GLOBAL EDITION

BUSINESS STATISTICS

A Decision-Making Approach

ELEVENTH EDITION

David F. Groebner
Patrick W. Shannon
Phillip C. Fry

BUSINESS
Statistics

A Decision-Making Approach

6-44. A car salesperson states that on an average two customers will buy a car from her every month. Assume that the probability distribution of the number of months a customer will buy a car from her is exponentially distributed.
   a. What is the probability that a customer will buy a car from her in less than 3 months?
   b. What is the probability that a customer will buy a car from her in more than 2 months?
   c. What is the probability that a customer will buy a car from her between 2 and 2.5 months?

6-45. A California water company has determined that the average customer billing is $1,250 per year and the amounts billed have an exponential distribution.
   a. Calculate the probability that a randomly chosen customer would spend more than $5,000.
   b. Compute the probability that a randomly chosen customer would spend more than the average amount spent by all customers of this company.

6-46. A customer spent half an hour to travel to an automotive service center. The duration of waiting in a particular automotive service center is uniformly distributed between 40 minutes and 3 hours.
   a. Determine the density function for the waiting time in the particular automotive service center.
   b. Find the mean and standard deviation for the waiting time in the particular automotive service center.
   c. What is the probability that the waiting time in the service center for a particular customer is more than 1.5 hours?
   d. What is the probability that the waiting time in the service center for a particular customer is between 1 and 2.5 hours?

6-47. Currently, a manufacturing company experiences an average of 0.75 equipment-related shutdown per shift and the downtimes are exponentially distributed. In testing, a new maintenance program reduced the frequency of equipment-related shutdowns to an average of 0.20 per shift, exponentially distributed. The new maintenance program is expensive, but the company will install it if it can help achieve the target of four consecutive shifts without an equipment-related shutdown.
   a. Under the current system, what is the probability that the plant would run four or more consecutive shifts without an equipment-related shutdown?
   b. Using the computer decision model, what is the probability that the plant could run four or more consecutive shifts without an equipment-related

shutdown? Has the decision model helped the company achieve its goal?
   c. What would be the maximum average number of shutdowns allowed per day such that the probability of experiencing four or more consecutive shifts without an equipment-related shutdown is greater than or equal to 0.70?

Computer Software Exercises

6-48. The Schoonover Manufacturing Company monitors the downtimes on its key fabricating machines. Data for the downtimes on one particular machine are in the file called **Schoonover**. Assuming that the downtimes are exponentially distributed, use the data in the file to find the probability that a specific downtime will exceed 1.0 minute.

6-49. As part of their human development-focused activities, UNDP designed the Human Development Index (HDI). That index measures how well males and females in all UN countries can use their capabilities to develop. The file **UNDP Female Income** contains one relevant variable: female income per capita data in 2021 in 178 countries.
   a. Calculate the mean and standard deviation of the income data.
   b. Assume that the female incomes are exponentially distributed with the same mean as that of the sample. Determine the probability that female income exceeds $10,000 in a randomly chosen country.

6-50. The San Luis Obispo, California, Transit Program provides daily fixed-route transit service to the general public within the city limits and to Cal Poly State University's staff and students. The most heavily traveled route schedules a city bus to arrive at Cal Poly at 8:54 A.M. The file titled **Late** lists plausible differences between the actual and scheduled times of arrival rounded to the nearest minute for this route.
   a. Produce a relative frequency histogram for these data. Does it seem plausible the data came from a population that has a uniform distribution?
   b. Provide the density for this uniform distribution.
   c. Classes start 10 minutes after the hour and classes are a 5-minute walk from the drop-off point. Determine the probability that a randomly chosen bus on this route would cause the students on board to be late for class. Assume the differences form a continuous uniform distribution with a range the same as the sample.
   d. Determine the median difference between the actual and scheduled arrival times.

6 Overview

Summary

6.1

The Normal Probability Distribution (pg. 239–252)

OUTCOME 1 Convert a normal distribution to a standard normal distribution.

OUTCOME 2 Determine probabilities using the standard normal distribution.

OUTCOME 3 Calculate values of the random variable associated with specified probabilities from a normal distribution.

- The **normal distribution** is a symmetric, bell-shaped probability distribution. Half the probability lies to the right and half lies to the left of the mean.
- To find probabilities associated with a normal distribution, we convert to a **standard normal distribution** by first converting values of the random variables to **standardized z-values**. The probabilities associated with a range of values for the random variable are found using the normal distribution table in Appendix D or using Excel.

6.2

Other Continuous Probability Distributions (pg. 252–258)

OUTCOME 4 Calculate probabilities associated with a uniformly distributed random variable.

OUTCOME 5 Determine probabilities using an exponential probability distribution.

- Although the normal distribution is by far the most frequently used continuous probability distribution, two other continuous distributions are the uniform distribution and the exponential distribution.
- With the **uniform distribution**, the probability over any interval is the same as the probability over any other interval of the same width. We can compute the probabilities for the uniform distribution using Equation 6.3.
- The **exponential distribution** is based on a single parameter, λ, and is often used to describe random service times or the time between customer arrivals in waiting-line applications. We can compute the probability over a range of values for an exponential distribution using either Equation 6.7 or the exponential table in Appendix E. Excel also has a function for calculating exponential probabilities.
- Figure 6.15 summarizes the discrete probability distributions introduced in Chapter 5 and the continuous probability distributions introduced in this chapter.

FIGURE 6.15 Probability Distribution Summary

259
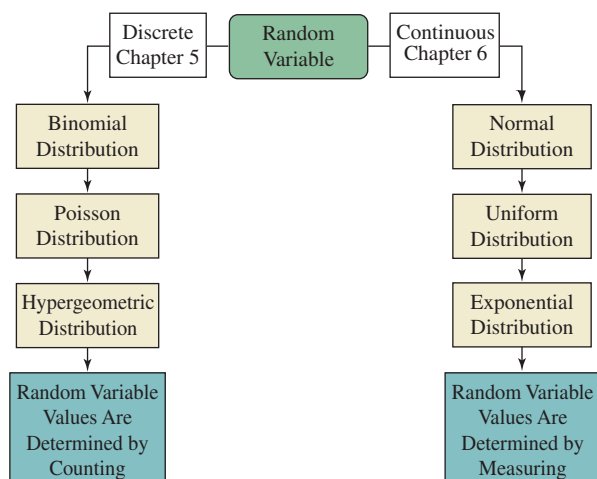
Equations

(6.1) Normal Probability Density Function pg. 240

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

(6.2) Standardized Normal z-Value pg. 241

$$z = \frac{x - \mu}{\sigma}$$

(6.3) Continuous Uniform Density Function pg. 253

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

(6.4) Mean of a Uniform Distribution pg. 254

$$E(x) = \mu = \frac{a+b}{2}$$

(6.5) Standard Deviation of a Uniform Distribution pg. 254

$$\sigma = \sqrt{\frac{(b-a)^2}{12}}$$

(6.6) Exponential Density Function pg. 255

$$f(x) = \lambda e^{-\lambda x}, \quad x \ge 0$$

(6.7) Exponential Probability pg. 255

$$P(0 \le x \le a) = 1 - e^{-\lambda a}$$

Key Terms

Exponential distribution pg. 254
Normal distribution pg. 239
Standard normal distribution pg. 240

Standardized z-value pg. 241
Uniform distribution pg. 252

Chapter Exercises

Conceptual Questions

6-51. Discuss the difference between discrete and continuous probability distributions. Discuss two situations in which a variable of interest may be considered either continuous or discrete.

6-52. Recall the Empirical Rule from Chapter 3. It states that if the data distribution is bell-shaped, then the interval $\mu \pm \sigma$ contains approximately 68% of the values, $\mu \pm 2\sigma$ contains approximately 95%, and $\mu \pm 3\sigma$ contains virtually all of the data values. The bell-shaped distribution referenced is the normal distribution.
  a. Verify that a standard normal distribution contains approximately 68% of the values in the interval $\mu \pm \sigma$.
  b. Verify that a standard normal distribution contains approximately 95% of the values in the interval $\mu \pm 2\sigma$.
  c. Verify that a standard normal distribution contains virtually all of the data in the interval $\mu \pm 3\sigma$.

6-53. The probability that a value from a normally distributed random variable will exceed the mean is 0.50. The same is true for the uniform distribution. Why is this not necessarily true for the exponential distribution? Discuss and show examples to illustrate your point.

6-54. Suppose you tell one of your fellow students that when working with a continuous distribution, it does not make sense to try to compute the probability of any specific value, since it will be zero. She says that when the experiment is performed, some value must occur; the probability can't be zero. Your task is to respond to her statement and, in doing so, explain why it is appropriate to find the probability for specific ranges of values for a continuous distribution.

6-55. The exponential distribution has a characteristic that is called the "memoryless" property. This means $P(X > x) = P(X > x + x_0 | X > x_0)$. To illustrate this, consider the calls coming into 911. Suppose that the distribution of the time between occurrences has an exponential distribution with a mean of one half hour $(=0.5)$.
  a. Calculate the probability that no calls come in during the first hour.
  b. Now suppose that you are monitoring the call frequency, and you note that a call does not come in during the first two hours. Determine the probability that no calls will come in during the next hour.

6-56. Revisit Exercise 6-55, but examine whether it would matter when you started monitoring the 911 calls if the time between occurrences had a uniform distribution with a mean of 2 and a range of 4.
  a. Calculate the probability that no call comes in during the first hour.
  b. Now suppose that you are monitoring the call frequency, and you note that no call comes in during the first two hours. Determine the probability that no calls will arrive during the next hour.

6-57. Suppose that, on average, 20 customers arrive every hour at a 24-hour coffee shop. Assume that the time between customer arrivals is exponentially distributed. Determine

a. the probability that a customer arrives within the next 2 minutes.
b. the probability that the time between two arriving customers will be between 1 and 4 minutes.
c. the probability that 5 or more minutes will pass between customer arrivals.

6-58. Suppose the time required to receive confirmation that an electronic bank transfer has occurred is uniformly distributed between 20 and 70 seconds.
a. What is the probability that a randomly selected bank transfer confirmation will take between 25 and 40 seconds?
b. What is the probability that a randomly selected bank transfer confirmation will take between 35 and 60 seconds?
c. What proportion of bank transfer confirmations will require between 30 and 40 seconds?

Business Applications

6-59. Drive-through services are very popular and convenient to consumers today. It provides a business that allows customers to purchase products without leaving their cars. McDonald's has customers arriving at the drive-through window at an average rate of three customers per minute and that their arrivals follow the Poisson model. Use the appropriate exponential distribution to find the probability that the next customer will arrive within 1.5 minutes.

6-60. The Four Brothers Lumber Company is considering buying a machine that planes lumber to the correct thickness. The machine is advertised to produce "6-inch lumber" having a thickness that is normally distributed, with a mean of 6 inches and a standard deviation of 0.1 inch.
a. If building standards in the industry require a 99% chance of a board being between 5.85 and 6.15 inches, should Four Brothers purchase this machine? Why or why not?
b. To what level would the company that manufactures the machine have to reduce the standard deviation for the machine to conform to industry standards?

6-61. Two automatic dispensing machines are being considered for use in a fast-food chain. The first dispenses an amount of liquid that has a normal distribution with a mean of 11.9 ounces and a standard deviation of 0.07 ounce. The second dispenses an amount of liquid that has a normal distribution with a mean of 12.0 ounces and a standard deviation of 0.05 ounce. Acceptable amounts of dispensed liquid are between 11.9 and 12.0 ounces. Calculate the relevant probabilities and determine which machine should be selected.

6-62. Domino's Pizza receives majority of its orders by telephone. The call center supervisor conducted a study on the duration an order had been taken. He found that the amount of time a customer was waiting to place an order was about 10 minutes. Assume that the distribution of time spent in taking orders is approximately exponentially distributed with a mean of 5 minutes. The supervisor is considering adding extra telephone order services to decrease the customers' waiting time. Help the supervisor determine the probability that a second telephone order is coming before the first order has ended. Justify whether the extra telephone order service is necessary.

6-63. Assume that after the first 12 hours, the average remaining useful life of a particular battery (before recharging is required) is 9 hours and that the remaining time is exponentially distributed. What is the probability that a randomly sampled battery of this type will last between 15 and 17 hours?

6-64. Suppose eye shadow and eyeliner have a shelf life of up to three years, and the shelf lives of these two products are exponentially distributed with an average of one year.
a. Calculate the probability that the shelf life of eye shadow will be longer than three years.
b. Determine the probability that at least one of these products will have a shelf life of more than three years.
c. Determine the probability that a purchased eyeliner that is useful after one year will be useful after three years.

6-65. The Shadow Mountain Golf Course is preparing for a major LPGA golf tournament. Since parking near the course is extremely limited (room for only 500 cars), the course officials have contracted with the local community to provide parking and a bus shuttle service. Sunday, the final day of the tournament, will have the largest crowd, and the officials estimate there will be between 8,000 and 12,000 cars needing parking spaces but think no value is more likely than another. The tournament committee is discussing how many parking spots to contract from the city. If they want to limit the chance of not having enough provided parking to 10%, how many spaces do they need from the city on Sunday?

6-66. A product made by a window manufacturer is constructed of two pieces of glass 0.125 inch thick, with a thin layer of vinyl sandwiched between them. The average thickness of the laminated safety glass is 0.25 inch. The thickness of the glass does not vary from the mean by more than 0.10 inch. Assume the thickness of the glass has a uniform distribution.
a. Provide the density for this uniform distribution.
b. If the glass has a thickness that is more than 0.05 inch below the mean, it must be discarded for safety considerations. Determine the probability that a randomly selected glass is discarded due to safety considerations.
c. If the glass thickness is more than 0.075 inch above the mean, it will create installation problems and must be discarded. Calculate the probability that a randomly selected glass will be rejected due to installation concerns.

6-67. A traffic control camera at a busy intersection records, on average, 5 traffic violations per hour. Assume that the

random variable number of recorded traffic violations follows a Poisson distribution.
a. What is the probability that the next recorded violation will occur within 5 minutes?
b. How likely is it that no traffic violations will be recorded within the next 7 minutes?

6-68. A statistics professor uses the following grading curve to assign final grades for her large section course:

Grade	Total Points
A	Greater then $\mu + 1.8\sigma$
B	$\mu + 1.4\sigma$ to $\mu + 1.8\sigma$
C	$\mu - 1.2\sigma$ to $\mu + 1.4\sigma$
D	$\mu - 1.8\sigma$ to $\mu - 1.2\sigma$
F	Below $\mu - 1.8\sigma$

a. What percent (proportion) of students will receive a final grade of "A"?
b. What percent (proportion) of students will receive a final grade of "B"?
c. What percent (proportion) of students will receive a grade of "C" or better?

6-69. The personnel manager for a large company is interested in the distribution of sick-leave hours for employees of her company. A recent study revealed the distribution to be approximately normal, with a mean of 58 hours per year and a standard deviation of 14 hours.

An office manager in one division has reason to believe that during the past year, two of his employees have taken excessive sick leave relative to everyone else. The first employee used 74 hours of sick leave, and the second used 90 hours. What would you conclude about the office manager's claim and why?

6-70. Assume that the number of operating hours before servicing is required for a particular snowmobile engine is exponentially distributed with an average of 240 hours. Determine the probability that a randomly selected engine
a. will run at least 275 hours before servicing is required.
b. will run at more than 200 hours before servicing is required.

6-71. Assume that the amount of time eighth-graders take to complete an assessment examination is 78 minutes with a standard deviation of 12 minutes.
a. What proportion of eighth-graders complete the assessment examination in 72 minutes or less?
b. What proportion of eighth-graders complete the assessment examination in 82 minutes or more?
c. For what number of minutes would 90% of all eighth-graders complete the assessment examination?

6-72. Assume that the time required to assemble a product is normally distributed with a mean of 200 minutes and a standard deviation of 20 minutes, find
a. the probability that the product's assembly time would be less than 190 minutes.

b. the probability that the product's assembly time would be greater than 215 minutes.
c. the probability that the product's assembly time would be between 195 and 205 minutes.

6-73. If the average amount of time customers spent online at an Internet retailer's site is assumed to be normally distributed with mean of 36 minutes and a standard deviation of 6 minutes, determine
a. the probability a customer spends less than 30 minutes on the site.
b. the probability a customer spends more than 44 minutes on the site.
c. the probability a customer spends between 33 and 42 minutes on the site.
d. the amount of time in minutes spent on the site by no more than 5% of all customers.

6-74. A manufacturing process applies a coating to a product in order to protect it from scratches. If the thickness of the coating follows a uniform distribution between 15 and 25 microns, determine
a. the mean thickness of the protective coating.
b. the standard deviation of the thickness of the protective coating.
c. the probability that the protective coating is between 18 and 22 microns thick.
d. the probability that the protective coating is less than 23 microns thick.

Computer Software Exercises

6-75. The Cozine Corporation runs the landfill operation outside Little Rock, Arkansas. Each day, each of the company's trucks makes several trips from the city to the landfill. On each entry, the truck is weighed. The data file **Cozine** contains a sample of 200 truck weights. Determine the mean and standard deviation for the garbage truck weights. Assume that these sample values are representative of the population of all Cozine garbage trucks, and that the weights are normally distributed.
a. Determine the probability that a truck will arrive at the landfill weighing in excess of 46,000 pounds.
b. Compare the probability in part a to the proportion of trucks in the sample that weighed more than 46,000 pounds. What does this imply to you?
c. Suppose the managers are concerned that trucks are returning to the landfill before they are fully loaded. If they have set a minimum weight of 38,000 pounds before the truck returns to the landfill, what is the probability that a truck will fail to meet the minimum standard?

6-76. The Hydronics Company's research and development department came up with two weight-loss products. To determine whether these products are effective, the company has conducted a test. A total of 300 people who were 30 pounds or more overweight were recruited to participate in the study. Of these, 100 people were given a placebo supplement, 100 people were given Product 1, and 100 people were given Product 2. As

might be expected, some people dropped out of the study before the four-week study period was completed. The weight loss (or gain) for each individual is listed in the data file called **Hydronics**.

  a. Develop a frequency histogram for the weight loss (or gain) for those people on Product 1. Does it appear from this graph that weight loss is approximately normally distributed?

  b. Referring to part a and assuming that a normal distribution does apply, compute the mean and standard deviation weight loss for the Product 1 users.

  c. Referring to parts a and b, assume that the weight-change distribution for Product 1 users is normally distributed and that the sample mean and standard deviation are used to directly represent the population mean and standard deviation. What is the probability that a Product 1 user will lose over 12 pounds in a four-week period?

  d. Referring to your answer in part c, would it be appropriate for the company to claim that Product 1 users can expect to lose as much as 12 pounds in four weeks? Discuss.

6-77. Midwest Fan Manufacturing Inc. is a distributor of quality ventilation equipment. Midwest Fan's products include the AXC range hood exhaust fans. The file titled **Fan Life** contains the length of life of 125 randomly chosen AXC fans that were used in an accelerated life-testing experiment.

  a. Produce a relative frequency histogram for the data. Does it seem plausible the data came from a population that has an exponential distribution?

  b. Calculate the mean and standard deviation of the fans' length of life.

  c. Calculate the median length of life of the fans.

  d. Determine the probability that a randomly chosen fan will have a life longer than 25,000 hours.

6-78. The World Happiness Report is a United Nations Sustainable Development Solutions Network publication. The reports contain so-called ladder scores for national happiness (it asks respondents to think of a ladder, with the best possible life for them being a 10 and the worst possible life being a 0). Data of ladder scores of 145 countries is in **Ladder Happiness 2022**.

  a. Produce a relative frequency histogram for these data. Does it seem plausible the data were sampled from a population that was normally distributed?

  b. Calculate the mean and standard deviation of the ladder scores.

  c. Calculate the 90th percentile of the ladder scores.

  d. Malaysia has a 2022 Ladder score of 5.711. Determine the percentile of the Malaysian ladder score, assuming normally distributed population scores with mean and standard deviation as calculated in b.

6-79. The Future-Vision Digital and Satellite TV Company recently surveyed its customers. A total of 548 responses were received. Among other things, the respondents were asked to indicate their household income. The data from the survey are found in a file named **Future-Vision**.

  a. Develop a frequency histogram for the income variable. Does it appear from the graph that income is approximately normally distributed? Discuss.

  b. Compute the mean and standard deviation for the income variable.

  c. Referring to parts a and b and assuming that income is normally distributed and the sample mean and standard deviation are good substitutes for the population values, what is the probability that a Future-Vision customer will have an income exceeding $40,000?

  d. Suppose that Future-Vision managers are thinking about offering a monthly discount to customers who have a household income below a certain level. If the management wants to grant discounts to no more than 7% of the customers, what income level should be used for the cutoff?

6-80. Championship Billiards, owned by D & R Industries, in Lincolnwood, Illinois, provides some of the finest billiard fabrics, cushion rubber, and component parts in the industry. It sells billiard cloth in bolts and half-bolts. A half-bolt of billiard cloth has an average length of 35 yards with widths of either 62 or 66 inches. The file titled **Half Bolts** contains the lengths of 120 randomly selected half-bolts.

  a. Produce a relative frequency histogram for these data. Does it seem plausible the data came from a population that has a uniform distribution?

  b. Provide the density, $f(x)$, for this uniform distribution.

  c. A billiard retailer, Sticks & Stones Billiard Supply, is going to recover the pool tables in the local college pool hall, which has eight tables. It takes approximately 3.8 yards per table. If Championship ships a randomly chosen half-bolt, determine the probability that it will contain enough cloth to recover the eight tables.

Case 6.1   State Entitlement Programs

Franklin Joiner, director of health, education, and welfare, had just left a meeting with the state's newly elected governor and several of the other recently appointed department heads. One of the governor's campaign promises was to try to halt the rising cost of a certain state entitlement program. In several speeches, the governor indicated the state of Idaho should allocate funds only to those individuals ranked in the bottom 10% of the state's income distribution. Now the governor wants to know how much one could earn before being disqualified from the program, and he also wants to know the range of incomes for the middle 95% of the state's income distribution.