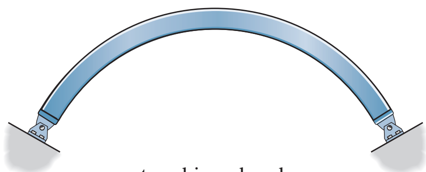
A typical arch is shown in Fig. 5–9, which specifies some of the nomenclature used to define its geometry. Depending upon the application, several types of arches can be selected to support a loading. A ***fixed arch***, Fig. 5–10*a*, is often made of reinforced concrete. Although it may require less material to construct than other types of arches, it must have solid foundation abutments since it is statically indeterminate to the third degree and, consequently, additional stresses can be introduced into the arch due to relative settlement of its supports. A ***two-hinged arch***, Fig. 5–10*b*, is commonly made of metal or timber. It is statically indeterminate to the first degree, and although it is not as rigid as a fixed arch, it is somewhat insensitive to settlement. We could make this structure statically determinate by replacing one of the hinges with a roller. Doing so, however, would remove the capacity of the arch to resist bending along its span, and as a result it would serve as a curved beam, and *not* as an arch. A ***three-hinged arch***, Fig. 5–10*c*, which is also made of metal or timber, is statically determinate. Unlike statically indeterminate arches, a three-hinged arch is not affected by settlement or a change in length due to temperature. Finally, if two- and three-hinged arches are to be constructed without the need for larger foundation abutments, and if clearance is not a problem, then pin and roller supports can be connected with a tie rod, Fig. 5–10*d*. This ***tied arch*** allows the structure to behave as a rigid unit, since the tie rod carries the horizontal component of thrust at the supports. It is also unaffected by relative settlement of the supports.



fixed arch

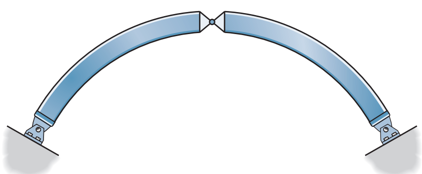
(a)





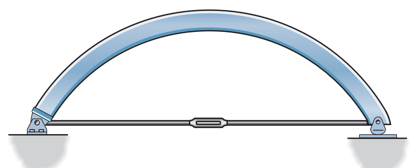
two-hinged arch

(b)



three-hinged arch

(c)



tied arch

(d)

Fig. 5–10