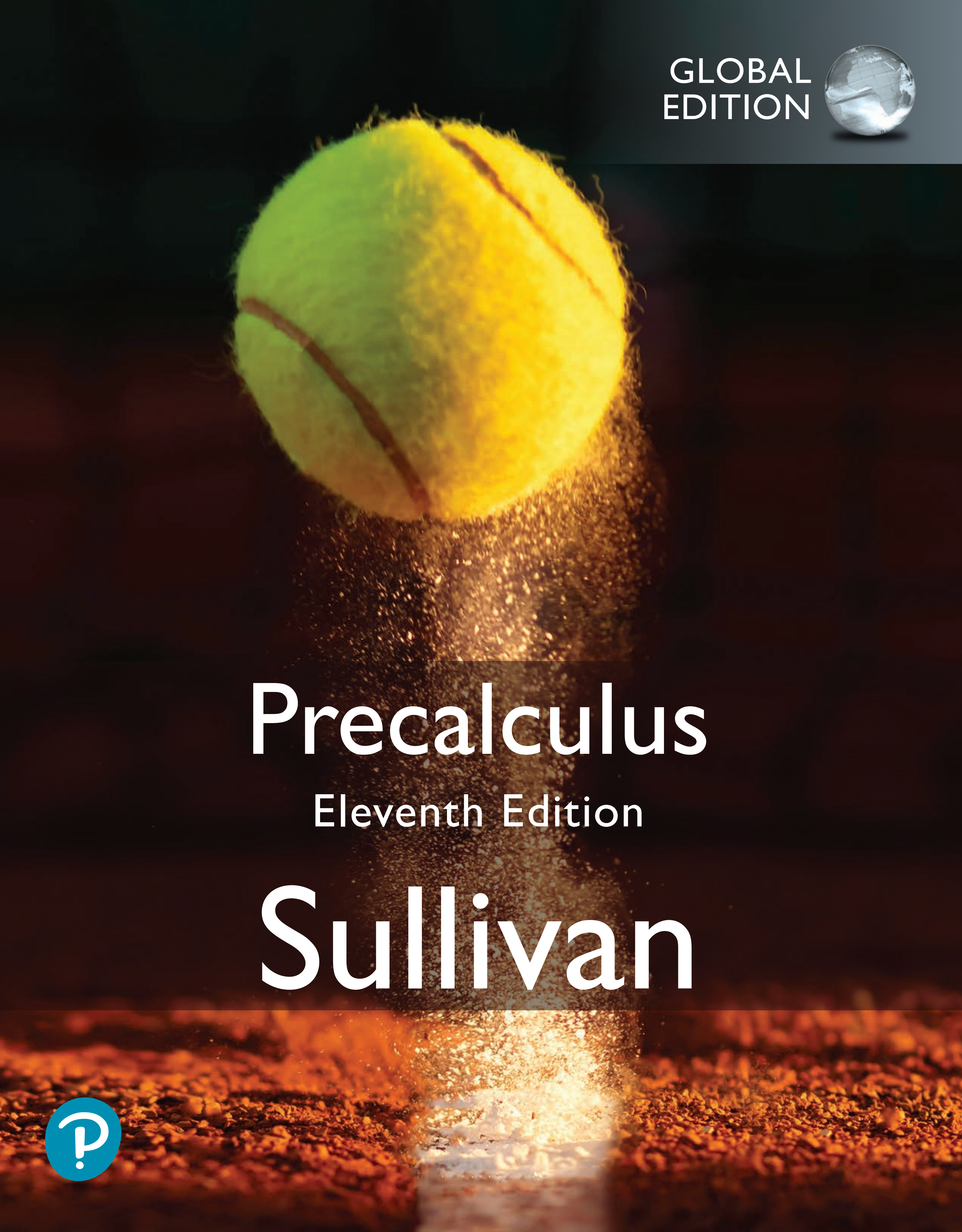

GLOBAL EDITION

Precalculus

Eleventh Edition

Sullivan

To the Student

As you begin, you may feel anxious about the number of theorems, definitions, procedures, and equations. You may wonder if you can learn it all in time. Don't worry–your concerns are normal. This textbook was written with you in mind. If you attend class, work hard, and read and study this text, you will build the knowledge and skills you need to be successful. Here's how you can use the text to your benefit.

Read Carefully

When you get busy, it's easy to skip reading and go right to the problems. Don't … the text has a large number of examples and clear explanations to help you break down the mathematics into easy-to-understand steps. Reading will provide you with a clearer understanding, beyond simple memorization. Read before class (not after) so you can ask questions about anything you didn't understand. You'll be amazed at how much more you'll get out of class if you do this.

Use the Features

I use many different methods in the classroom to communicate. Those methods, when incorporated into the text, are called "features." The features serve many purposes, from providing timely review of material you learned before (just when you need it) to providing organized review sessions to help you prepare for quizzes and tests. Take advantage of the features and you will master the material.

To make this easier, we've provided a brief guide to getting the most from this text. Refer to "Prepare for Class," "Practice," and "Review" at the front of the text. Spend fifteen minutes reviewing the guide and familiarizing yourself with the features by flipping to the page numbers provided. Then, as you read, use them. This is the best way to make the most of your text.

Please do not hesitate to contact me through Pearson, with any questions, comments, or suggestions for improving this text. I look forward to hearing from you, and good luck with all of your studies.

Best Wishes!

Michael Sullivan

In Problems 45–72, solve each exponential equation. Express irrational solutions in exact form.

45. $5^{-x} = 25$
46. $2^{x-5} = 8$
47. $2^x = 10$
48. $3^x = 14$
49. $2^{-x} = 1.5$
50. $8^{-x} = 1.2$
51. $0.3(4^{0.2x}) = 0.2$
52. $5(2^{3x}) = 8$
53. $2^{x+1} = 5^{1-2x}$
54. $3^{1-2x} = 4^x$
55. $\left(\frac{4}{3}\right)^{1-x} = 5^x$
56. $\left(\frac{3}{5}\right)^x = 7^{1-x}$
57. $1.2^x = (0.5)^{-x}$
58. $0.3^{1+x} = 1.7^{2x-1}$
59. $e^{x+3} = \pi^x$
60. $\pi^{1-x} = e^x$
61. $3^{2x} + 3^x - 2 = 0$
62. $2^{2x} + 2^x - 12 = 0$
63. $2^{2x} + 2^{x+2} - 12 = 0$
64. $3^{2x} + 3^{x+1} - 4 = 0$
65. $16^x + 4^{x+1} - 3 = 0$
66. $9^x - 3^{x+1} + 1 = 0$
67. $36^x - 6 \cdot 6^x = -9$
68. $25^x - 8 \cdot 5^x = -16$
69. $2 \cdot 49^x + 11 \cdot 7^x + 5 = 0$
70. $3 \cdot 4^x + 4 \cdot 2^x + 8 = 0$
71. $3^x - 14 \cdot 3^{-x} = 5$
72. $4^x - 10 \cdot 4^{-x} = 3$

In Problems 73–86, use a graphing utility to solve each equation. Express your answer rounded to two decimal places.

73. $\log_2(x-1) - \log_6(x+2) = 2$
74. $\log_5(x+1) - \log_4(x-2) = 1$
75. $e^x = -x$
76. $e^{2x} = x + 2$
77. $e^x = x^3$
78. $e^x = x^2$
79. $\ln(2x) = -x + 2$
80. $\ln x = -x$
81. $\ln x = -x^2$
82. $\ln x = x^3 - 1$
83. $e^x - \ln x = 4$
84. $e^x + \ln x = 4$
85. $e^{-x} = -\ln x$
86. $e^{-x} = \ln x$

Applications and Extensions

87. $f(x) = \log_2(x+3)$ and $g(x) = \log_2(3x+1)$.
- **(a)** Solve $f(x) = 3$. What point is on the graph of f?
- **(b)** Solve $g(x) = 4$. What point is on the graph of g?
- **(c)** Solve $f(x) = g(x)$. Do the graphs of f and g intersect? If so, where?
- **(d)** Solve $(f+g)(x) = 7$.
- **(e)** Solve $(f-g)(x) = 2$.

88. $f(x) = \log_3(x+5)$ and $g(x) = \log_3(x-1)$.
- **(a)** Solve $f(x) = 2$. What point is on the graph of f?
- **(b)** Solve $g(x) = 3$. What point is on the graph of g?
- **(c)** Solve $f(x) = g(x)$. Do the graphs of f and g intersect? If so, where?
- **(d)** Solve $(f+g)(x) = 3$.
- **(e)** Solve $(f-g)(x) = 2$.

89. **(a)** If $f(x) = 3^{x+1}$ and $g(x) = 2^{x+2}$, graph f and g on the same Cartesian plane.
- **(b)** Find the point(s) of intersection of the graphs of f and g by solving $f(x) = g(x)$. Round answers to three decimal places. Label any intersection points on the graph drawn in part (a).
- **(c)** Based on the graph, solve $f(x) > g(x)$.

90. **(a)** If $f(x) = 5^{x-1}$ and $g(x) = 2^{x+1}$, graph f and g on the same Cartesian plane.
- **(b)** Find the point(s) of intersection of the graphs of f and g by solving $f(x) = g(x)$. Label any intersection points on the graph drawn in part (a).
- **(c)** Based on the graph, solve $f(x) > g(x)$.

91. **(a)** Graph $f(x) = 3^x$ and $g(x) = 10$ on the same Cartesian plane.
- **(b)** Shade the region bounded by the y-axis, $f(x) = 3^x$, and $g(x) = 10$ on the graph drawn in part (a).
- **(c)** Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

92. **(a)** Graph $f(x) = 2^x$ and $g(x) = 12$ on the same Cartesian plane.
- **(b)** Shade the region bounded by the y-axis, $f(x) = 2^x$, and $g(x) = 12$ on the graph drawn in part (a).
- **(c)** Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

93. **(a)** Graph $f(x) = 2^{x+1}$ and $g(x) = 2^{-x+2}$ on the same Cartesian plane.
- **(b)** Shade the region bounded by the y-axis, $f(x) = 2^{x+1}$, and $g(x) = 2^{-x+2}$ on the graph drawn in part (a).
- **(c)** Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

94. **(a)** Graph $f(x) = 3^{-x+1}$ and $g(x) = 3^{x-2}$ on the same Cartesian plane.
- **(b)** Shade the region bounded by the y-axis, $f(x) = 3^{-x+1}$, and $g(x) = 3^{x-2}$ on the graph drawn in part (a).
- **(c)** Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

95. **(a)** Graph $f(x) = 2^x - 4$.
- **(b)** Find the zero of f.
- **(c)** Based on the graph, solve $f(x) < 0$.

96. **(a)** Graph $g(x) = 3^x - 9$.
- **(b)** Find the zero of g.
- **(c)** Based on the graph, solve $g(x) > 0$.

97. **A Population Model** The population of the world in 2018 was 7.63 billion people and was growing at a rate of 1.1% per year. Assuming that this growth rate continues, the model $P(t) = 7.63(1.011)^{t-2018}$ represents the population P (in billions of people) in year t.
- **(a)** According to this model, when will the population of the world be 9 billion people?
- **(b)** According to this model, when will the population of the world be 12.5 billion people?

Source: U.S. Census Bureau

98. **A Population Model** The population of a certain country in 1999 was 287 million people. In addition, the population of the country was growing at a rate of 1.0% per year. Assuming that this growth rate continues, the model $P(t) = 287(1.010)^{t-1999}$ represents the population P (in millions of people) in year t.
- **(a)** According to this model, when will the population of the country reach 307 million people?
- **(b)** According to this model, when will the population of the country reach 394 million people?

Source: U.S. Census Bureau

99. Depreciation The value V of a Honda Civic LX that is t years old can be modeled by $V(t) = 19{,}705(0.848)^t$.

(a) According to the model, when will the car be worth \$14,000?

(b) According to the model, when will the car be worth \$10,000?

(c) According to the model, when will the car be worth \$7500?

Source: *Kelley Blue Book*

100. Depreciation The value V of a Chevy Cruze LT that is t years old can be modeled by $V(t) = 19{,}200(0.82)^t$.

(a) According to the model, when will the car be worth \$12,000?

(b) According to the model, when will the car be worth \$9000?

(c) According to the model, when will the car be worth \$3000?

Source: *Kelley Blue Book*

Challenge Problems *In Problems 101–105, solve each equation. Express irrational solutions in exact form.*

101. $(\sqrt[3]{2})^{2-x} = 2^{x^2}$

102. $\log_2(x+1) - \log_4 x = 1$

103. $\ln x^2 = (\ln x)^2$

104. $\log_2 x^{\log_2 x} = 4$

105. $\sqrt{\log x} = 2\log\sqrt{3}$

Explaining Concepts: Discussion and Writing

106. Fill in the reason for each step in the following two solutions.

Solve: $\log_3(x-1)^2 = 2$

Solution A

$\log_3(x-1)^2 = 2$

$(x-1)^2 = 3^2 = 9$ ________

$(x-1) = \pm 3$ ________

$x - 1 = -3$ or $x - 1 = 3$ ________

$x = -2$ or $x = 4$ ________

Solution B

$\log_3(x-1)^2 = 2$

$2\log_3(x-1) = 2$ ________

$\log_3(x-1) = 1$ ________

$x - 1 = 3^1 = 3$ ________

$x = 4$ ________

Both solutions given in Solution A check. Explain what caused the solution $x = -2$ to be lost in Solution B.

Retain Your Knowledge

Problems 107–116, are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

107. Solve: $4x^3 + 3x^2 - 25x + 6 = 0$

108. Determine whether the function is one-to-one:

$$\{(0,-4),(2,-2),(4,0),(6,2)\}$$

109. For $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{x+5}{x-3}$, find $f \circ g$. Then find the domain of $f \circ g$.

110. Find the domain of $f(x) = \sqrt{x+3} + \sqrt{x-1}$.

111. Solve: $x - \sqrt{x+7} = 5$

112. Find the real zero of the function $f(x) = 5x - 30$.

113. If $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{5}{x+2}$, find $(f+g)(x)$.

114. Find the distance between the center of the circle

$$(x-2)^2 + (y+3)^2 = 25$$

and the vertex of the parabola $y = -2(x-6)^2 + 9$.

115. Find the average rate of change $f(x) = \log_2 x$ from 4 to 16.

116. Rationalize the numerator: $\dfrac{\sqrt{x+6} - \sqrt{x}}{6}$

'Are You Prepared?' Answers

1. $\{-3, 10\}$ **2.** $\{-2, 0\}$ **3.** $\{-1.43\}$ **4.** $\{-1.77\}$

5.7 Financial Models

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Simple Interest (Section A.8, pp. A63–A64)

Now Work the **'Are You Prepared?'** problems on page 367.

OBJECTIVES
1 Determine the Future Value of a Lump Sum of Money (p. 361)
2 Calculate Effective Rates of Return (p. 364)
3 Determine the Present Value of a Lump Sum of Money (p. 365)
4 Determine the Rate of Interest or the Time Required to Double a Lump Sum of Money (p. 366)

1 Determine the Future Value of a Lump Sum of Money

Interest is money paid for the use of money. The total amount borrowed (whether by an individual from a bank in the form of a loan or by a bank from an individual in the form of a savings account) is called the **principal**. The **rate of interest**, expressed as a percent, is the amount charged for the use of the principal for a given period of time, usually on a yearly (that is, per annum) basis.

THEOREM Simple Interest Formula

If a principal of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is

$$I = Prt \qquad \textbf{(1)}$$

Interest charged according to formula (1) is called **simple interest**.

In problems involving interest, the term **payment period** is defined as follows.

Annually:	Once per year	**Monthly:**	12 times per year
Semiannually:	Twice per year	**Daily:**	365 times per year*
Quarterly:	Four times per year		

When the interest due at the end of a payment period is added to the principal so that the interest computed at the end of the next payment period is based on this new principal amount (old principal + interest), the interest is said to have been **compounded**. **Compound interest** is interest paid on the principal and on previously earned interest.

EXAMPLE 1 **Computing Compound Interest**

A credit union pays interest of 2% per annum compounded quarterly on a certain savings plan. If \$1000 is deposited in the plan and the interest is left to accumulate, how much is in the account after 1 year?

Solution Use the simple interest formula, $I = Prt$. The principal P is \$1000 and the rate of interest is $2\% = 0.02$. After the first quarter of a year, the time t is $\frac{1}{4}$ year, so the interest earned is

$$I = Prt = \$1000 \cdot 0.02 \cdot \frac{1}{4} = \$5$$

*Some banks use a 360-day "year." Why do you think they do?

(continued)

The new principal is $P + I = \$1000 + \$5 = \$1005$. At the end of the second quarter, the interest on this principal is

$$I = \$1005 \cdot 0.02 \cdot \frac{1}{4} = \$5.03$$

At the end of the third quarter, the interest on the new principal of $\$1005 + \$5.03 = \$1010.03$ is

$$I = \$1010.03 \cdot 0.02 \cdot \frac{1}{4} = \$5.05$$

Finally, after the fourth quarter, the interest is

$$I = \$1015.08 \cdot 0.02 \cdot \frac{1}{4} = \$5.08$$

After 1 year the account contains $\$1015.08 + \$5.08 = \$1020.16$.

The pattern of the calculations performed in Example 1 leads to a general formula for compound interest. For this purpose, let P represent the principal to be invested at a per annum interest rate r that is compounded n times per year, so the time of each compounding period is $\frac{1}{n}$ year. (For computing purposes, r is expressed as a decimal.) The interest earned after each compounding period is given by formula (1).

$$\text{Interest} = \text{principal} \cdot \text{rate} \cdot \text{time} = P \cdot r \cdot \frac{1}{n} = P \cdot \frac{r}{n}$$

The amount A after one compounding period is

$$A = P + P \cdot \frac{r}{n} = P \cdot \left(1 + \frac{r}{n}\right)$$

After two compounding periods, the amount A, based on the new principal $P \cdot \left(1 + \frac{r}{n}\right)$, is

$$A = \underbrace{P \cdot \left(1 + \frac{r}{n}\right)}_{\text{New principal}} + \underbrace{P \cdot \left(1 + \frac{r}{n}\right) \cdot \frac{r}{n}}_{\text{Interest on new principal}} \underset{\uparrow}{=} P \cdot \left(1 + \frac{r}{n}\right) \cdot \left(1 + \frac{r}{n}\right) = P \cdot \left(1 + \frac{r}{n}\right)^2$$

Factor out $P \cdot \left(1 + \frac{r}{n}\right)$

After three compounding periods, the amount A is

$$A = P \cdot \left(1 + \frac{r}{n}\right)^2 + P \cdot \left(1 + \frac{r}{n}\right)^2 \cdot \frac{r}{n} = P \cdot \left(1 + \frac{r}{n}\right)^2 \cdot \left(1 + \frac{r}{n}\right) = P \cdot \left(1 + \frac{r}{n}\right)^3$$

Continuing this way, after n compounding periods (1 year), the amount A is

$$A = P \cdot \left(1 + \frac{r}{n}\right)^n$$

Because t years will contain $n \cdot t$ compounding periods, the amount after t years is

$$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

THEOREM Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate r, expressed as a decimal, compounded n times per year is

$$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt} \qquad \textbf{(2)}$$

In equation (2), the amount A is typically referred to as the **accumulated value** of the account, and P is called the **present value**.

Exploration

To observe the effects of compounding interest monthly on an initial deposit of $1, graph $Y_1 = \left(1 + \frac{r}{12}\right)^{12x}$ with $r = 0.06$ and $r = 0.12$ for $0 \le x \le 30$. What is the future value of $1 in 30 years when the interest rate per annum is $r = 0.06$ (6%)? What is the future value of $1 in 30 years when the interest rate per annum is $r = 0.12$ (12%)? Does doubling the interest rate double the future value?

For example, to rework Example 1, use $P = \$1000$, $r = 0.02$, $n = 4$ (quarterly compounding), and $t = 1$ year to obtain

$$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.02}{4}\right)^{4 \cdot 1} = \$1020.15$$

The result obtained here differs slightly from that obtained in Example 1 because of rounding.

Now Work PROBLEM 7

EXAMPLE 2 Comparing Investments Using Different Compounding Periods

Investing $1000 at an annual rate of 10% compounded annually, semiannually, quarterly, monthly, and daily will yield the following amounts after 1 year:

Annual compounding $(n = 1)$:
$$A = P \cdot (1 + r) = \$1000(1 + 0.10) = \$1100.00$$

Semiannual compounding $(n = 2)$:
$$A = P \cdot \left(1 + \frac{r}{2}\right)^2 = \$1000(1 + 0.05)^2 = \$1102.50$$

Quarterly compounding $(n = 4)$:
$$A = P \cdot \left(1 + \frac{r}{4}\right)^4 = \$1000(1 + 0.025)^4 = \$1103.81$$

Monthly compounding $(n = 12)$:
$$A = P \cdot \left(1 + \frac{r}{12}\right)^{12} = \$1000\left(1 + \frac{0.10}{12}\right)^{12} = \$1104.71$$

Daily compounding $(n = 365)$:
$$A = P \cdot \left(1 + \frac{r}{365}\right)^{365} = \$1000\left(1 + \frac{0.10}{365}\right)^{365} = \$1105.16$$

From Example 2, note that the effect of compounding more frequently is that the amount after 1 year is higher. This leads to the following question: What would happen to the amount after 1 year if the number of times that the interest is compounded were increased without bound?

Let's find the answer. Suppose that P is the principal, r is the per annum interest rate, and n is the number of times that the interest is compounded each year. The amount A after 1 year is

$$A = P \cdot \left(1 + \frac{r}{n}\right)^n$$

Rewrite this expression as follows:

$$A = P \cdot \left(1 + \frac{r}{n}\right)^n = P \cdot \left(1 + \frac{1}{\frac{n}{r}}\right)^n = P \cdot \left[\left(1 + \frac{1}{\frac{n}{r}}\right)^{n/r}\right]^r \underset{h = \frac{n}{r}}{\uparrow}= P \cdot \left[\left(1 + \frac{1}{h}\right)^h\right]^r \quad (3)$$

Need to Review?
The number e is defined on page 323.

Now suppose that the number n of times that the interest is compounded per year gets larger and larger; that is, suppose that $n \to \infty$. Then $h = \frac{n}{r} \to \infty$, and the expression in brackets in equation (3) equals e. That is, $A \to Pe^r$.

Table 8 compares $\left(1 + \frac{r}{n}\right)^n$, for large values of n, to e^r for $r = 0.05$, $r = 0.10$, $r = 0.15$, and $r = 1$. As n becomes larger, the closer $\left(1 + \frac{r}{n}\right)^n$ gets to e^r. No matter how frequent the compounding, the amount after 1 year has the upper bound Pe^r.

Table 8

	$\left(1+\frac{r}{n}\right)^n$			
	$n = 100$	$n = 1000$	$n = 10{,}000$	e^r
$r = 0.05$	1.0512580	1.0512698	1.051271	1.0512711
$r = 0.10$	1.1051157	1.1051654	1.1051704	1.1051709
$r = 0.15$	1.1617037	1.1618212	1.1618329	1.1618342
$r = 1$	2.7048138	2.7169239	2.7181459	2.7182818

When interest is compounded so that the amount after 1 year is Pe^r, the interest is said to be **compounded continuously**.

THEOREM Continuous Compounding

The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is

$$A = Pe^{rt} \qquad (4)$$

EXAMPLE 3

Using Continuous Compounding

The amount A that results from investing a principal P of \$1000 at an annual rate r of 10% compounded continuously for a time t of 1 year is

$$A = \$1000e^{0.10} = \$1000 \cdot 1.10517 = \$1105.17$$

Now Work PROBLEM 13

2 Calculate Effective Rates of Return

Suppose that you have \$1000 to invest and a bank offers to pay you 3 percent annual interest compounded monthly. What simple interest rate is needed to earn an equal amount after one year? To answer this question, first determine the value after one year of the \$1000 investment that earns 3 percent compounded monthly.

$$A = \$1000\left(1 + \frac{0.03}{12}\right)^{12} \quad \text{Use } A = P\left(1+\frac{r}{n}\right)^n \text{ with } P = \$1000, r = 0.03, n = 12.$$
$$= \$1030.42$$

So the interest earned is \$30.42. Using $I = Prt$ with $t = 1$, $I = \$30.42$, and $P = \$1000$, the annual simple interest rate is $0.03042 = 3.042\%$. This interest rate is known as the *effective rate of interest*.