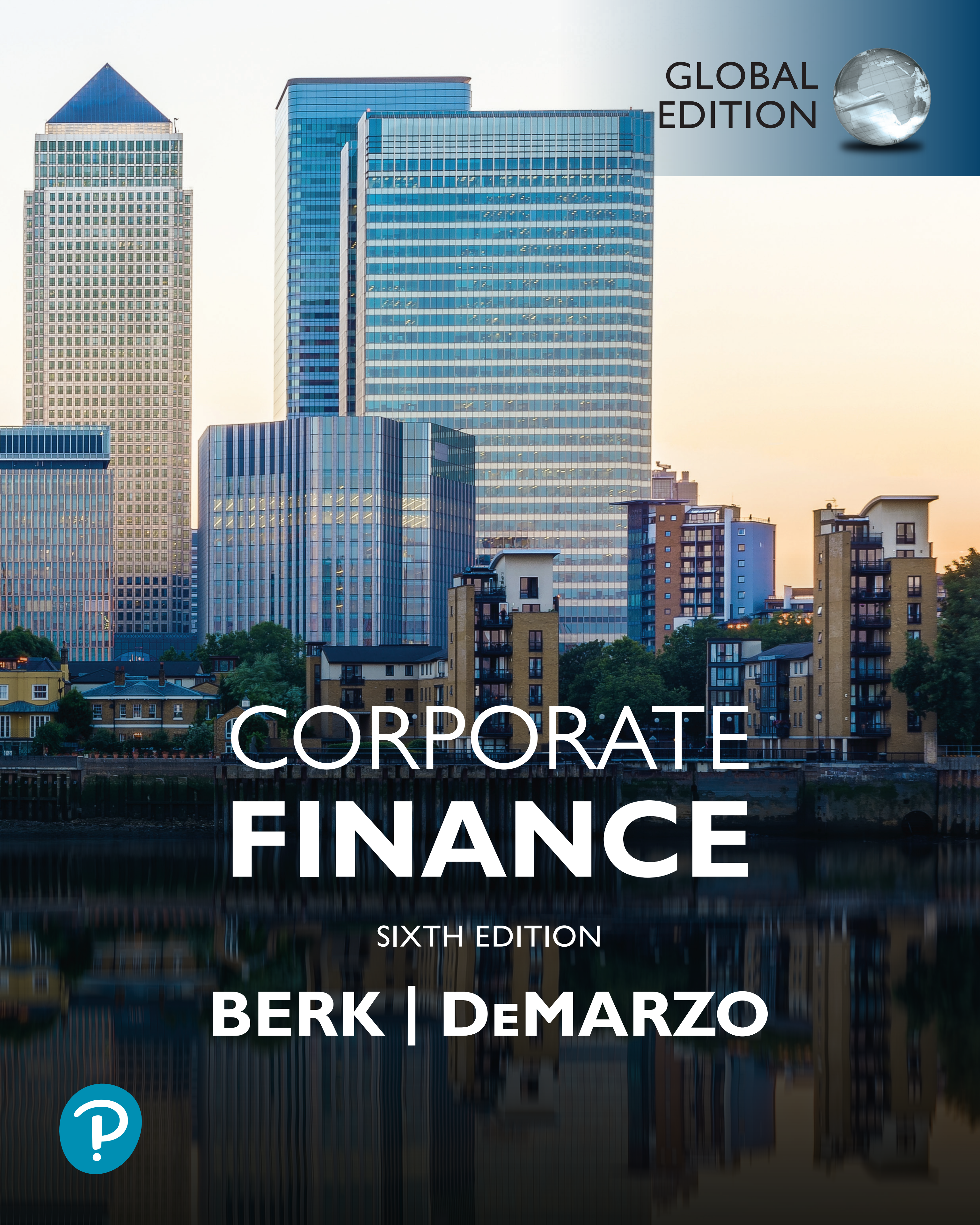

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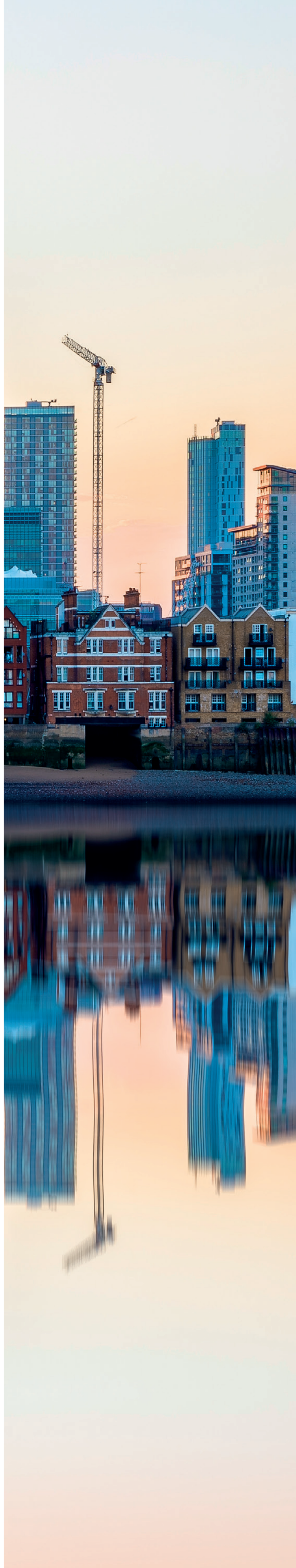

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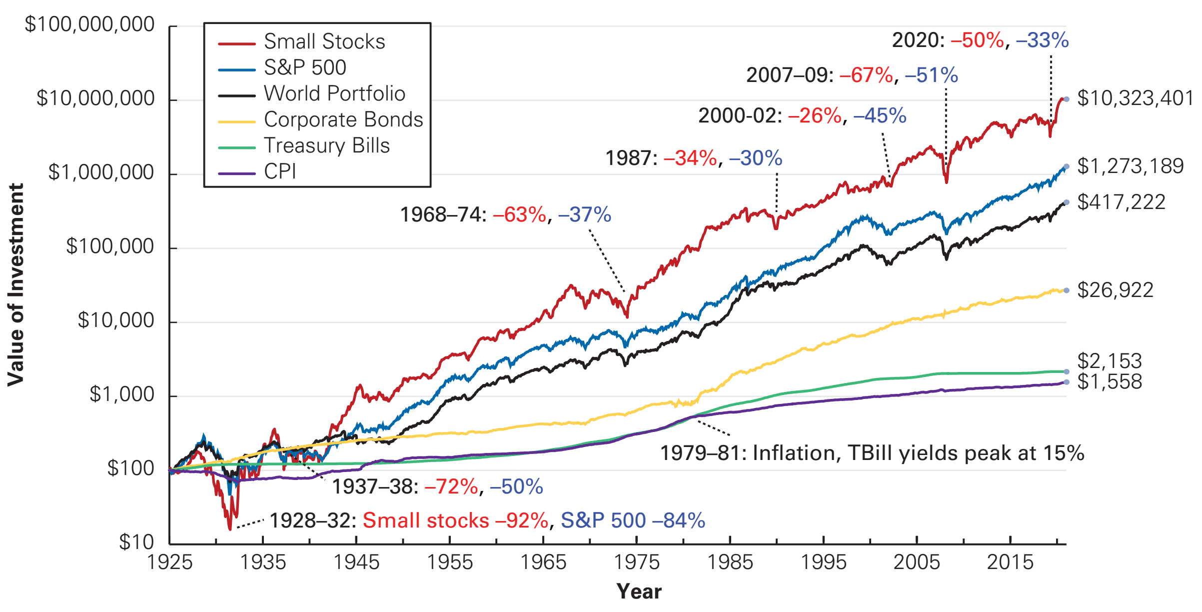
FIGURE 10.1 **Value of $100 Invested in 1925 in Stocks, Bonds, or Bills**



The chart shows the growth in value of $100 invested in 1925 if it were invested in U.S. large stocks, small stocks, world stocks, corporate bonds, or Treasury bills, with the level of the consumer price index (CPI) shown as a reference. Returns were calculated at year-end assuming all dividends and interest are reinvested and excluding transactions costs. Note that while stocks have generally outperformed bonds and bills, they have also endured periods of significant losses (numbers shown represent peak to trough decline, with the decline in small stocks in red and the S&P 500 in blue).

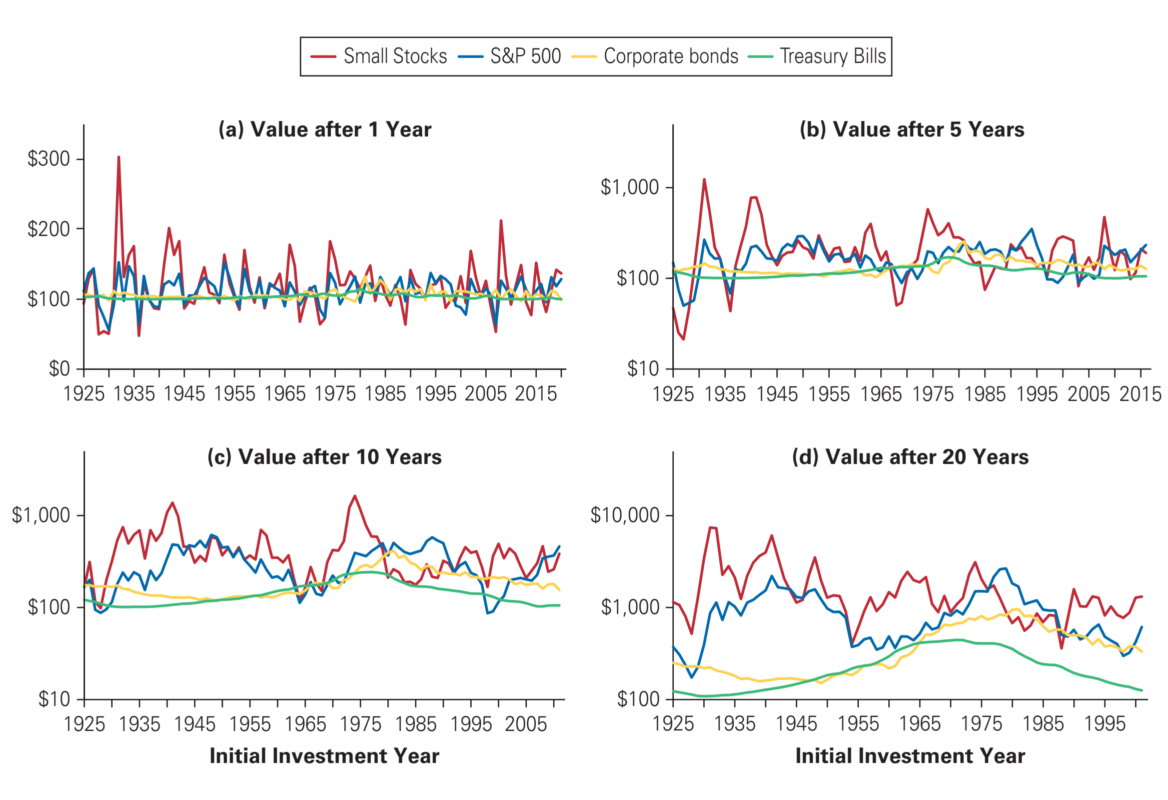
Source: Chicago Center for Research in Security Prices, Standard and Poor's, MSCI, and Global Financial Data

precisely when the value of their savings eroded. Thus, while the stock portfolios had the best performance over this 96-year period, that performance came at a cost—the risk of large losses in a downturn. On the other hand, Treasury bills enjoyed steady—albeit modest—gains each year.

Few people ever make an investment for 96 years, as depicted in Figure 10.1. To gain additional perspective on the risk and return of these investments, Figure 10.2 shows the results for more realistic investment horizons and different initial investment dates. Panel (a), for example, shows the value of each investment after one year and illustrates that if we rank the investments by the volatility of their annual increases and decreases in value, we obtain the same ranking we observed with regard to performance: Small stocks had the most variable returns, followed by the S&P 500, the world portfolio, corporate bonds, and finally Treasury bills.

Panels (b), (c), and (d) of Figure 10.2 show the results for 5-, 10-, and 20-year investment horizons, respectively. Note that as the horizon lengthens, the relative performance of the stock portfolios improves. That said, even with a 10-year horizon there were periods during which stocks underperformed Treasuries. And while investors in small stocks most often came out ahead, this was not assured even with a 20-year horizon: For investors in

FIGURE 10.2 Value of $100 Invested in Alternative Assets for Differing Horizons



Each panel shows the result of investing $100, in each investment opportunity, for horizons of 1, 5, 10, or 20 years, plotted as a function of the year when the investment was initially made. Dividends and interest are reinvested and transaction costs are excluded. Note that small stocks show the greatest variation in performance at the one-year horizon, followed by large stocks and then corporate bonds. For longer horizons, the relative performance of stocks improved, but they remained riskier.

Source Data: Chicago Center for Research in Security Prices, Standard and Poor's, MSCI, and Global Financial Data

the early 1980s, small stocks did worse than both the S&P 500 *and* corporate bonds over the subsequent 20 years. Finally, stock investors with long potential horizons might find themselves in need of cash in intervening years, and be forced to liquidate at a loss relative to safer alternatives.

In Chapter 3, we explained why investors are averse to fluctuations in the value of their investments, and that investments that are more likely to suffer losses in downturns must compensate investors for this risk with higher expected returns. Figures 10.1 and 10.2 provide compelling historical evidence of this relationship between risk and return, just as we should expect in an efficient market. Given this clear evidence that investors do not like risk and thus demand a risk premium to bear it, our goal in this chapter is to quantify this relationship. We want to explain *how much* investors demand (in terms of a higher expected return) to bear a given level of risk. To do so, we must first develop tools that will allow us to measure risk and return. That is our objective in the next section.

CONCEPT CHECK

1. For an investment horizon from 1926 to 2021, which of the following investments had the highest return: the S&P 500, small stocks, world portfolio, corporate bonds, or Treasury bills? Which had the lowest return?
2. For an investment horizon of just one year, which of these investments was the most variable? Which was the least variable?

10.2 Common Measures of Risk and Return

When a manager makes an investment decision or an investor purchases a security, they have some view as to the risk involved and the likely return the investment will earn. Thus, we begin our discussion by reviewing the standard ways to define and measure risks.

Probability Distributions

Different securities have different initial prices, pay different cash flows, and sell for different future amounts. To make them comparable, we express their performance in terms of their returns. The return indicates the percentage increase in the value of an investment per dollar initially invested in the security. When an investment is risky, there are different returns it may earn. Each possible return has some likelihood of occurring. We summarize this information with a **probability distribution**, which assigns a probability, p_R, that each possible return, R, will occur.

Let's consider a simple example. Suppose BFI stock currently trades for $100 per share. You believe that in one year there is a 25% chance the share price will be $140, a 50% chance it will be $110, and a 25% chance it will be $80. BFI pays no dividends, so these payoffs correspond to returns of 40%, 10%, and −20%, respectively. Table 10.1 summarizes the probability distribution for BFI's returns.

We can also represent the probability distribution with a histogram, as shown in Figure 10.3.

Expected Return

Given the probability distribution of returns, we can compute the expected return. We calculate the **expected (or mean) return** as a weighted average of the possible returns, where the weights correspond to the probabilities.[3]

Expected (Mean) Return

$$\text{Expected Return} = E[R] = \sum_R p_R \times R \tag{10.1}$$

TABLE 10.1 Probability Distribution of Returns for BFI

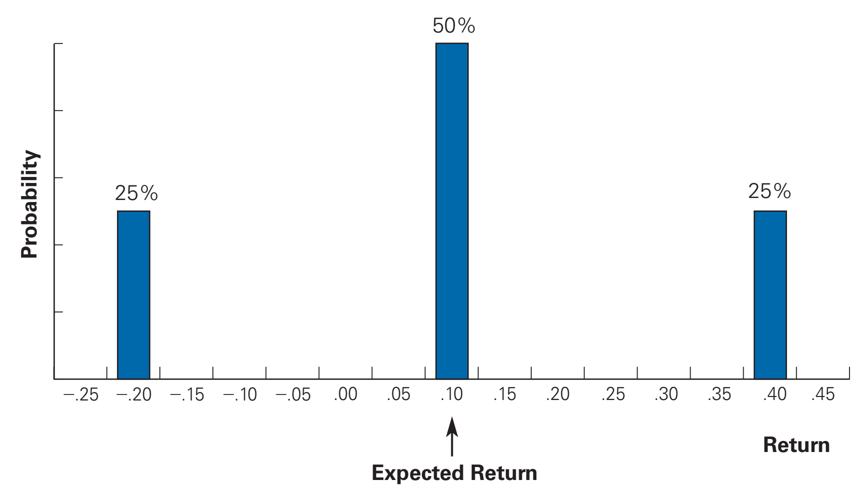
		Probability Distribution	
Current Stock Price ($)	**Stock Price in One Year ($)**	**Return, R**	**Probability, p_R**
	140	0.40	25%
100	110	0.10	50%
	80	−0.20	25%

[3] The notation Σ_R means that we calculate the sum of the expression (in this case, $p_R \times R$) over all possible returns R.

FIGURE 10.3

Probability Distribution of Returns for BFI

The height of a bar in the histogram indicates the likelihood of the associated outcome.



The expected return is the return we would earn on average if we could repeat the investment many times, drawing the return from the same distribution each time. In terms of the histogram, the expected return is the "balancing point" of the distribution, if we think of the probabilities as weights. The expected return for BFI is

$$E[R_{BFI}] = 25\%(-0.20) + 50\%(0.10) + 25\%(0.40) = 10\%$$

This expected return corresponds to the balancing point in Figure 10.3.

Variance and Standard Deviation

Two common measures of the risk of a probability distribution are its *variance* and *standard deviation*. The **variance** is the expected squared deviation from the mean, and the **standard deviation** is the square root of the variance.

Variance and Standard Deviation of the Return Distribution

$$Var(R) = E\left[(R - E[R])^2\right] = \sum_R p_R \times (R - E[R])^2$$
$$SD(R) = \sqrt{Var(R)} \tag{10.2}$$

If the return is risk-free and never deviates from its mean, the variance is zero. Otherwise, the variance increases with the magnitude of the deviations from the mean. Therefore, the variance is a measure of how "spread out" the distribution of the return is. The variance of BFI's return is

$$Var(R_{BFI}) = 25\% \times (-0.20 - 0.10)^2 + 50\% \times (0.10 - 0.10)^2 + 25\% \times (0.40 - 0.10)^2$$
$$= 0.045$$

The standard deviation of the return is the square root of the variance, so for BFI,

$$SD(R) = \sqrt{Var(R)} = \sqrt{0.045} = 21.2\% \tag{10.3}$$

In finance, we refer to the standard deviation of a return as its **volatility**. While the variance and the standard deviation both measure the variability of the returns, the standard deviation is easier to interpret because it is in the same units as the returns themselves.[4]

EXAMPLE 10.1 Calculating the Expected Return and Volatility

Problem

Suppose AMC stock is equally likely to have a 45% return or a −25% return. What are its expected return and volatility?

Solution

First, we calculate the expected return by taking the probability-weighted average of the possible returns:

$$E[R] = \sum_R p_R \times R = 50\% \times 0.45 + 50\% \times (-0.25) = 10.0\%$$

To compute the volatility, we first determine the variance:

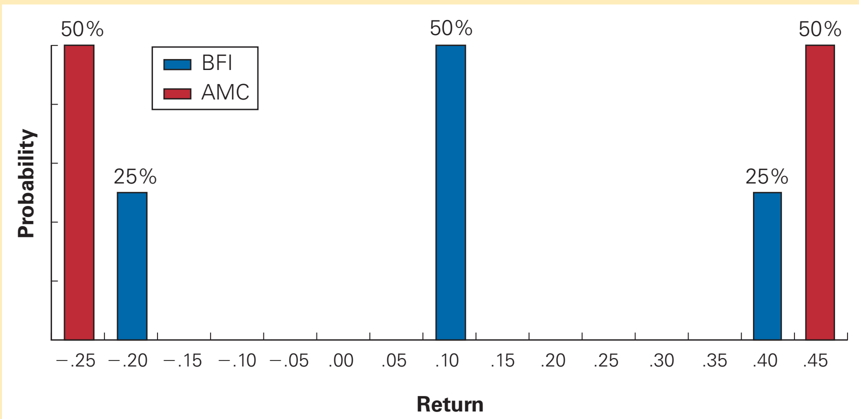
$$Var(R) = \sum_R p_R \times (R - E[R])^2 = 50\% \times (0.45 - 0.10)^2 + 50\% \times (-0.25 - 0.10)^2$$
$$= 0.1225$$

Then, the volatility or standard deviation is the square root of the variance:

$$SD(R) = \sqrt{Var(R)} = \sqrt{0.1225} = 35\%$$

Note that both AMC and BFI have the same expected return, 10%. However, the returns for AMC are more spread out than those for BFI—the high returns are higher and the low returns are lower, as shown by the histogram in Figure 10.4. As a result, AMC has a higher variance and volatility than BFI.

FIGURE 10.4



Probability Distribution for BFI and AMC Returns While both stocks have the same expected return, AMC's return has a higher variance and standard deviation.

[4] While variance and standard deviation are the most common measures of risk, they do not differentiate upside and downside risk. Alternative measures that focus on downside risk include the semivariance (variance of the losses only) and the expected tail loss (the expected loss in the worst x% of outcomes). Because they often produce the same ranking (as in Example 10.1, or if returns are normally distributed) but are more complicated to apply, these alternative measures tend to be used only in special applications.

If we could observe the probability distributions that investors anticipate for different securities, we could compute their expected returns and volatilities and explore the relationship between them. Of course, in most situations we do not know the explicit probability distribution, as we did for BFI. Without that information, how can we estimate and compare risk and return? A popular approach is to extrapolate from historical data, which is a sensible strategy if we are in a stable environment and believe that the distribution of future returns should mirror that of past returns. Let's look at the historical returns of stocks and bonds, to see what they reveal about the relationship between risk and return.

CONCEPT CHECK

1. How do we calculate the expected return of a stock?
2. What are the two most common measures of risk, and how are they related to each other?

10.3 Historical Returns of Stocks and Bonds

In this section, we explain how to compute average returns and volatilities using historical stock market data. The distribution of past returns can be helpful when we seek to estimate the distribution of returns investors may expect in the future. We begin by first explaining how to compute historical returns.

Computing Historical Returns

Of all the possible returns, the **realized return** is the return that actually occurs over a particular time period. How do we measure the realized return for a stock? Suppose you invest in a stock on date t for price P_t. If the stock pays a dividend, Div_{t+1}, on date $t+1$, and you sell the stock at that time for price P_{t+1}, then the realized return from your investment in the stock from t to $t+1$ is

$$\begin{aligned} R_{t+1} &= \frac{Div_{t+1} + P_{t+1}}{P_t} - 1 = \frac{Div_{i+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t} \\ &= \text{Dividend Yield} + \text{Capital Gain Rate} \end{aligned} \quad (10.4)$$

That is, as we discussed in Chapter 9, the realized return, R_{t+1}, is the total return we earn from dividends and capital gains, expressed as a percentage of the initial stock price.[5]

Calculating Realized Annual Returns. If you hold the stock beyond the date of the first dividend, then to compute your return you must specify how you invest any dividends you receive in the interim. To focus on the returns of a single security, let's assume that *you reinvest all dividends immediately and use them to purchase additional shares of the same stock or security*. In this case, we can use Eq. 10.4 to compute the stock's return between dividend payments, and then compound the returns from each dividend interval to compute the return over a longer horizon. For example, if a stock pays dividends at the end of each quarter, with realized returns $R_{Q1}, \ldots, R_{Q4}$ each quarter, then its annual realized return, R_{annual}, is

$$1 + R_{\text{annual}} = (1 + R_{Q1})(1 + R_{Q2})(1 + R_{Q3})(1 + R_{Q4}) \quad (10.5)$$

[5] We can compute the realized return for any security in the same way, by replacing the dividend payments with any cash flows paid by the security (e.g., with a bond, coupon payments would replace dividends).