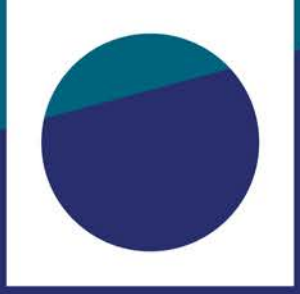

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MATHEMATICS FOR ECONOMICS AND BUSINESS

TENTH EDITION

Pearson

MATHEMATICS

FOR ECONOMICS AND BUSINESS

Solution

The raw figures shown in Table 3.8 give the impression that house prices increased steadily in value throughout this period, with a quite substantial gain during the first year. However, if inflation was very high, then the gain in real terms would have been quite small. Indeed, if the rate of inflation were to exceed the percentage rise of these nominal data, then the price of a house would actually fall in real terms. To analyse this situation we will use Table 3.9, which shows the rates of inflation during this period. Notice that since the house prices listed in Table 3.9 are quoted at the end of each year, we are not interested in the rate of inflation during Year 1.

We are told in the question to choose Year 2 as the base year and calculate the value of the house at 'Year 2 prices'. The value of the house at the end of Year 2 is obviously \$250 000 since no adjustment needs to be made. At the end of Year 3, the house is worth \$261 000. However, during that year, inflation was 7.1%. To adjust this price to 'Year 2 prices' we simply divide by the scale factor 1.071, since we are going backwards in time. We get

$$\frac{261\,000}{1.071} = 243\,697$$

In real terms the house has fallen in value by over \$6000.

To adjust the price of the house in Year 4 we first need to divide by 1.035 to backtrack to Year 3, and then divide again by 1.071 to reach Year 2. We get

$$\frac{281\,000}{1.035 \times 1.071} = 253\,499$$

In real terms there has at least been a reasonable gain during Year 4.

For the Year 5 price, the adjusted value is

$$\frac{297\,000}{1.023 \times 1.035 \times 1.071} = 261\,909$$

and, for Year 1, the adjusted value is

$$202\,000 \times 1.107 = 223\,614$$

going forward in time, so multiply

Table 3.10 lists both the nominal and the 'constant Year 2' values of the house (rounded to the nearest thousand) for comparison. It shows quite clearly that, apart from the gain during Year 2, the increase in value is not as spectacular as the nominal prices suggest.

Table 3.10

	Year				
	1	2	3	4	5
Nominal house price (\$000)	202	250	261	281	297
Year 2 house price (\$000)	224	250	244	253	262

Practice Problem

8. Table 3.11 shows the average annual salary (in thousands of dollars) of employees in a small firm, together with the annual rate of inflation for that year. Adjust these salaries to the prices prevailing at the end of Year 2 and so give the real values of the employees' salaries at constant 'Year 2 prices'. Comment on the rise in earnings during this period.

Table 3.11

	Year				
	1	**2**	**3**	**4**	**5**
Salary ($000)	34.6	36.2	39.6	47.0	52.0
Inflation (%)		4.9	4.3	4.0	3.5

Key Terms

Index number The scale factor of a variable measured from the base year multiplied by 100.

Inflation The percentage increase in the level of prices over a 12-month period.

Laspeyres index An index number for groups of data which are weighted by the quantities used in the base year.

Nominal data Monetary values prevailing at the time that they were measured.

Paasche index An index number for groups of data which are weighted by the quantities used in the current year.

Real data Monetary values adjusted to take inflation into account.

Scale factor The multiplier that gives the final value in percentage problems.

Time series A sequence of numbers indicating the variation of data over time.

Exercise 3.1

1. Express the following percentages as fractions in their simplest form:

(a) 35% **(b)** 88% **(c)** 250% **(d)** $17\frac{1}{2}\%$ **(e)** 0.2%

2. Calculate each of the following:

(a) 5% of 24 **(b)** 8% of 88 **(c)** 48% of 4563 **(d)** 112% of 56

3. A firm has 132 full-time and 88 part-time employees.

(a) What percentage of staff are full-time?

(b) During the next year, 8 additional full-time staff are employed. If the percentage of full-time staff is now 56%, how many additional part-time staff were recruited during the year?

4. Write down the scale factors corresponding to

(a) an increase of 19%;

(b) an increase of 250%;

(c) a decrease of 2%;

(d) a decrease of 43%.

5. Write down the percentage changes corresponding to the following scale factors:

(a) 1.04 **(b)** 1.42 **(c)** 0.86

(d) 3.45 **(e)** 1.0025 **(f)** 0.04

6. Find the new quantities when

(a) \$16.25 is increased by 12%;

(b) the population of a town, currently at 113 566, rises by 5%;

(c) a good priced by a firm at \$87.90 is subject to a sales tax of 20%;

(d) a good priced at \$2300 is reduced by 30% in a sale;

(e) a car, valued at \$23 000, depreciates by 32%.

7. A student discount card reduces a bill in a restaurant from \$124 to \$80.60. Work out the percentage discount.

8. A shop sells books at '20% below the recommended retail price (r.r.p.)'. If it sells a book for \$12.40, find

(a) the r.r.p.;

(b) the cost of the book after a further reduction of 15% in a sale;

(c) the overall percentage discount obtained by buying the book from the shop in the sale compared with the manufacturer's r.r.p.

9. A TV costs \$900 including 20% sales tax. Find the new price if tax is reduced to 15%.

10. An antiques dealer tries to sell a vase at 45% above the \$18 000 which the dealer paid at auction.

(a) What is the new sale price?

(b) By what percentage can the dealer now reduce the price before making a loss?

11. Find the single percentage increase or decrease equivalent to

(a) a 10% increase followed by a 25% increase;

(b) a 34% decrease followed by a 65% increase;

(c) a 25% increase followed by a 25% decrease.

Explain in words why the overall change in part (c) is not 0%.

12. Table 3.12 gives the annual rate of inflation during a 5-year period.

Table 3.12

	Year				
	1	2	3	4	5
Annual rate of inflation (%)	1.8	2.1	2.9	2.4	2.7

If a nominal house price at the end of Year 1 was \$10.8 million, find the real house price adjusted to prices prevailing at the end of Year 4. Round your answer to three significant figures.

13. The price of a good during the last 5 years is:

\$25 \$30 \$36 \$43 \$50

Calculate the index numbers using the last year as the base year and hence comment on the rise in prices during this period.

14. Table 3.13 shows the monthly index of sales of a good during the first four months of the year.

Table 3.13

	Jan	Feb	Mar	Apr
Index	100	120	145	150

(a) Which month is chosen as the base year?

(b) If the sales in February are 3840, what are the sales in April?

(c) What is the index number in May if sales are 4256?

15. Table 3.14 shows the index numbers associated with transport costs during a 20-year period. The public transport costs reflect changes to bus and train fares, whereas private transport costs include purchase, service, petrol, tax and insurance costs of cars.

Table 3.14

	Year				
	2000	2005	2010	2015	2020
Public transport	100	130	198	224	245
Private transport	100	125	180	199	221

(1) Which year is chosen as the base year?

(2) Find the percentage increases in the cost of public transport from

(a) 2000 to 2005 **(b)** 2005 to 2010 **(c)** 2010 to 2015 **(d)** 2015 to 2020

(3) Repeat part (2) for private transport.

(4) Comment briefly on the relative rise in public and private transport costs during this 20-year period.

16. Table 3.15 shows the number of items (in thousands) produced from a factory production line during the course of a year. Taking the second quarter as the base quarter, calculate the associated index numbers. Suggest a possible reason for the fluctuations in output.

Table 3.15

	Quarter			
	Q1	Q2	Q3	Q4
Output	13.5	1.4	2.5	10.5

17. Table 3.16 shows the prices of a good for each year between Year 1 and Year 6.

Table 3.16

Year	1	2	3	4	5	6
Price ($)	40	48	44	56	60	71

(a) Work out the index numbers, correct to one decimal place, taking Year 2 as the base year.

(b) If the index number for Year 7 is 135, calculate the corresponding price. You may assume that the base year is still Year 2.

(c) If the index number in Year 3 is approximately 73, find the year that is used as the base year.

Exercise 3.1*

1. Total revenue from daily ticket sales to a theme park is $1 352 400. A total of 12 000 tickets were sold and 65% of these were child's tickets with a 30% discount off the adult price. Work out the cost of an adult ticket.

2. The cost of a computer is $6000 including 20% sales tax. In a generous gesture, the government decides to reduce the rate to just 17.5%. Find the cost of the computer after the tax has changed.

3. A coat originally costing $150 is reduced by 25% in a sale and, since nobody bought the coat, a further reduction of 20% of the sale price is applied.

(a) Find the final cost of the coat after both reductions.

(b) Find the overall percentage reduction and explain why this is not the same as a single reduction of 45%.

4. A furniture store has a sale of 40% off on selected items.
A sales assistant, Carol, reduces the price of a sofa originally costing $1200.

(a) What is the new price?
The manager does not want this sofa to be in the sale and the following day tells another sales assistant, Michael, to restore the sofa back to the original price. He does not know what the original price was and decides to show off his mathematical knowledge by taking the answer to part (a) and multiplying it by 1.4.

(b) Explain carefully why this does not give the correct answer of $1200.

(c) Suggest an alternative calculation that would give the right answer.

5. During 2022 the price of a good increased by 8%. In the sales on 1 January 2023 all items are reduced by 25%.

(a) If the sale price of the good is $688.50, find the original price at the beginning of 2022.

(b) Find the overall percentage change.

(c) What percentage increase would be needed to restore the cost to the original price prevailing on 1 January 2022? Give your answer to one decimal place.

6. Table 3.17 shows government expenditure (in billions of dollars) on education for four consecutive years, together with the rate of inflation for each year.

(a) Taking Year 1 as the base year, work out the index numbers of the nominal data given in the third row of the table.

(b) Find the values of expenditure at constant Year 1 prices and hence recalculate the index numbers of real government expenditure.

(c) Give an interpretation of the index numbers calculated in part (b).

Table 3.17

	Year			
	1	2	3	4
Spending ($bn)	236	240	267	276
Inflation (%)		4.7	4.2	3.4

7. Index numbers associated with the growth of unemployment during an 8-year period are shown in Table 3.18.

(a) What are the base years for the two indices?

(b) If the government had not switched to index 2, what would be the values of index 1 in Years 7 and 8?

(c) What values would index 2 have been in Years 1, 2, 3, 4 and 5?

(d) If unemployment was 1.2 million in Year 4, how many people were unemployed in Years 1 and 8?

Table 3.18

	Year							
	1	2	3	4	5	6	7	8
Index 1	100	95	105	110	119	127		
Index 2						100	112	118

8. The prices of a good at the end of each year between Year 1 and Year 6 are listed in Table 3.19, which also shows the annual rate of inflation.

Table 3.19

	Year					
	1	2	3	4	5	6
Price ($)	230	242	251	257	270	284
Inflation (%)		4	3	2.5	2	2

(a) Find the values of the prices adjusted to the end of Year 2, correct to two decimal places. Hence, calculate the index numbers of the real data with Year 2 as the base year. Give your answers correct to one decimal place.

(b) If the index number of the real price for Year 7 is 109 and the rate of inflation for that year is 2.5%, work out the nominal value of the price in Year 7. Give your answer rounded to the nearest whole number.

(c) If the index number of the real data in Year 0 is 95.6 and the nominal price is $215, find the rate of inflation for Year 1. Give your answer correct to one decimal place.