GLOBAL EDITION

Introductory Circuit Analysis

Fourteenth Edition

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Summary of Equations to Accompany

INTRODUCTORY CIRCUIT ANALYSIS, Fourteenth Edition, Global Edition

by Robert L. Boylestad / Brian A. Olivari

ac

Sinusoidal Alternating Waveforms

Sine wave $v = V_m \sin\alpha$, $\alpha = \omega t = 2\pi f t$, $f = 1/T$, 1 radian $= 57.3°$, radians $= (\pi/180°) \times$ (degrees), degrees $= (180°/\pi) \times$ (radians)
Identities $\sin(\omega t + 90°) = \cos\omega t$, $\sin\omega t = \cos(\omega t - (\pi/2))$, $\sin(-\alpha) = -\sin\alpha$, $\cos(-\alpha) = \cos\alpha$
Average value $G =$ algebraic sum of areas/length of curve
Effective (rms) value $I_{\text{rms}} = 0.707 I_m$, $I_m = \sqrt{2} I_{\text{rms}}$,

$$I_{\text{rms}} = \sqrt{\text{area}[i(t)]^2/T}$$
$$V_{\text{rms}} = 0.707\, V_m$$
$$V_m = \sqrt{2}\, V_{\text{rms}}$$
$$V_{\text{rms}} = \sqrt{\text{area}[v(t)]^2/T}$$

The Basic Elements and Phasors

***R*:** $I_m = V_m/R$, in phase ***L*:** $X_L = \omega L$, v_L leads i_L by 90°
***C*:** $X_C = 1/\omega C$, i_C leads v_C by 90° **Power** $P = (V_m I_m/2)\cos\theta = V_{\text{rms}} I_{\text{rms}} \cos\theta$ ***R*:** $P = V_{\text{rms}} I_{\text{rms}} = I^2{}_{\text{rms}} R = V^2{}_{\text{rms}}/R$
Power factor $F_p = \cos\theta = P/V_{\text{rms}} I_{\text{rms}}$ **Rectangular form** $\mathbf{C} = A \pm jB$
Polar form $C = C\angle\theta$ **Conversions** $C = \sqrt{A^2 + B^2}$, $\theta = \tan^{-1}(B/A)$, $A = C\cos\theta$, $B = C\sin\theta$ **Operations** $j = \sqrt{-1}$, $j^2 = -1$, $1/j = -j$, $\mathbf{C}_1 \pm \mathbf{C}_2 = (\pm A_1 \pm A_2) + j(\pm B_1 \pm B_2)$, $\mathbf{C}_1 \cdot \mathbf{C}_2 = C_1 C_2 \angle(\theta_1 + \theta_2)$, $\mathbf{C}_1/\mathbf{C}_2 = (C_1/C_2)\angle(\theta_1 - \theta_2)$

Series and Parallel ac Circuits

Elements $R\angle 0°$, $X_L\angle 90°$, $X_C\angle -90°$
Series $\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N$, $\mathbf{I}_s = \mathbf{E}/\mathbf{Z}_T$, $F_p = R/Z_T$
Voltage divider rule $\mathbf{V}_x = \mathbf{Z}_x\mathbf{E}/\mathbf{Z}_T$ **Parallel** $\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N$, $\mathbf{Z}_T = \mathbf{Z}_1\mathbf{Z}_2/(\mathbf{Z}_1 + \mathbf{Z}_2)$, $G\angle 0°$, $B_L\angle -90°$, $B_C\angle 90°$, $F_P = \cos\theta_T = G/Y_T$ **Current divider rule** $\mathbf{I}_1 = \mathbf{Z}_2\mathbf{I}_T/(\mathbf{Z}_1 + \mathbf{Z}_2)$, $\mathbf{I}_2 = \mathbf{Z}_1\mathbf{I}_T/(\mathbf{Z}_1 + \mathbf{Z}_2)$
Equivalent circuits $R_s = R_p X_p^2/(X_p^2 + R_p^2)$, $X_s = R_p^2 X_p/(X_p^2 + R_p^2)$, $R_p = (R_s^2 + X_s^2)/R_s$, $X_p = (R_s^2 + X_s^2)/X_s$

Series-Parallel ac Networks

Employ block impedances and obtain general solution for reduced network. Then substitute numerical values. General approach similar to that for dc networks.

Methods of Analysis and Selected Topics (ac)

Source conversion $\mathbf{E} = \mathbf{I}\mathbf{Z}_p$, $\mathbf{Z}_s = \mathbf{Z}_p$, $\mathbf{I} = \mathbf{E}/\mathbf{Z}_s$
Bridge networks $\mathbf{Z}_1/\mathbf{Z}_3 = \mathbf{Z}_2/\mathbf{Z}_4$
Δ-Y, Y-Δ conversions See dc coverage, replacing *R* by *Z*.

Network Theorems (ac)

Review dc content on other side.
Thévenin's theorems (dependent sources) $\mathbf{E}_{OC} = \mathbf{E}_{Th}$, $\mathbf{Z}_{Th} = \mathbf{E}_{OC}/\mathbf{I}_{SC}$, $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$ **Norton's theorems** (dependent sources) $\mathbf{I}_{SC} = \mathbf{I}_N$, $\mathbf{Z}_N = \mathbf{E}_{OC}/\mathbf{I}_{SC}$, $\mathbf{Z}_N = \mathbf{E}_g/\mathbf{I}_g$ **Maximum power transfer theorem** $Z_L = Z_{Th}$, $\theta_L = -\theta_{Th_Z}$, $P_{\max} = E^2{}_{Th}/4R_{Th}$

Power (ac)

R: $P = VI = V_m I_m/2 = I^2 R = V^2/R$ **Apparent power** $S = VI$, $P = S\cos\theta$, $F_p = \cos\theta = P/S$ **Reactive power** $Q = VI\sin\theta$
L: $Q_L = VI = I^2 X_L = V^2/X_L$, C: $Q_C = VI = I^2 X_C = V^2/X_C$, $S_T = \sqrt{P_T^2 + Q_T^2}$, $F_p = P_T/S_T$

Resonances

Series $X_L = X_C$, $f_S = 1/(2\pi\sqrt{LC})$, $Z_{TS} = R$, $Q_l = X_L/R_l$, $Q_S = X_L/R = (1/R)\sqrt{L/C}$, $V_L = Q_S E$, $V_{Cs} = Q_S E$, $P_{\text{HPF}} = (1/2)P_{\max}$, $f_1 = (1/2\pi)[-R/2L + (1/2)\sqrt{(R/L)^2 + 4/LC}]$, f_2 (use $-R/2L$), $BW = f_2 - f_1 = R/2\pi L = f_S/Q_S$ **Parallel** $X_{Lp} = X_C$, $X_{Lp} = (R_l^2 + X_L^2)/X_L$, $f_p = [1/(2\pi\sqrt{LC})]\sqrt{1 - (R_l^2 C/L)}$, $Z_{TP} = R_s \parallel R_p$, $R_p = (R_l^2 + X_L^2)/R_l$, $Q_p = (R_s \parallel R_p)/X_{Lp}$, $BW = f_2 - f_1 = f_p/Q_p$
***Q* ≥ 10:** $Z_{Tp} \cong R_s \parallel Q^2 R_l$, $X_{Lp} \cong X_L$, $X_L = X_C$, $f_p \cong 1/(2\pi\sqrt{LC})$, $Q_p = Q_l$, $I_L = I_C \cong QI_T$, $BW = f_p/Q_p = R_l/2\pi L$

Decibels, Filters, and Bode Plots

Logarithms $N = b^x$, $x = \log_b N$, $\log_e x = 2.3\log_{10} x$, $\log_{10} ab = \log_{10} a + \log_{10} b$, $\log_{10} a/b = \log_{10} a - \log_{10} b$, $\log_{10} a^n = n\log_{10} a$, $\text{dB} = 10\log_{10} P_2/P_1$, $\text{dB}_v = 20\log_{10} V_2/V_1$
***R-C* filters** (high-pass) $f_c = 1/(2\pi RC)$,

$$\mathbf{V}_o/\mathbf{V}_i = R\sqrt{R^2 + X_C^2}\angle\tan^{-1}(X_C/R)$$

(low-pass) $f_c = 1/(2\pi RC)$,

$$\mathbf{V}_o/\mathbf{V}_i = X_C/\sqrt{R^2 + X_C^2}\angle -\tan^{-1}\frac{R}{X_C}$$

Octave 2 : 1, 6 dB/octave **Decade** 10 : 1, 20 dB/decade

Transformers

Mutual inductance $M = k\sqrt{L_p L_s}$ **Iron-core** $E_p = 4.44 f N_p \Phi_m$, $E_S = 4.44 f N_S \Phi_m$, $E_p/E_S = N_p/N_S$, $a = N_p/N_S$, $I_p/I_S = N_S/N_p$, $\mathbf{Z}_p = a^2\mathbf{Z}_L$, $E_p I_p = E_S I_S$, $P_i = P_O$ (ideal)
Air-core $\mathbf{Z}_i = \mathbf{Z}_p + (\omega M)^2/(\mathbf{Z}_S + \mathbf{Z}_L)$

Polyphase Systems

Y-Y system $I_{\phi g} = I_L = I_{\phi L}$, $V_\phi = E_\phi$, $E_L = \sqrt{3}V_\phi$ **Y-Δ system** $V_\phi = E_L$, $I_L = \sqrt{3}I_\phi$ **Δ-Δ system** $V_\phi = E_L = E_\phi$, $I_L = \sqrt{3}I_\phi$
Δ-Y system $E_L = \sqrt{3}V_\phi$, $I_\phi = I_L$, $E_L = E_\phi$ **Power** $P_T = 3P_\phi$, $Q_T = 3Q_\phi$, $S_T = 3S_\phi = \sqrt{3}E_L I_L$, $F_p = P_T/S_T$

Pulse Waveforms and the *R-C* Response

% tilt $= [(V_1 - V_2)/V] \times 100\%$ with $V = (V_1 + V_2)/2$
Pulse repetition frequency (prf) $= 1/T$
Duty cycle $= (t_p/T) \times 100\%$
$V_{\text{av}} =$ (duty cycle)(peak value) $+ (1 -$ duty cycle$) \times (V_b)$
***R-C* circuits** $v_C = V_i + (V_f - V_i)(1 - e^{-t/RC})$
Compensated attenuator $R_p C_p = R_s C_s$

Nonsinusoidal Circuits

Fourier series $f(\alpha) = A_0 + A_1\sin\omega t + A_2\sin 2\omega t + \cdots + A_n\sin n\omega t + B_1\cos\omega t + B_2\cos 2\omega t + \cdots + B_n\cos n\omega t$
Even function $f(\alpha) = f(-\alpha)$, no B_n terms **Odd function** $f(\alpha) = -f(-\alpha)$, no A_n terms, no odd harmonics if $f(t) = f[(T/2) + t]$, no even harmonics if $f(t) = -f((T/2) + t)$
Effective (rms) value

$$V_{(\text{rms})} = \sqrt{V_0^2 + (V_{m_1}^2 + \cdots + V'^2_{m_n} + V'^2_{m_1} + \cdots + V'^2_{m_n})/2}$$

Power $P_T = V_0 I_0 + V_1 I_1\cos\theta + \cdots + V_n I_n\cos\theta_n = I_{\text{rms}}{}^2 R = V_{\text{rms}}{}^2/R$

Standard Resistor Values

Ohms (Ω)					Kilohms (kΩ)		Megohms (MΩ)	
0.10	**1.0**	**10**	**100**	**1000**	**10**	**100**	**1.0**	**10.0**
0.11	1.1	11	110	1100	11	110	1.1	11.0
0.12	**1.2**	**12**	**120**	**1200**	**12**	**120**	**1.2**	**12.0**
0.13	1.3	13	130	1300	13	130	1.3	13.0
0.15	**1.5**	**15**	**150**	**1500**	**15**	**150**	**1.5**	**15.0**
0.16	1.6	16	160	1600	16	160	1.6	16.0
0.18	**1.8**	**18**	**180**	**1800**	**18**	**180**	**1.8**	**18.0**
0.20	2.0	20	200	2000	20	200	2.0	20.0
0.22	**2.2**	**22**	**220**	**2200**	**22**	**220**	**2.2**	**22.0**
0.24	2.4	24	240	2400	24	240	2.4	
0.27	**2.7**	**27**	**270**	**2700**	**27**	**270**	**2.7**	
0.30	3.0	30	300	3000	30	300	3.0	
0.33	**3.3**	**33**	**330**	**3300**	**33**	**330**	**3.3**	
0.36	3.6	36	360	3600	36	360	3.6	
0.39	**3.9**	**39**	**390**	**3900**	**39**	**390**	**3.9**	
0.43	4.3	43	430	4300	43	430	4.3	
0.47	**4.7**	**47**	**470**	**4700**	**47**	**470**	**4.7**	
0.51	5.1	51	510	5100	51	510	5.1	
0.56	**5.6**	**56**	**560**	**5600**	**56**	**560**	**5.6**	
0.62	6.2	62	620	6200	62	620	6.2	
0.68	**6.8**	**68**	**680**	**6800**	**68**	**680**	**6.8**	
0.75	7.5	75	750	7500	75	750	7.5	
0.82	**8.2**	**82**	**820**	**8200**	**82**	**820**	**8.2**	
0.91	9.1	91	910	9100	91	910	9.1	

EXAMPLE 8.29 Write the nodal equations and find the voltage across the 2 Ω resistor for the network in Fig. 8.72.

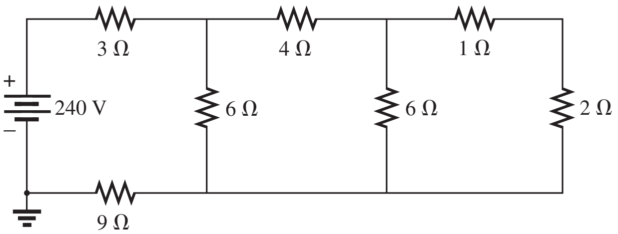

FIG. 8.72
Example 8.29.

Solution: The nodal voltages are chosen as shown in Fig. 8.73. We have

$$V_1: \left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{4\,\Omega}\right)V_1 - \left(\frac{1}{4\,\Omega}\right)V_2 + 0 = 20\text{ A}$$

$$V_2: \left(\frac{1}{4\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{1\,\Omega}\right)V_2 - \left(\frac{1}{4\,\Omega}\right)V_1 - \left(\frac{1}{1\,\Omega}\right)V_3 = 0$$

$$V_3: \left(\frac{1}{1\,\Omega} + \frac{1}{2\,\Omega}\right)V_3 - \left(\frac{1}{1\,\Omega}\right)V_2 + 0 = 0$$

and

$$\begin{aligned} 0.5V_1 - 0.25V_2 + \quad 0 &= 20 \\ -0.25V_1 + \frac{17}{12}V_2 - \quad 1\,V_3 &= 0 \\ 0 - \quad 1\,V_2 + 1.5V_3 &= 0 \end{aligned}$$

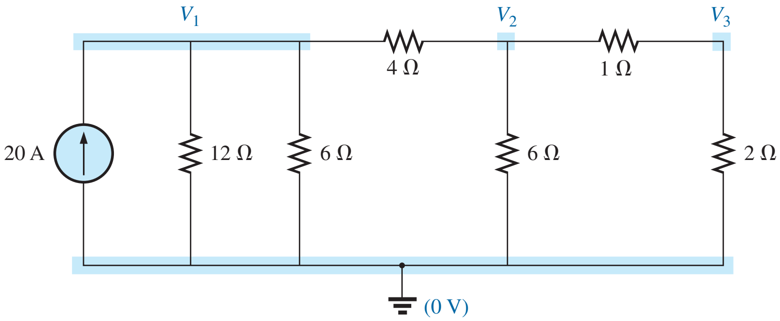

FIG. 8.73
Converting the voltage source to a current source and defining the nodes for the network in Fig. 8.72.

Note the symmetry present about the major axis. Application of determinants reveals that

$$V_3 = V_{2\,\Omega} = \mathbf{10.67\ V}$$

8.8 BRIDGE NETWORKS

This section introduces the **bridge network**, a configuration that has a multitude of applications. In the following chapters, this type of network is used in both dc and ac meters. Electronics courses introduce these in the discussion of rectifying circuits used in converting a varying signal to one of a steady nature (such as dc). A number of other areas of application also require some knowledge of ac networks; these areas are discussed later.

The bridge network may appear in one of the three forms as indicated in Fig. 8.74. The network in Fig. 8.74(c) is also called a *symmetrical* *lattice network* if $R_2 = R_3$ and $R_1 = R_4$. Fig. 8.74(c) is an excellent example of how a planar network can be made to appear nonplanar. For the purposes of investigation, let us examine the network in Fig. 8.75 using mesh and nodal analysis.

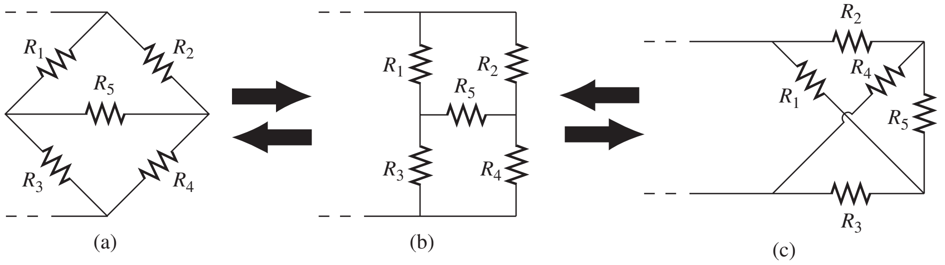

FIG. 8.74
Various formats for a bridge network.

Mesh analysis (Fig. 8.76) yields

$$\begin{aligned}(3\,\Omega + 4\,\Omega + 2\,\Omega)I_1 - (4\,\Omega)I_2 - (2\,\Omega)I_3 &= 20\text{ V}\\(4\,\Omega + 5\,\Omega + 2\,\Omega)I_2 - (4\,\Omega)I_1 - (5\,\Omega)I_3 &= 0\\(2\,\Omega + 5\,\Omega + 1\,\Omega)I_3 - (2\,\Omega)I_1 - (5\,\Omega)I_2 &= 0\end{aligned}$$

and

$$\begin{aligned}9I_1 - 4I_2 - 2I_3 &= 20\\-4I_1 + 11I_2 - 5I_3 &= 0\\-2I_1 - 5I_2 + 8I_3 &= 0\end{aligned}$$

with the result that

$$\begin{aligned}I_1 &= \mathbf{4\ A}\\I_2 &= \mathbf{2.67\ A}\\I_3 &= \mathbf{2.67\ A}\end{aligned}$$

The net current through the 5 Ω resistor is

$$I_{5\,\Omega} = I_2 - I_3 = 2.67\text{ A} - 2.67\text{ A} = 0\text{ A}$$

Nodal analysis (Fig. 8.77) yields

$$\begin{aligned}\left(\frac{1}{3\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{2\,\Omega}\right)V_1 - \left(\frac{1}{4\,\Omega}\right)V_2 - \left(\frac{1}{2\,\Omega}\right)V_3 &= \frac{20}{3}\text{ A}\\\left(\frac{1}{4\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{5\,\Omega}\right)V_2 - \left(\frac{1}{4\,\Omega}\right)V_1 - \left(\frac{1}{5\,\Omega}\right)V_3 &= 0\\\left(\frac{1}{5\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{1\,\Omega}\right)V_3 - \left(\frac{1}{2\,\Omega}\right)V_1 - \left(\frac{1}{5\,\Omega}\right)V_2 &= 0\end{aligned}$$

and

$$\begin{aligned}\left(\frac{1}{3\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{2\,\Omega}\right)V_1 - \left(\frac{1}{4\,\Omega}\right)V_2 - \left(\frac{1}{2\,\Omega}\right)V_3 &= 6.67\text{ A}\\-\left(\frac{1}{4\,\Omega}\right)V_1 + \left(\frac{1}{4\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{5\,\Omega}\right)V_2 - \left(\frac{1}{5\,\Omega}\right)V_3 &= 0\\-\left(\frac{1}{2\,\Omega}\right)V_1 - \left(\frac{1}{5\,\Omega}\right)V_2 + \left(\frac{1}{5\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{1\,\Omega}\right)V_3 &= 0\end{aligned}$$

Note the symmetry of the solution.

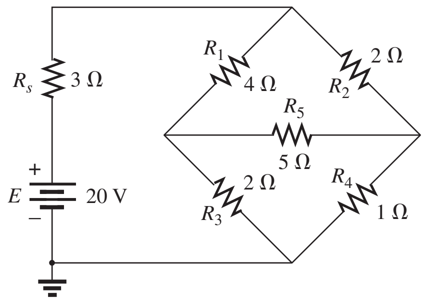

FIG. 8.75
Standard bridge configuration.

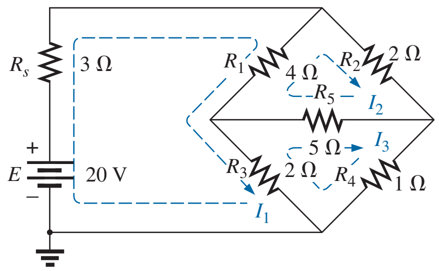

FIG. 8.76
Assigning the mesh currents to the network in Fig. 8.75.

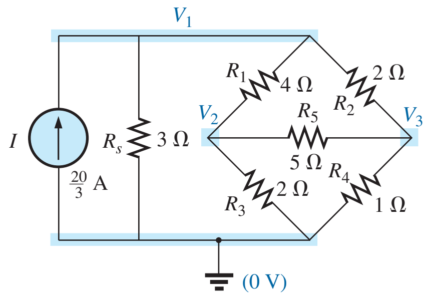

FIG. 8.77
Defining the nodal voltages for the network in Fig. 8.75.

TI-89 Calculator Solution

With the TI-89 calculator, the top part of the determinant is determined by the sequence in Fig. 8.78 (take note of the calculations within parentheses):

det([6.67,−1/4,−1/2;0,(1/4+1/2+1/5),−1/5;0,−1/5,(1/5+1/2+1/1)]) (ENTER)	10.51E0

FIG. 8.78
TI-89 solution for the numerator of the solution for V_1.

with the bottom of the determinant determined by the sequence in Fig. 8.79.

det([(1/3+1/4+1/2),−1/4,−1/2;−1/4,(1/4+1/2+1/5),−1/5;−1/2,−1/5,(1/5+1/2+1/1)]) (ENTER)	1.31E0

FIG. 8.79
TI-89 solution for the denominator of the equation for V_1.

Finally, the simple division in Fig. 8.80 provides the desired result.

10.51/1.31 (ENTER)	8.02

FIG. 8.80
TI-89 solution for V_1.

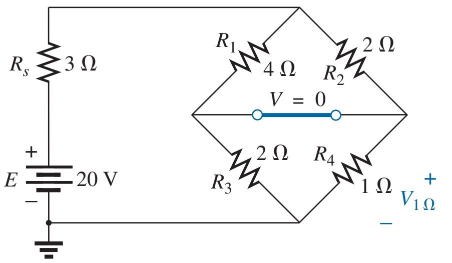

FIG. 8.81
Substituting the short-circuit equivalent for the balance arm of a balanced bridge.

and $$V_1 = \mathbf{8.02\ V}$$

Similarly, $$V_2 = \mathbf{2.67\ V} \quad \text{and} \quad V_3 = \mathbf{2.67\ V}$$

and the voltage across the 5 Ω resistor is

$$V_{5\,\Omega} = V_2 - V_3 = 2.67\ \text{A} - 2.67\ \text{A} = \mathbf{0\ V}$$

Since $V_{5\,\Omega} = 0$ V, we can insert a short in place of the bridge arm without affecting the network behavior. (Certainly $V = IR = I \cdot (0) = 0$ V.) In Fig. 8.81, a short circuit has replaced the resistor R_5, and the voltage across R_4 is to be determined. The network is redrawn in Fig. 8.82, and

$$V_{1\,\Omega} = \frac{(2\,\Omega \| 1\,\Omega)20\ \text{V}}{(2\,\Omega \| 1\,\Omega) + (4\,\Omega \| 2\,\Omega) + 3\,\Omega} \qquad \text{(voltage divider rule)}$$

$$= \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{8}{6} + 3} = \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{4}{3} + \frac{9}{3}}$$

$$= \frac{2(20\ \text{V})}{2 + 4 + 9} = \frac{40\ \text{V}}{15} = \mathbf{2.67\ V}$$

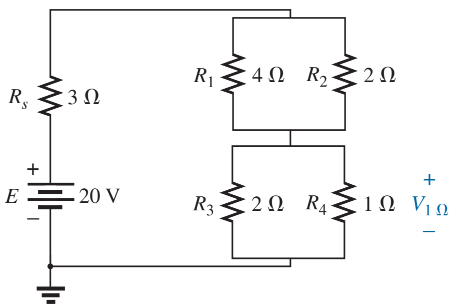

FIG. 8.82
Redrawing the network in Fig. 8.81.

as obtained earlier.

We found through mesh analysis that $I_{5\,\Omega} = 0$ A, which has as its equivalent an open circuit as shown in Fig. 8.83(a). (Certainly $I = V/R = 0/(\infty\ \Omega) = 0$ A.) The voltage across the resistor R_4 is again determined and compared with the result above.

The network is redrawn after combining series elements as shown in Fig. 8.83(b), and

$$V_{3\,\Omega} = \frac{(6\ \Omega \| 3\ \Omega)(20\ \text{V})}{6\ \Omega \| 3\ \Omega + 3\ \Omega} = \frac{2\ \Omega(20\ \text{V})}{2\ \Omega + 3\ \Omega} = 8\ \text{V}$$

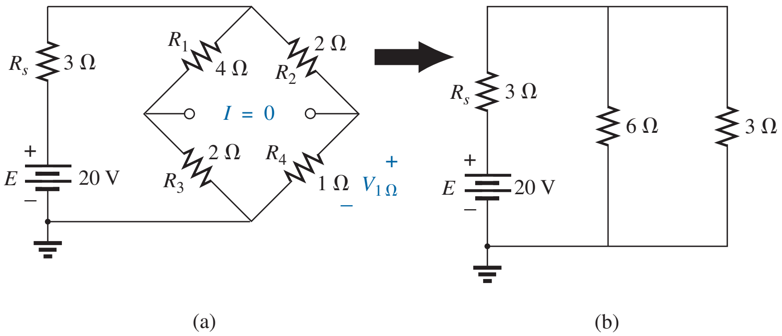

FIG. 8.83
Substituting the open-circuit equivalent for the balance arm of a balanced bridge.

and $$V_{1\,\Omega} = \frac{1\,\Omega(8\text{ V})}{1\,\Omega + 2\,\Omega} = \frac{8\text{ V}}{3} = \mathbf{2.67\ V}$$

as above.

The condition $V_{5\,\Omega} = 0$ V or $I_{5\,\Omega} = 0$ A exists only for a particular relationship between the resistors of the network. Let us now derive this relationship using the network in Fig. 8.84, in which it is indicated that $I = 0$ A and $V = 0$ V. Note that resistor R_s of the network in Fig. 8.83 does not appear in the following analysis.

The bridge network is said to be *balanced* when the condition of $I = 0$ A or $V = 0$ V exists.

If $V = 0$ V (short circuit between a and b), then

$$V_1 = V_2$$

and $$I_1R_1 = I_2R_2$$

or $$I_1 = \frac{I_2R_2}{R_1}$$

In addition, when $V = 0$ V,

$$V_3 = V_4$$

and $$I_3R_3 = I_4R_4$$

If we set $I = 0$ A, then $I_3 = I_1$ and $I_4 = I_2$, with the result that the above equation becomes

$$I_1R_3 = I_2R_4$$

Substituting for I_1 from above yields

$$\left(\frac{I_2R_2}{R_1}\right)R_3 = I_2R_4$$

or, rearranging, we have

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}} \qquad \textbf{(8.2)}$$

This conclusion states that if the ratio of R_1 to R_3 is equal to that of R_2 to R_4, the bridge is balanced, and $I = 0$ A or $V = 0$ V. A method of memorizing this form is indicated in Fig. 8.85.

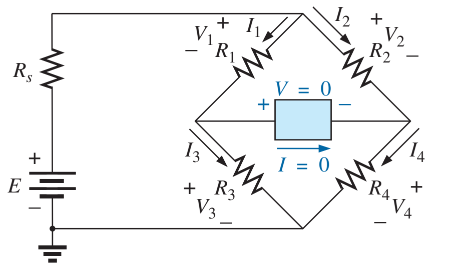

FIG. 8.84
Establishing the balance criteria for a bridge network.

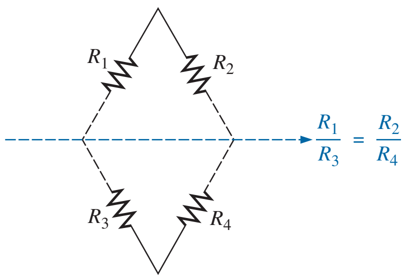

FIG. 8.85
A visual approach to remembering the balance condition.

For the example above, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 2\ \Omega$, $R_4 = 1\ \Omega$, and

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \rightarrow \frac{4\ \Omega}{2\ \Omega} = \frac{2\ \Omega}{1\ \Omega} = 2$$

The emphasis in this section has been on the balanced situation. Understand that if the ratio is not satisfied, there will be a potential drop across the balance arm and a current through it. The methods just described (mesh and nodal analysis) will yield any and all potentials or currents desired, just as they did for the balanced situation.

8.9 Y-Δ (T-π) AND Δ-Y (π-T) CONVERSIONS

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the **wye (Y)** and **delta (Δ) configurations** depicted in Fig. 8.86(a). They are also referred to as the **tee (T)** and **pi (π)**, respectively, as indicated in Fig. 8.86(b). Note that the pi is similar to an inverted delta.

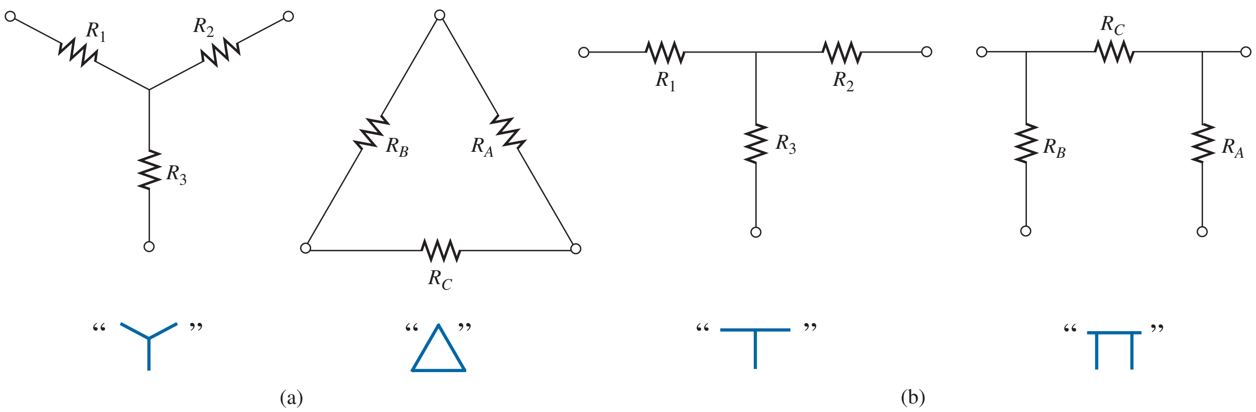

FIG. 8.86
The Y (T) and Δ(π) configurations.

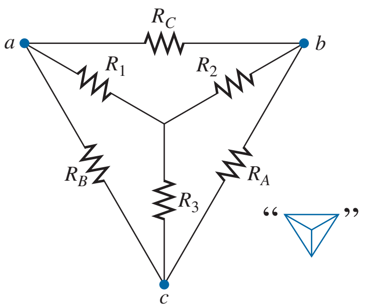

FIG. 8.87
Introducing the concept of Δ-Y or Y-Δ conversions.

The purpose of this section is to develop the equations for converting from Δ to Y, or vice versa. This type of conversion normally leads to a network that can be solved using techniques such as those described in Chapter 7. In other words, in Fig. 8.87, with terminals *a, b*, and *c* held fast, if the wye (Y) configuration were desired *instead of* the inverted delta (Δ) configuration, all that would be necessary is a direct application of the equations to be derived. The phrase *instead of* is emphasized to ensure that it is understood that only one of these configurations is to appear at one time between the indicated terminals.

It is our purpose (referring to Fig. 8.87) to find some expression for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C, and vice versa, that will ensure that the resistance between any two terminals of the Y configuration will be the same with the Δ configuration inserted in place of the Y configuration (and vice versa). If the two circuits are to be equivalent, the total resistance between any two terminals must be the same. Consider terminals *a-c* in the Δ-Y configurations in Fig. 8.88.

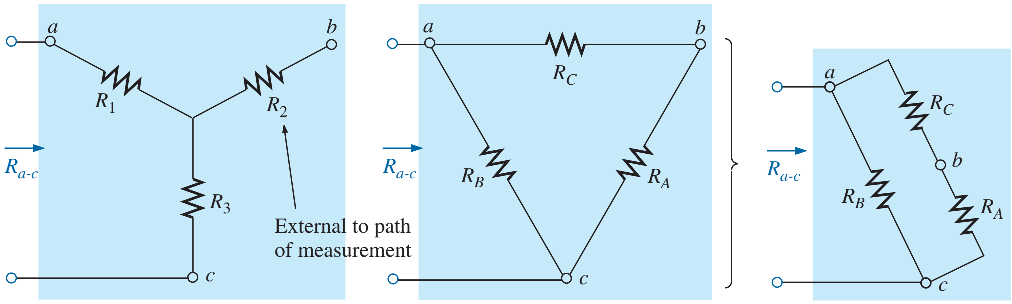

FIG. 8.88
Finding the resistance $R_{a\text{-}c}$ for the Y and Δ configurations.

Let us first assume that we want to convert the Δ (R_A, R_B, R_C) to the Y (R_1, R_2, R_3). This requires that we have a relationship for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C. If the resistance is to be the same between terminals *a-c* for both the Δ and the Y, the following must be true:

$$R_{a\text{-}c}(\text{Y}) = R_{a\text{-}c}(\Delta)$$

so that
$$R_{a\text{-}c} = R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_B + (R_A + R_C)} \tag{8.3a}$$

Using the same approach for *a-b* and *b-c*, we obtain the following relationships:

$$R_{a\text{-}b} = R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_C + (R_A + R_B)} \tag{8.3b}$$

and
$$R_{b\text{-}c} = R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + (R_B + R_C)} \tag{8.3c}$$

Subtracting Eq. (8.3a) from Eq. (8.3b), we have

$$(R_1 + R_2) - (R_1 + R_3) = \left(\frac{R_CR_B + R_CR_A}{R_A + R_B + R_C}\right) - \left(\frac{R_BR_A + R_BR_C}{R_A + R_B + R_C}\right)$$

so that
$$R_2 - R_3 = \frac{R_AR_C - R_BR_A}{R_A + R_B + R_C} \tag{8.4}$$

Subtracting Eq. (8.4) from Eq. (8.3c) yields

$$(R_2 + R_3) - (R_2 - R_3) = \left(\frac{R_AR_B + R_AR_C}{R_A + R_B + R_C}\right) - \left(\frac{R_AR_C - R_BR_A}{R_A + R_B + R_C}\right)$$

so that
$$2R_3 = \frac{2R_BR_A}{R_A + R_B + R_C}$$

resulting in the following expression for R_3 in terms of R_A, R_B, and R_C:

$$\boxed{R_3 = \frac{R_AR_B}{R_A + R_B + R_C}} \tag{8.5a}$$