GLOBAL EDITION

Elementary and Middle School Mathematics

Teaching Developmentally

ELEVENTH EDITION

John A. Van de Walle
Karen S. Karp
Jennifer M. Bay-Williams

ELEVENTH EDITION

GLOBAL EDITION

Elementary and Middle School Mathematics

Teaching Developmentally

John A. Van de Walle
Late of Virginia Commonwealth University

Karen S. Karp
Johns Hopkins University

Jennifer M. Bay-Williams
University of Louisville

With contributions by
Jonathan Wray
Howard County Public Schools

Pearson

Harlow, England • London • New York • Boston • San Francisco • Toronto • Sydney • Dubai • Singapore • Hong Kong
Tokyo • Seoul • Taipei • New Delhi • Cape Town • São Paulo • Mexico City • Madrid • Amsterdam • Munich • Paris • Milan

Activity 9.2 How Many Feet in the Bed?

CCSS-M: 1.OA.A.1; 1.OA.C.6; 2.OA.B.2

Read *How Many Feet in the Bed?* by Diane Johnston Hamm (1991). On the second time reading the book, ask students how many more feet are in the bed each time a new person gets in. Ask students to record the equation (e.g., $6 + 2$) and tell how many feet in total. Two less can be considered as family members get out of the bed. Find opportunities to make the connection between counting on and adding using a number line. For multilingual learners, be sure that they know what the phrases “two more” and “two less” mean (and clarify the meaning of *foot,* which is also used for measuring). Acting out this situation with students in the classroom can be a great illustration for both multilingual learners and students with special needs in math.

In the context of this activity, the expression is set up with the larger number first. However, when the expression is reversed (e.g., $2 + 6$), students may initially count on from the first number rather than from the larger number. An explicit focus on strategies (e.g., counting on from the larger number) helps students, particularly those with special needs in math, select the more efficient strategy (Dennis, Sorrells, & Falcomata, 2016).

Students must be able to add 1 or 2 automatically *through 20*. Even though $14 + 1$ is not considered a basic fact, students must know it to use the Near Doubles strategy (e.g., $7 + 8 = 7 + 7 + 1 = 14 + 1$). Activity 9.3 engages students in adding 0, 1, or 2 to be the first one to get a bean (or counter) to 19.

Activity 9.3 Game: First Bean to 19

CCSS-M: 1.OA.C.5; 1.OA.C.6; 2.OA.B.2

Materials: A die labeled +0, +1, +2, +0, +1, +2 (or a spinner with these three options), two beans or counters (one for each player), and a First Bean to 19 game board (see Figure 9.3)

Students take turns. On their turn, they roll the die and move their bean to the new spot on the number path. They say and record the addition sentence. For example, if a student is on 11 and rolls a 1, they say, “Eleven plus 1 equals twelve” or “One more than eleven is twelve.” They can record equations in their notebook or a recording page.

Player 1

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

Player 2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

FIGURE 9.3 First Bean to 19 game board

Activity 9.4 Game: Dart to the Start

CCSS-M: 1.OA.C.5; 1.OA.C.6; 2.OA.B.2

Materials: A die labeled −0, −1, −2, −0, −1, −2 (or a spinner with these three options), two beans or counters (one for each player), and a First Bean to 19 game board (see Figure 9.3)

This game is played just like First Bean to 19 (Activity 9.3), only in reverse, with the beans placed at 19 and the first reaching 0 winning the game. As always, encourage students to say aloud the subtraction move they make. If they are on 16 and roll −2, they may say "Sixteen minus 2 equals fourteen."

Standards for Mathematical Practice
MP7. Look for and make use of structure

While zeros can be easy to compute, the concept of adding zero must be developed through concrete experiences. Asking students to generalize from a set of problems is a good way to reinforce reasoning and avoid overgeneralization.

Activity 9.5 Zero Has an Identity

CCSS-M: 1.OA.C.5; 1.OA.C.6; 2.OA.B.2

Give students a set of expressions to solve. Have students model each one on their Part-Part-Whole graphic organizer, using counters. Ask students what they notice about the problems, and what they notice across problems. Ask them to create their own stories and/or to illustrate the problems to go with each problem.

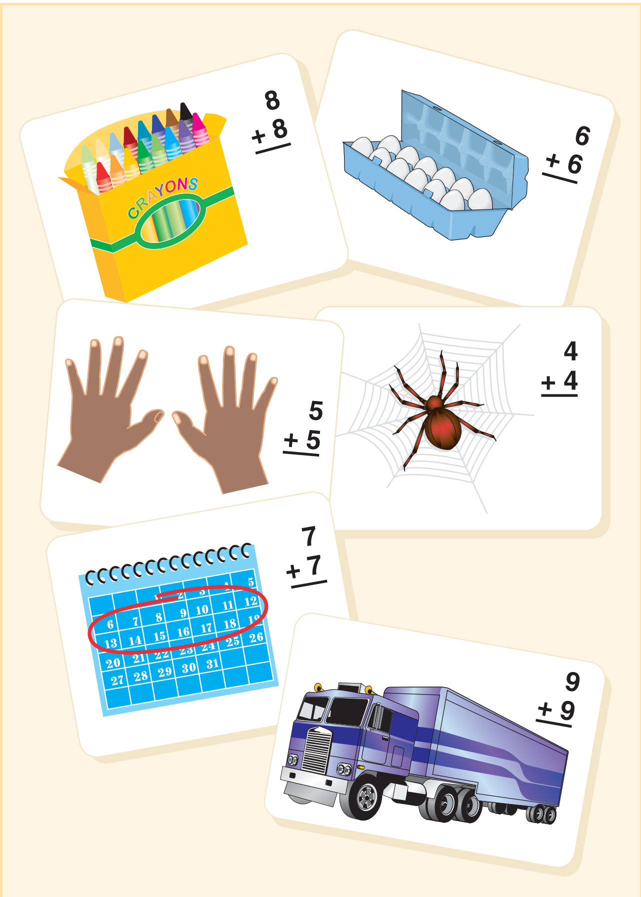

FIGURE 9.4 Real-life situations for doubles facts

Doubles

There are ten doubles facts from $0 + 0$ to $9 + 9$. These facts are foundational and provide the basis for the Near Doubles strategy. Many students find doubles easy, perhaps from reciting doubles. Picture cards that show real-life doubles can help students learn doubles if they do not know them (see Figure 9.4). Story problems can focus on pairs of the same addends: "Alex and Zack each found 7 seashells at the beach. How many did they find together?" And quick images with doubles, using ten-frames, can help students see that $7 + 7$ equals $5 + 5 + 2 + 2$, or $10 + 4$.

Activity 9.6 Double the Ducks

CCSS-M: 1.OA.C.5; 1.OA.C.6; 2.OA.B.2

Read *Double the Ducks* (Murphy, 2002), a story that begins with 5 ducks, each bringing home a friend. Have students work with a partner. Give each pair a set of counters. The first partner selects some of those counters to be the ducks. Their partner tells how many ducks there will be when each duck brings a friend. To support students with special needs in math and to make strategies more visible for all students, ducks (counters) can be placed on a ten-frame.

Activity 9.7 Calculator Doubles

CCSS-M: 1.OA.C.6; 2.OA.B.2

Students work in pairs with a calculator. Students enter the double maker (2×) in the calculator. One student says a double—for example, "Seven plus seven." The other student presses 7, says what the double is, and then presses = to see the correct double (14) on the display. The students then switch roles and reset the calculator (2×). For multilingual students, you can encourage saying the double in their native language or in both their native language and in English. (Note that this activity with the calculator can easily be adapted to other fact sets.)

Combinations of 10. Perhaps the most important strategy for students to know is the combinations that equal 10. It is a foundational fact from which students can derive many other facts. Consider story situations such as the following and ask students to tell possible answers.

There are 10 fishing boats, some at the dock and some at sea.
How many boats might be at the dock and at sea?

There are many children's books focused on the concept of 10. While they might be counting books, you can incorporate questions like, "How many more to equal 10?" Such books could be used as a context for Activity 9.8.

Activity 9.8 How Many More to Equal 10?

CCSS-M: 1.OA.B.4; 1.OA.C.6; 2.OA.B.2

Display counters on one ten-frame (see Figure 9.5) and ask, "How many more to equal 10?" This activity can be repeated using different start numbers. Eventually, display a blank ten-frame and say a number less than 10. Students start with that number and complete the "10 fact." If you say, "four," they say, "four plus six equals ten." This can be completed as whole class or with students working with a partner. Students who are still in Phase 1 of learning the facts (using counting strategies) or students with special needs in math can cover the open spaces on the ten-frame to confirm the missing part. This activity can be implemented in a learning center to provide additional experiences with learning Combinations of 10.

Blackline Master: Ten-Frame

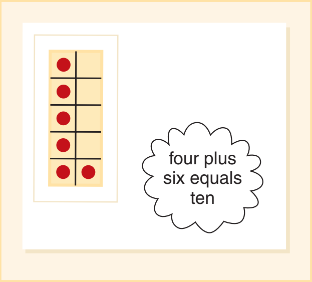

FIGURE 9.5 Combinations of 10 on a ten-frame

Activity 9.9 Game: Go Fish for 10s

CCSS-M: 1.OA.B.4; 1.OA.C.6; 2.OA.B.2

Materials: Mini-ten-frames as a deck of cards, or a regular deck of cards, removing kings and jacks (queens = 0, aces = 1)

The classic game of Go Fish is a great fit for Combinations of 10. Place students in groups of 3 or 4. Deal five cards to each player. In the classic game, on a player's turn, they ask one friend if they have a card that matches one of theirs (e.g., a 6). If the friend has a 6, they have to give it to the player. The player has a match, and lays it down in front of them and they get another turn. In this Fish for 10s version, when the child has a 6, they select a friend and ask, "Do you have a 4?" If the friend does, they hand it over, and the player puts down the match (a combination of 10). If the friend does not have the card, they say, "Go Fish." The player draws a card. If it is not a 4, their turn is over. If it is a 4, they get to go again. Play continues until all the cards are gone. If a player runs out of cards, they draw three new cards.

Blackline Master: Little Ten-Frames

10 + _______. Adding onto 10 is not a basic fact, but it is a foundational fact set because it is necessary for both the Make 10 and Pretend-a-10 strategies. Using a full ten-frame and a partial ten-frame can help students with this group of facts. Just like the foundational facts of one more and two more, the goal is to move from counting on to just knowing. A number line or hundred chart can help students notice the relationship in these sums that will help them move to automaticity.

Activity 9.10 Game: Teen Time

CCSS-M: 1.OA.B.4; 1.OA.C.6; 2.OA.B.2

Materials: Ten-sided die (or deck of cards), 15 counters per player and a Teen Time game board per player

Students can play in groups of 2–5. Students place all 15 counters on whichever numbers they want on their game board. On their turn, a player rolls the die (e.g., 7), adds that number to 10, saying "Ten plus seven is seventeen," and removes a counter from that sum (17). If there is no counter on that number, they lose their turn. First player to remove all their counters wins!

Teen Time game board:

10	11	12	13	14	15	16	17	18	19

***TECHNOLOGY* Note.** Several websites offer virtual visuals to help students with Combinations of 10 and 10 + __ facts. For example, DreamBox (dreambox.com) teacher tools include quick looks for numbers within 10, using ten-frames and math racks, and an app for finding combinations that equal 10 (and other totals as well). Math Learning Center (https://www.mathlearningcenter.org/) apps include ten-frames, number lines, and math racks (number racks), where you can create the visuals you want to share with students. ■

Reasoning Strategies for Addition

Once students have their foundational fact sets to automaticity (Phase 3), they are ready to learn strategies (Phase 2) for the rest of the facts. These strategies eventually become the foundation for reasoning with multi-digit addition, fraction addition, and decimal addition. Learning these strategies is not only to help students learn their addition facts; it helps them develop procedural fluency in general. Whereas in the preceding section games were used to develop automaticity, in this section, games will be used to develop decision-making skills. Students need to decide when they want to use a Near Doubles strategy, for example, and when they want to use a Pretend-a-10 strategy. Therefore, in this section, we briefly explain each strategy and then share games that will engage students in choosing and using the strategies.

Near-Doubles. The Near-Doubles strategy involves using a double fact to figure out the given problem. For instance, to solve $6 + 7$, a student might use either their $6 + 6$ or their $7 + 7$ fact to solve the problem: $6 + 6 + 1$ or $7 + 7 - 1$. This strategy is therefore sometimes called "doubles-plus-one" or "doubles-minus-one," but using the more general phrase communicates that there are options. Also, a student might solve $6 + 8$ by thinking $6 + 6 + 2$—also a Near Doubles strategy. A good way to develop this strategy is with paired quick looks (see Figure 9.1). Begin with a doubles fact quick image, such as $6 + 6$, and follow it with a quick image of $6 + 7$ or $8 + 6$. Transition to using a string of expressions, pairing doubles facts with facts that lend to the Near Doubles strategy. For example:

4 + 4	5 + 5	8 + 8	7 + 7
4 + 6	5 + 7	6 + 8	6 + 8

Another way to help students see a double in a non-double fact is to do a matching activity, as illustrated here in Activity 9.11.

Activity 9.11 Game: Flip and Find

CCSS-M: 1.OA.B.4; 1.OA.C.6; 2.OA.B.2

MATERIALS: A doubles reference mat and a set of fact equation cards that lend to using the Near Doubles strategy. Another option is to cut up a practice page to make the cards.

Students play with a partner. They turn the cards face-down next to the doubles reference mat (see Figure 9.6). On their turn, a player *flips* the card face-up and *finds* a double they will use to add. The player shares their thinking out loud and then places the card under that double fact. A way to increase the challenge is for one partner to share one way, and the other partner to find a different double to solve it. For example, for $7 + 6$, one student might use $6 + 6 + 1$, and another might use $7 + 7 - 1$. After all cards are placed, players examine the facts that are in the same pile and discuss what the facts have in common.

Make 10. All of the basic facts with sums between 11 and 18 can be solved by using the Make 10 strategy. Students use their known facts that equal 10 and then add the rest of the number to 10. For example, to solve $6 + 8$, a student might start with the larger number (8) and see that 8 is 2 away from 10; therefore, they take 2 from the 6 to equal 10 and then add on the remaining 4 to get 14. Make 10 is also aptly called *break apart to make ten (BAMT)* (Sarama & Clements, 2009).

This reasoning strategy is extremely important and is heavily emphasized in high-performing countries (Korea, China, Taiwan, and Japan), where students learn facts sooner and more accurately than U.S. students do (Henry & Brown, 2008). Yet this strategy is not emphasized enough in the United States. A study of California first graders found that this strategy contributed more to developing fluency than using doubles did (even though using doubles had been emphasized by teachers and textbooks in the study) (Henry & Brown, 2008).

The Make 10 strategy can also be applied to larger numbers. For example, for $28 + 7$, students can make 30, seeing that $28 + 7 = 30 + 5$. Thus, this reasoning strategy deserves significant attention when teaching addition (and subtraction) facts.

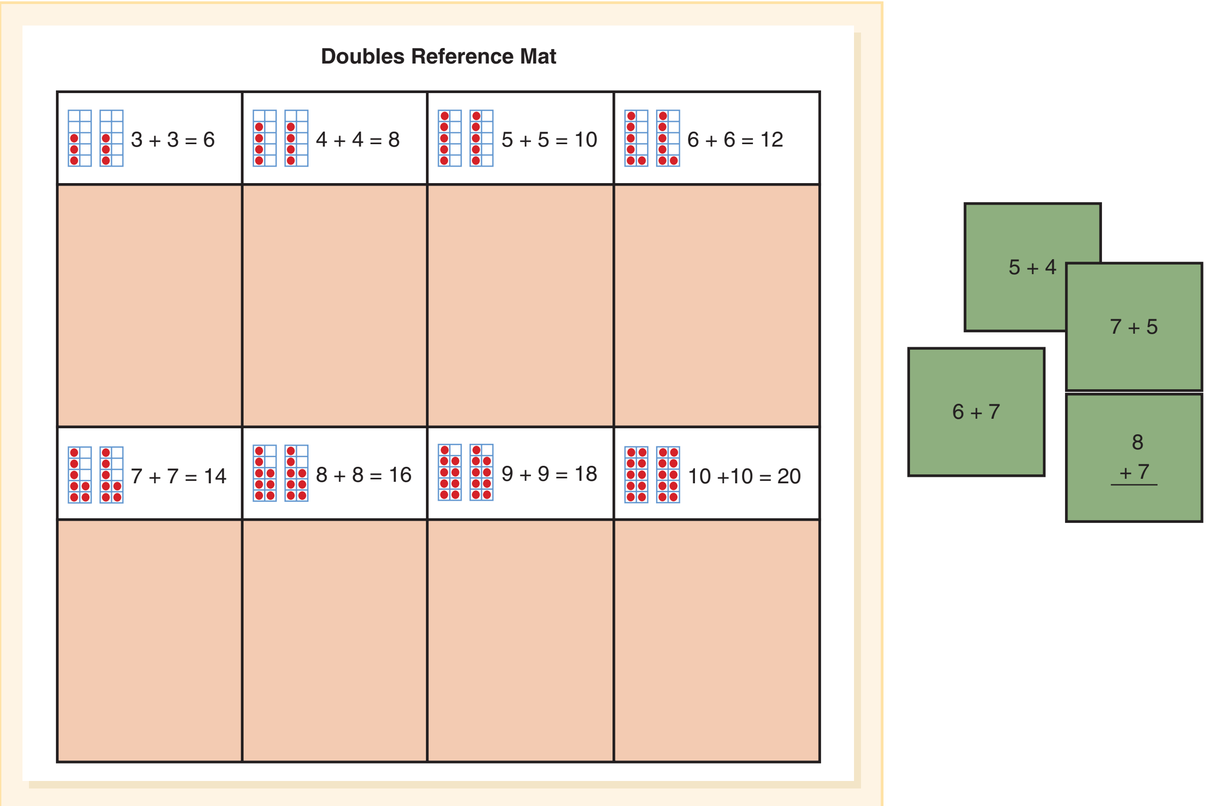

FIGURE 9.6 Doubles reference mat and example cards

Quick images or manipulating double ten-frames can help students develop the Make 10 strategy, the focus of Activity 9.12.

Blackline Master: Double Ten-Frames

Activity 9.12 Move It, Move It

CCSS-M: 1.OA.B.3; 1.OA.C.6; 2.OA.B.2

Distribute a mat with a double ten-frame and an activity page with ten-frames under two heads, one titled "Starting Problem" and the other titled "After I 'Moved It.'" Flash cards with facts are placed next to the double ten-frame, or a fact can be given orally. The students cover each frame with counters to represent the problem ($9 + 6$ would mean covering nine places on one frame and six on the other). Ask students to "move it"—to decide a way to move the counters so that they can find the total without counting. Ask students to explain what they did and connect to the new equation. For example, $9 + 6$ may have become $10 + 5$ by moving one counter to the first ten-frame. Emphasize strategies that are working for that student (5 as an anchor and/or Combinations of 10 and/or Make 10).

Blackline Master: Double Ten-Frames

Pretend-a-10. Pretend-a-10 (also called Use 10) is a compensation strategy. The student pretends one of the addends is a 10, adds, and then compensates for how they changed one addend. For example, for 9 + 6, a student thinks, $10 + 6$ equals 16, 9 is one less than 10, so the sum is also one less, 15. Notice this does not require decomposing or recomposing a number, only knowing how far away one addend is from 10. Therefore, it can be less difficult to implement than Make 10. Students who learned this strategy and practiced it while playing games did substantially better than their peers on addition facts involving adding 8s or 9s (Baroody et al., 2016).

Paired stories again work well to develop this strategy. For example:

Story 1: Ten horses are in the red barn and 6 horses are in the brown barn. How many horses are in the barns?
Story 2: Today, 9 horses are in the red barn and 6 horses are in the brown barn. How many horses are in the barns? *Use your work from the first problem to help you solve this problem.*

Reasoning Strategies for Subtraction

Subtraction facts prove to be more difficult than addition, perhaps because they require knowing the related addition fact strategies and perhaps because students do not have sufficient opportunity to learn and practice these strategies. Without such opportunities, however, students will continue to rely on counting strategies, a slow and often error-prone method.

Think-Addition. As the label implies, in this strategy students use known addition facts to produce the unknown quantity or part of the subtraction. For 13 − 8, a student thinks, "What plus 8 equals 13?" Students taught this strategy do significantly better than their peers on subtraction (Baroody et al., 2016). Considering subtraction as think-addition rather than take-away is conceptually different, because students are really comparing the two numbers to see how far apart they are. A student might think aloud, saying: "What do I add to the part I see to get to the whole?" For example, for 14 − 8, rather than take away 8, a student thinks, "What do I add to 8 to get to 14?"

One way to build the relationship between addition and subtraction is to illustrate facts on triangle or T cards as illustrated in Figure 9.7. The T is particularly useful, because