



Example 4.14 A Fuel Cell to Generate Electricity from Methane Problem Statement

"A Fuel Cell in Every Car" is the headline of an article in *Chemical and Engineering News* (March 5, 2001, p. 43). In essence, a fuel cell is an open system into which fuel and air are fed, and out of which come electricity and waste products. Figure E4.14 is a sketch of a fuel cell in which a continuous flow of methane (CH₄) and air (O_2 plus N_2) produces electricity plus CO_2 and H_2O . Special membranes and catalysts are needed to promote the oxidation of the CH₄.

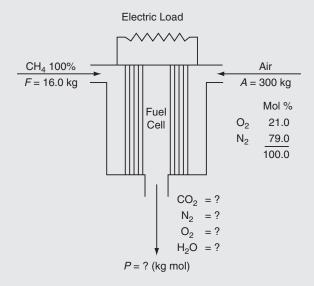


Figure E4.14

Based on the data given in Figure E4.14, you are asked to calculate the composition of the products in *P*.

Solution

Steps 1-4

This is a steady-state process with reaction. Can you assume that a complete reaction occurs? Yes. How? No CH_4 or CO appears in P. The system is the fuel cell (open, steady state). Because the process output is a gas, the composition will be in mole fractions (or moles); hence, it is more convenient to use kilogram moles rather than mass in this problem even though the quantities of CH_4 and air are

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Example 4.14 A Fuel Cell to Generate Electricity from Methane (Continued)

stated in kilograms. You can carry out the necessary conversions from kilograms to kilogram moles as follows:

$$\frac{300 \text{ kg } A}{29.0 \text{ kg } A} = 10.35 \text{ kg mol } A \text{ in}$$

$$\frac{16.0 \text{ kg CH}_4}{16.0 \text{ kg CH}_4} = 1.00 \text{ kg mol CH}_4 \text{ in}$$

$$\frac{10.35 \text{ kg mol } A}{1 \text{ kg mol } A} = 2.17 \text{ kg mol O}_2 \text{ in } A$$

$$\frac{10.35 \text{ kg mol } A}{1 \text{ kg mol } A} = 8.18 \text{ kg mol N}_2 \text{ in } A$$

The chemical reaction equation for this system can be assumed to be

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Step 5

We will pick a convenient basis.

Basis:
$$16.0 \text{ kg CH}_4$$
 entering = 1 kg mol CH_4 plus $300 \text{ kg } A$ entering = $10.35 \text{ kg mol of air}$

Steps 6 and 7

The degree-of-freedom analysis is as follows:

Variables: 10 F, P, A,
$$n_{\text{CO}_2}^P$$
, $n_{\text{N}_2}^P$, $n_{\text{O}_2}^P$, $n_{\text{H}_2\text{O}}^P$, $n_{\text{O}_2}^A$, $n_{\text{N}_2}^A$ plus ξ

Given the basis and the quantities calculated above, four of these variables $\left(F,\,A,\,n_{\mathrm{N}_{2}}^{A}\,,\,n_{\mathrm{N}_{2}}^{A}\right)$ can be assigned values. Therefore, there are six unknowns remaining.

Equations: 6

Five (5) independent species balances: CH_4 , O_2 , N_2 , CO_2 , H_2O

One (1) independent implicit equation for $P: \sum n_i^P = P$

Therefore, the degrees of freedom are zero.

Step 8The species mole balances are as follows:

Compound	Out		In		$v_i \xi$		g mol
CH_4	$n_{\mathrm{CH_4}}^P$	=	1.0	-	ξ	=	0
O_2	$n_{\mathrm{O}_2}^P$	=	2.17	-	25	=	0.17
N_2	$n_{ m N_2}^P$	=	8.18	_	0(<i>§</i>)	=	8.18
CO ₂	$n_{\mathrm{CO}_2}^P$	=	0	+	ξ	=	1.0
H ₂ O	$n_{ m H_2O}^P$	=	0	+	2 <i>ξ</i>	=	2.0

Step 9

The solution of this set of equations gives

$$\begin{split} n_{\text{CH}_4}^P &= 0, \ n_{\text{O}_2}^P = 0.17, \ n_{\text{N}_2}^P = 8.18, \ n_{\text{CO}_2}^P = 1.0, \\ n_{\text{CO}_2}^P &= 1.0, \ n_{\text{H}_2\text{O}}^P = 2.0, \ P = 11.35 \end{split}$$

and the mole percentage composition of P is

$$y_{\rm O_2} = 1.5\%$$
, $y_{\rm N_2} = 72.1\%$, $y_{\rm CO_2} = 8.8\%$, and $y_{\rm H_2O} = 17.6\%$

You could also use element balances without knowing the reaction to get the same solution using four element balances and one implicit equation for $P(\xi \text{ would no longer be a variable})$.

Step 10

You can check the answer by determining the total mass of the exit gas and comparing it to total mass in (316 kg), but we will omit this step here to save space.

Example 4.15 Combustion of Coal

Problem Statement

A local utility burns coal having the following composition on a dry basis. (Note that the coal analysis below is a convenient one for our calculations but is not necessarily the only type of analysis that is reported for coal. Some analyses contain much less information about each element.)

Component	Percent
С	83.05
Н	4.45
O	3.36
N	1.08
S	0.70
Ash	7.36
Total	100.0

The average Orsat analysis of the gas from the stack during a 24 hr test was

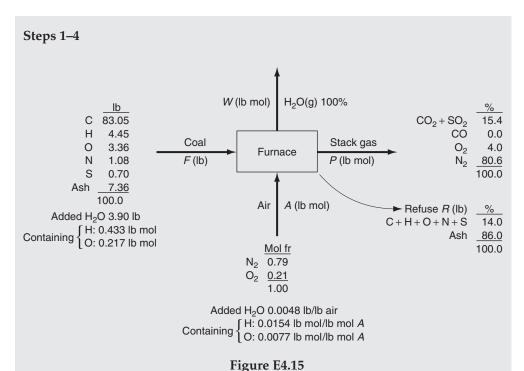
Component	Percent		
$CO_2 + SO_2$	15.4		
CO	0.0		
O_2	4.0		
N_2	80.6		
Total	100.0		

Moisture (H_2O) in the fuel was 3.90%, and the air on the average contained 0.0048 lb H_2O /lb dry air. The refuse showed 14.0% unburned coal, with the remainder being ash. The unburned coal in the refuse can be assumed to be of the same composition as the coal that serves as fuel.

What is the percent excess air used for this process as shown in Figure E4.15?

Solution

Note that for this problem you are asked only for the percent excess air used. Therefore, Figure E4.15 contains more information than is required to solve this problem.



g

Step 5

Pick a basis of F = 100 lb as convenient.

Steps 6-9

We need to solve for the feed rate of air (*A*), but to do this, we first need to solve for the unknown flow rates: *R* and *P*. Note that ash is a tie component. Applying a material balance for ash yields

$$7.36 = 0.86R \Rightarrow R = 8.56 \text{ lb}$$

Note that a portion of the coal does not react and leaves the process in R. We will assume here that the portion of R that is not ash has the same composition as F. Therefore, we simply subtract this from F to determine the amount of coal that is combusted:

Amount of coal combusted = 100. - 0.14R = 98.8 lb

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Example 4.15 Combustion of Coal (*Continued***)**

Because all of the C and S in the coal that combusts ends up in the stack gas (*P*), we can form the following combined mole balance for C and S:

moles of C combusted moles of S combusted moles of C + S in flue gas
$$\frac{(0.8305)(98.8)}{12} + \frac{(0.007)(98.8)}{32} = 0.154P$$

Solving for P yields P = 44.54 lb. Now we can perform a nitrogen balance to determine A using the amount of combusted coal (98.8 lb):

$$\frac{N_2 \text{ in } F}{28}$$
 $\frac{N_2 \text{ in } A}{28}$ $\frac{N_2 \text{ in } P}{28}$ + 0.79 $\frac{N_2 \text{ in } P}{28}$

Solving yields A = 45.39 lb mol.

To calculate the percent excess air, because of the oxygen in the coal available for combustion and the existence of unburned combustibles including O, we use the total oxygen in and the required oxygen as shown previously in Equation (4.15):

% excess air =
$$100 \left(\frac{O_2 \text{ entering} - O_2 \text{ required}}{O_2 \text{ required}} \right)$$

The required O_2 is equal to the stoichiometric requirements for complete combustion of C, H, and S minus the O_2 present in the coal (not based on what actually combusted):