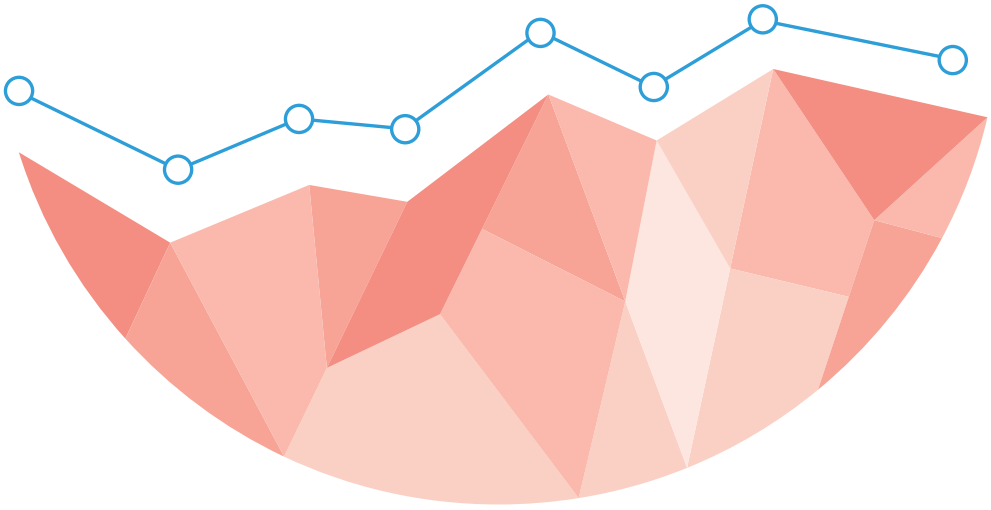
GLOBAL EDITION

THIRTEENTH EDITION

Mathematics with Applications

in the Management, Natural, and Social Sciences

Lial • Hungerford • Holcomb • Mullins



Mathematics
with Applications
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5.3 Exercises

Auto Loans *Find the monthly payment necessary to amortize the given auto loans. (See Example 1.)*

1. $12,500, 4.5% interest; 5 years
2. $8900, 5.1% interest; 4 years
3. $29,675, 3.1% interest; 6 years
4. $56,000, 3.8% interest; 5 years
5. $18,250, 2.9% interest; 4 years
6. $9900, 5.1% interest; 3 years

Mortgages *Find the monthly payment necessary to amortize the given mortgage loans. (See Example 2.)*

7. $225,000 at 3.68% for 30 years
8. $330,000 at 3.50% for 30 years
9. $140,000 at 2.63% for 15 years
10. $180,000 at 2.50% for 10 years

Student Loans *For each of the following student loan situations,*

(a) *calculate the monthly payment necessary to amortize the given loan amount.*

(b) *find the amount of money saved over the lifetime of the loan if an additional $100 is added to the monthly payment.*

(See Example 5.)

11. $21,000 at 2.75% for 10 years
12. $30,500 at 2.95% for 15 years
13. $12,500 at 2.80% for 10 years
14. $15,900 at 3.25% for 20 years

Credit Card Payments *For each of the following credit card situations,*

(a) *calculate the time it will take to pay off the debt making only the given payment each month.*

(b) *calculate the total interest paid.*

(c) *find the amount of money saved over the lifetime of the debt if twice the amount of the given monthly payment is paid each month.*

(See Example 6.)

15. $8000 purchase at 17.1% interest with a monthly payment of $194
16. $6500 purchase at 18.5% interest with a monthly payment of $165.21
17. $12,750 purchase at 15.2% interest with a monthly payment of $289
18. $3500 purchase at 16.75% interest with a monthly payment of $83.85

Loans *Find the monthly payment and estimate the remaining balance at the given time. Assume interest is on the unpaid balance. (See Examples 2–5.)*

19. Six-year new car loan for $26,799 at 3.74%; remaining balance after 2 years
20. Three-year used car loan for $15,875 at 2.89%; remaining balance after 1 year
21. Thirty-year mortgage for $210,000 at 3.63%; remaining balance after 12 years
22. Fifteen-year mortgage for $195,000 at 2.75%; remaining balance after 4.5 years
23. Ten-year student loan for $28,500 at 2.82%; remaining balance after 5 years
24. Ten-year student loan for $15,700 at 3.15%; remaining balance after 6 years
25. Fifteen-year medical school student loan for $125,000 at 3.25%; remaining balance after 8 years
26. Twenty-year medical school student loan for $210,000 at 3.00%; remaining balance after 12 years
27. Fifteen-year law school student loan for $125,000 at 3.45%; remaining balance after 9 years
28. Twenty-year law school student loan for $148,000 at 3.35%; remaining balance after 12 years

Amortization *Use the amortization table in Example 3(c) to answer the questions in Exercises 29–32.*

29. How much of the 5th payment is interest?
30. How much of the 10th payment is used to reduce the debt?
31. How much interest is paid in the first 5 months of the loan?
32. How much interest is paid in the last 5 months of the loan?

Lottery *Find the cash value of the lottery jackpot (to the nearest dollar). Yearly jackpot payments begin immediately (30 for Mega Millions and 30 for Powerball). Assume the lottery can invest at the given interest rate. (See Example 7.)*

33. Powerball: $57.6 million; 5.1% interest
34. Powerball: $207 million; 5.78% interest
35. Mega Millions: $41.6 million; 4.735% interest
36. Mega Millions: $23.4 million; 4.23% interest

Work the following applied problems.

37. **Financing** An auto stereo dealer sells a stereo system for $600 down and monthly payments of $30 for the next 3 years. If the interest rate is 1.25% *per month* on the unpaid balance, find

    (a) the cost of the stereo system;

    (b) the total amount of interest paid.

38. Auto Loan Malik buys a used car costing $16,000. He agrees to make payments at the end of each monthly period for 4 years. He pays 4.35% interest, in monthly payments.

(a) What is the amount of each payment?

(b) Find the total amount of interest Malik will pay.

39. Land Mortgage A speculator agrees to pay $15,000 for a parcel of land; this amount, with interest, will be paid over 4 years with semiannual payments at an interest rate of 10%. Find the amount of each payment.

40. Auto Loan Raven buys a new car costing $26,750. What is the monthly payment if the interest rate is 4.2%, and the loan is for 60 months? Find the total amount of interest Raven will pay.

Student Loan ***A student education loan has two repayment options. The standard plan repays the loan in 10 years with equal monthly payments. The extended plan allows from 12 to 30 years to repay the loan. A student borrows $35,000 at 3.43% interest.***

41. Find the monthly payment and total interest paid under the standard plan.

42. Find the monthly payment and total interest paid under the extended plan with 20 years to pay off the loan.

Use the formula for the approximate remaining balance to work each problem. (See Examples 2(b) and 3(b).)

43. Business Loan When Teresa opened her law office, she bought $14,000 worth of law books and $7200 worth of office furniture. She paid $1200 down and agreed to amortize the balance with semiannual payments for 5 years at 12% interest.

(a) Find the amount of each payment.

(b) When her loan had been reduced below $5000, Teresa received a large tax refund and decided to pay off the loan. How many payments were left at this time?

44. Mortgage Kareem buys a house for $285,000. He pays $60,000 down and takes out a mortgage at 6.9% on the balance. Find his monthly payment and the total amount of interest he will pay if the length of the mortgage is

(a) 15 years.

(b) 20 years.

(c) 25 years.

(d) When will half of the 20-year loan be paid off?

45. Mortgage Susan will purchase a home for $257,000. She will use a down payment of 20% and finance the remaining portion at 3.9% interest for 30 years.

(a) What will be the monthly payment?

(b) How much will remain on the loan after making payments for 5 years?

(c) How much interest will be paid on the total amount of the loan over the course of 30 years?

46. Mortgage Manuel will purchase a $230,000 home with a 20-year mortgage. If he makes a down payment of 20% and the interest rate is 3.3%,

(a) what will the monthly payment be?

(b) how much will he owe after making payments for 8 years?

(c) how much in total interest will he pay over the course of the 20-year loan?

47. Credit Card Debt Jessica charges $8500 to a credit card with a 15.23% annual interest rate.

(a) If Jessica makes a payment of $192.88 each month, how many payments will it take to retire the balance?

(b) If Jessica doubles the payment to $385.76, how many payments will it take to retire the balance?

(c) If Jessica pays $385.76 each month instead of $192.88, how much will she save in interest charges?

48. Credit Card Debt Diamond charges $10,500 to a credit card with a 17.05% annual interest rate.

(a) If Diamond makes a payment of $254.19 each month, how many payments will it take to retire the balance?

(b) If Diamond doubles the payment to $508.38, how many payments will it take to retire the balance?

(c) If Diamond pays $508.38 each month instead of $254.19, how much will she save in interest charges?

49. Credit Card Debt Dakota charges $15,450 to a credit card with a 18.92% annual interest rate.

(a) If Dakota makes a payment of $398.10 each month, how many payments will it take to retire the balance?

(b) If Dakota makes a payment of $1194.30, how many payments will it take to retire the balance?

(c) If Dakota pays $1194.30 each month instead of $398.10, how much will he save in interest charges?

50. Credit Card Debt Booker charges $4500 to a credit card with a 16.84% annual interest rate.

(a) If Booker makes a payment of $108.15 each month, how many payments will it take to retire the balance?

(b) If Booker makes a payment of $324.45, how many payments will it take to retire the balance?

(c) If Booker pays $324.45 each month instead of $108.15, how much will he save in interest charges?

In Exercises 51–54, prepare an amortization schedule showing the first four payments for each loan. (See Example 3(c).)

51. Insurance Payment An insurance firm pays $4000 for a new printer for its computer. It amortizes the loan for the printer in four annual payments at 8% interest.

52. Commercial Auto Loan Large semitrailer trucks cost $72,000 each. Ace Trucking buys such a truck and agrees to pay for it by a loan that will be amortized with nine semiannual payments at 6% interest.

53. Business Loan One retailer charges $1048 for a certain computer. A firm of tax accountants buys eight of these computers. It makes a down payment of $1200 and agrees to amortize the balance with monthly payments at 12% interest for 4 years.

54. Business Loan Joan plans to borrow $20,000 to stock her small boutique. She will repay the loan with semiannual payments for 5 years at 7% interest.

✓Checkpoint Answers

1. $797.63 **2.** $141,857.05

3. **(a)** $1547.26
(b) 167 (166.6824613) payments
(c) $3937.45

4. $117.47; $2075.40

5. 63 (62.41372764) payments; $6,523.08 total interest

6. $31,456,910.67

5.4 Bonds and Treasury Bills

Simple interest is normally used for loans with a term of one year or less. A significant exception is the case of **corporate bonds** and similar financial instruments. When an investor buys a corporate bond, she is essentially lending money to the corporation that issues the bond. The corporation promises to return the principal to the investor at a specified maturity date *and* pay the investor a specified interest rate, usually semiannually (twice a year).

To determine the amount of money paid to the investor twice a year, we use the interest formula $I = PVrt$, from Section 5.1, with PV as the amount invested in the bond (principal or present value), r as the interest rate as a decimal, and $t = \frac{1}{2}$ because the bond pays twice a year.

EXAMPLE 1 **Corporate Bond** Suppose a bank issued 10-year bonds with an annual simple interest rate of 5.7%. The interest is paid semiannually. Santiago buys a $10,000 bond. Determine the following.

(a) How much interest will Santiago earn every 6 months?

Solution Use the simple interest formula $I = PVrt$ with $PV = 10{,}000$, $r = .057$, and $t = \frac{1}{2}$.

$$I = PVrt = (10{,}000)(.057)\left(\frac{1}{2}\right) = \$285.$$

(b) How much interest will he earn over the 10-year life of the bond?

Solution Either use the simple interest formula with $t = 10$, that is,

$$I = PVrt = (10{,}000)(.057)(10) = \$5700,$$

or take the answer in part (a), which will be paid out every 6 months for 10 years for a total of 20 times. Thus, Santiago would obtain $285(20) = $5700. ✓1

✓Checkpoint 1

For the given bonds, find the semiannual interest payment and the total interest paid over the life of the bond.

(a) $7500 30-year bond at 7.3% annual interest.

(b) $15,000 10-year bond at 9.0% annual interest.

Present Value

In interest problems, PV always represents the amount at the beginning of the period, and FV always represents the amount at the end of the period. To find a formula for PV, we begin with the future-value formula:

$$FV = PV(1 + rt).$$

Dividing each side by $1 + rt$ gives the following formula for the present value.

Present Value for Simple Interest

The **present value** PV of a future amount of FV dollars at a simple interest rate r for t years is

$$PV = \frac{FV}{1 + rt}.$$

EXAMPLE 2 **Present Value** Find the present value of $32,000 in 4 months at 9% interest.

Solution

$$PV = \frac{FV}{1 + rt} = \frac{32{,}000}{1 + (.09)\left(\frac{4}{12}\right)} = \frac{32{,}000}{1.03} \approx 31{,}067.96.$$

A deposit of $31,067.96 today at 9% interest would produce $32,000 in 4 months. These two sums, $31,067.96 today and $32,000.00 in 4 months, are equivalent (at 9%) because the first amount becomes the second amount in 4 months. ✓2

✓Checkpoint 2

Find the present value of the given future amounts. Assume 6% interest.

(a) $7500 in 1 year

(b) $89,000 in 5 months

(c) $164,200 in 125 days

EXAMPLE 3 **Court Settlement** Because of a court settlement, Jalinda owes $5000 to Chloe. The money must be paid in 10 months, with no interest. Suppose that Jalinda wants to pay the money today and that Chloe can invest it at an annual rate of 5%. What amount should Chloe be willing to accept to settle the debt?

Solution The $5000 is the future value in 10 months. So Chloe should be willing to accept an amount that will grow to $5000 in 10 months at 5% interest. In other words, she should accept an amount that is the present value of $5000 under these circumstances. Use the present-value formula with $FV = 5000$, $r = .05$, and $t = 10/12 = 5/6$:

$$PV = \frac{FV}{1 + rt} = \frac{5000}{1 + .05 * \frac{5}{6}} = 4800.$$

Chloe should be willing to accept $4800 today in settlement of the debt. ✓3

✓Checkpoint 3

Jerrell is owed $19,500 by Christine. The money will be paid in 11 months, with no interest. If the current interest rate is 10%, how much should Jerrell be willing to accept today in settlement of the debt?

EXAMPLE 4 **Selling a Note** Lester owes $6500 to Virginia. The loan is payable in one year at 6% interest. Virginia needs cash to pay medical bills, so 4 months before the loan is due, she sells the note (loan) to the bank. If the bank wants a return of 9% on its investment, how much should it pay Virginia for the note?

Solution First find the maturity value of the loan—the amount (with interest) that Lester must pay Virginia:

$$\begin{aligned} FV &= PV(1 + rt) && \text{Maturity-value formula} \\ &= 6500(1 + .06 * 1) && \text{Let } PV = 6500,\ r = .06, \text{ and } t = 1. \\ &= 6500(1.06) = \$6890. \end{aligned}$$

In 4 months, the bank will receive $6890. Since the bank wants a 9% return, compute the present value of this amount at 9% for 4 months:

$$\begin{aligned} PV &= \frac{FV}{1 + rt} && \text{Present-value formula} \\ &= \frac{6890}{1 + .09\left(\frac{4}{12}\right)} = \$6689.32. && \text{Let } FV = 6890,\ r = .09, \text{ and } t = 4/12. \end{aligned}$$

The bank pays Virginia $6689.32, and 4 months later it is worth $6890. ✓4

✓Checkpoint 4

A firm accepts a $21,000 note due in 8 months, with interest of 10.5%. Two months before it is due, the firm sells the note to a broker. If the broker wants a 12.5% return on the investment, how much should be paid for the note?

Discount

The preceding examples dealt with loans in which money is borrowed and simple interest is charged. For most simple interest loans, both the principal (amount borrowed) and the interest are paid at the end of the loan period. With a corporate bond (which is a loan to a company by the investor who buys the bond), interest is paid during the life of the bond and the principal is paid back at maturity. In both cases,

the borrower receives the principal
but pays back the principal *plus* the interest.

In a **simple discount loan,** however, the interest is deducted in advance from the amount of the loan, and the *balance* is given to the borrower. The *full value* of the loan must be paid back at maturity. Thus,

the borrower receives the principal *less* the interest
but pays back the principal.

The most common examples of simple discount loans are U.S. Treasury bills (T-bills), which are essentially short-term loans to the U.S. government by investors. T-bills are sold at a **discount** from their face value, and the Treasury pays back the face value of the T-bill at maturity. The discount amount is the interest deducted in advance from the face value. The Treasury receives the face value less the discount but pays back the full face value. If you are interested in more information on purchasing T-bills, visit www.treasurydirect.gov.

EXAMPLE 5 **Treasury Bills** Suppose an investor bought a 6-month $10,000 U.S. Treasury bill that sold at a discount rate of .39%. What is the amount of the discount? What is the price of the T-bill?

Solution The discount rate on a T-bill is always a simple annual interest rate. Consequently, the discount (interest) is found with the simple interest formula, using $PV = 10{,}000$ (face value), $r = .0039$ (discount rate), and $t = .5$ (because 6 months is half a year):

$$\text{Discount} = PVrt = 10{,}000 * .0039 * .5 = \$19.50.$$

So the price of the T-bill is:

$$\text{Face Value} - \text{Discount} = 10{,}000 - 19.50 = \$9980.50.$$ ✓5

In a simple discount loan such as a T-bill, the discount rate is not the actual interest rate that the borrower pays. In Example 5, the discount rate of .39% was applied to the face value of $10,000 rather than the $9980.50 that the Treasury (the borrower) received.

✓Checkpoint 5

The maturity times and discount rates for $15,000 T-bills are given. Find the discount amount and the price of each T-bill.

(a) one year; .56%

(b) six months; .34%

(c) three months; .26%

EXAMPLE 6 **Treasury Bills** Find the actual interest rate paid by the Treasury in Example 5.

Solution Use the formula for simple interest, $I = PVrt$, with r as the unknown. Here, $PV = 9980.50$ (the amount the Treasury received) and $I = 19.50$ (the discount amount). Because this is a 6-month T-bill, $t = .5$, and we have

$$\begin{aligned} I &= PVrt \\ 19.50 &= 9980.50(r)(.5) \\ 19.50 &= 4990.25r \qquad \text{Multiply the terms on the right side.} \\ r &= \frac{19.5}{4990.25} \approx .0039076. \qquad \text{Divide both sides by 4990.25.} \end{aligned}$$

The actual interest rate is then .39076%. ✓6

✓Checkpoint 6

Find the actual interest rate paid by the Treasury for each T-bill in Checkpoint 5.

Ordinary corporate or municipal bonds usually make semiannual simple interest payments. With a **zero-coupon bond,** however, there are no interest payments during the life of the bond. The investor receives a single payment when the bond matures, consisting of his original investment and the interest (compounded semiannually) that it has earned. Zero-coupon bonds are sold at a substantial discount from their face value, and the buyer receives the face value of the bond when it matures. The difference between the face value and the price of the bond is the interest earned.

EXAMPLE 7 **Bonds** Doug bought a 15-year zero-coupon bond paying 4.5% interest (compounded semiannually) for $12,824.50. What is the face value of the bond?

Solution Use the compound interest formula with $PV = 12{,}824.50$. Interest is paid twice a year, so the rate per period is $i = .045/2$, and the number of periods in 15 years is $n = 30$. The compound amount will be the face value:

$$FV = PV(1 + i)^n = 12{,}824.50(1 + .045/2)^{30} = 24{,}999.99618.$$

Rounding to the nearest cent, we see that the face value of the bond is $25,000. ✓7

✓Checkpoint 7

Find the face value of the zero-coupon bond. Assume the interest is compounded semiannually.

(a) 30-year bond at 6% sold for $2546

(b) 15-year bond at 5% sold for $16,686

EXAMPLE 8 **Bonds** A zero-coupon bond with a face value of $15,000 and a 6% interest rate (compounded semiannually) will mature in 9 years. What is a fair price to pay for the bond today?

Solution Think of the bond as a 9-year investment paying 6%, compounded semiannually, whose future value is $15,000. Its present value (what it is worth today) would be a fair price. So use the present-value formula from Section 5.1 with $FV = 15{,}000$. Since interest is compounded twice a year, the interest rate per period is $i = .06/2 = .03$ and the number of periods in 9 years is $n = 9(2) = 18$. Hence,

$$PV = \frac{FV}{(1 + i)^n} = \frac{15{,}000}{(1 + .03)^{18}} \approx 8810.919114.$$

So a fair price would be the present value of $8810.92. ✓8

✓Checkpoint 8

Find the fair price (present value) in Example 8 if the interest rate is 7.5%. Assume the interest is compounded semiannually.

5.4 Exercises

Corporate Bonds ***Suppose the interest rate provided for the given corporate bonds is recent. Find the semiannual interest payment and the total interest earned over the life of the bond. (See Example 1.)***

1. $5000 IBM, 30-year bond; 6.220%
2. $9000 Barrick Gold Corp., 10-year bond; 6.800%
3. $12,500 Morgan Stanley, 10-year bond; 4.100%
4. $4500 Goldman Sachs, 7-year bond; 2.134%
5. $6500 Amazon.com Corp, 5-year bond; 1.200%
6. $10,000 Wells Fargo, 12-year bond; 4.000%
7. What is meant by the *present value* of money?
8. In your own words, describe the *maturity value* of a loan.

Find the present value of each future amount. (See Examples 2 and 3.)

9. $15,000 for 9 months; money earns 6%
10. $48,000 for 8 months; money earns 5%
11. $15,402 for 120 days; money earns 6.3%
12. $29,764 for 310 days; money earns 7.2%

Treasury Bills ***For the given U.S. Treasury bills, find (a) the price of the T-bill, and (b) the actual interest rate paid by the Treasury. (See Examples 5 and 6.)***

13. Three-month $20,000 T-bill with discount rate of .27%
14. One-month $12,750 T-bill with discount rate of .23%
15. Six-month $15,500 T-bill with discount rate of .36%
16. One-year $7000 T-bill with discount rate of .44%