GLOBAL EDITION

Tenth Edition

# PRECALCULUS

## GRAPHICAL, NUMERICAL, ALGEBRAIC

Demana • Waits • Foley • Kennedy • Bock

# Precalculus

## Graphical, Numerical, Algebraic

Tenth Edition
Global Edition


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## Other Logistic Models

In Example 3, the bacteria cannot continue to grow exponentially forever because they cannot grow beyond the confines of the petri dish. In Example 7, though Florida's population is booming now, it will eventually level off, just as Pennsylvania's has done. Sunflowers and many other plants grow to a natural height following a logistic pattern. Chemical acid-base titration curves are logistic. Yeast cultures grow logistically. Contagious diseases and even rumors spread according to logistic models.

### EXAMPLE 8 Modeling a Rumor

Watauga High School has 1200 students. Bob, Carol, Ted, and Alice start a rumor, which spreads logistically so that $S(t) = 1200/(1 + 39e^{-0.9t})$ models the number of students who have heard the rumor by the end of Day $t$.

**(a)** How many students have heard the rumor by the end of Day 0?

**(b)** How long does it take for 1000 students to hear the rumor?

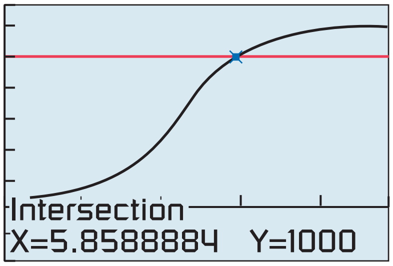
**SOLUTION**

**(a)** $S(0) = \dfrac{1200}{1 + 39e^{-0.9\cdot 0}} = \dfrac{1200}{1 + 39} = 30$. So, 30 students have heard the rumor by the end of Day 0.

**(b)** We need to solve $\dfrac{1200}{1 + 39e^{-0.9t}} = 1000$.

Figure 3.18 shows that the graph of $S(t) = 1200/(1 + 39e^{-0.9t})$ intersects $y = 1000$ when $t \approx 5.86$. So toward the end of Day 6 the rumor has reached the ears of 1000 students.

**Now try Exercise 45.**




[0, 10] by [−400, 1400]

**Figure 3.18** The spread of a rumor. (Example 8)

## QUICK REVIEW 3.2 *(For help, go to Section P.5.)*

**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

In Exercises 1 and 2, convert the percent to decimal form or the decimal into a percent.

**1.** 15%

**2.** 0.04

**3.** Show how to increase 23 by 7% using a single multiplication.

**4.** Show how to decrease 52 by 4% using a single multiplication.

In Exercises 5 and 6, solve the equation algebraically.

**5.** $40 \cdot b^2 = 160$

**6.** $243 \cdot b^3 = 9$

In Exercises 7–10, solve the equation numerically.

**7.** $782b^6 = 838$

**8.** $93b^5 = 521$

**9.** $672b^4 = 91$

**10.** $127b^7 = 56$

## SECTION 3.2 Exercises

In Exercises 1–6, tell whether the function is an exponential growth function or exponential decay function, and find the constant percentage rate of growth or decay.

**1.** $P(t) = 3.5 \cdot 1.09^t$

**2.** $P(t) = 4.3 \cdot 1.018^t$

**3.** $f(x) = 78{,}963 \cdot 0.968^x$

**4.** $f(x) = 5607 \cdot 0.9968^x$

**5.** $g(t) = 247 \cdot 2^t$

**6.** $g(t) = 43 \cdot 0.05^t$

In Exercises 7–18, determine the exponential function that satisfies the given conditions.

**7.** Initial value $=7$, increasing at a rate of 19% per year

**8.** Initial value $=52$, increasing at a rate of 2.3% per day

**9.** Initial value $=11$, decreasing at a rate of 60% per month

**10.** Initial value $=5$, decreasing at a rate of 0.59% per week

**11.** Initial population $=42{,}600$, increasing at a rate of 1.5% per year

**12.** Initial population $=502{,}000$, increasing at a rate of 1.7% per year

**13.** Initial height $=18$ cm, growing at a rate of 5.2% per week

**14.** Initial mass $=15$ g, decreasing at a rate of 4.6% per day

**15.** Initial mass $=0.6$ g, doubling every 3 days

**16.** Initial population $=250$, doubling every 7.5 hr

**17.** Initial mass $=592$ g, halving once every 6 years

**18.** Initial mass $=17$ g, halving once every 32 hr

In Exercises 19 and 20, determine a formula for the exponential function whose values are given in Table 3.11.

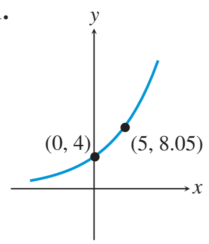
**19.** $f(x)$ **20.** $g(x)$

**Table 3.11** Values for Two Exponential Functions

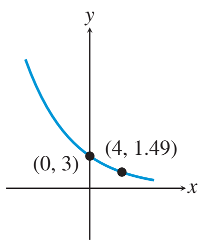
| $x$ | $f(x)$ | $g(x)$ |
|---|---|---|
| $-2$ | 1.472 | $-9.0625$ |
| $-1$ | 1.84 | $-7.25$ |
| 0 | 2.3 | $-5.8$ |
| 1 | 2.875 | $-4.64$ |
| 2 | 3.59375 | $-3.7123$ |

In Exercises 21 and 22, determine a formula for the exponential function whose graph is shown in the figure.

**21.**




**22.**




In Exercises 23–26, find the logistic function that satisfies the given conditions.

**23.** Initial value $=10$, limit to growth $=40$, passing through $(1, 20)$.

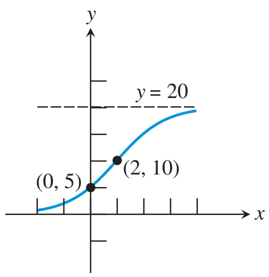
**24.** Initial value $=12$, limit to growth $=60$, passing through $(1, 24)$.

**25.** Initial population $=16$, maximum sustainable population $=128$, passing through $(5, 32)$.

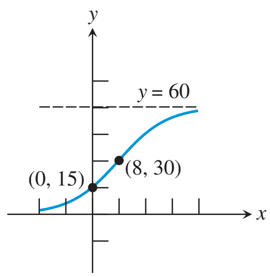
**26.** Initial height $=5$, limit to growth $=30$, passing through $(3, 15)$.

In Exercises 27 and 28, determine a formula for the logistic function whose graph is shown in the figure.

**27.**




**28.**




**29. Exponential Growth** In 2000 the population of Cairo, Egypt, was 13,626,000 and was increasing at the rate of 2.18% each year. At that rate, when will the population be 20 million?

**30. Exponential Growth** In 2000 the population of Delhi, India, was 15,692,000 and was increasing at the rate of 5.27% each year. At that rate, when should the population have reached 30 million?

**31. Exponential Growth** The population of Smallville in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year.

(a) Estimate the population in 1915 and 1940.

(b) Predict when the population reached 50,000.

**32. Exponential Growth** The population of River City in the year 1910 was 4200. Assume the population increased at a rate of 2.25% per year.

(a) Estimate the population in 1930 and 1945.

(b) Predict when the population reached 20,000.

**33. Radioactive Decay** The half-life of a certain radioactive substance is 14 days. There are 6.6 g present initially.

(a) Express the amount of substance remaining as a function of time $t$.

(b) When will there be less than 1 g remaining?

**34. Radioactive Decay** The half-life of a certain radioactive substance is 65 days. There are 3.5 g present initially.

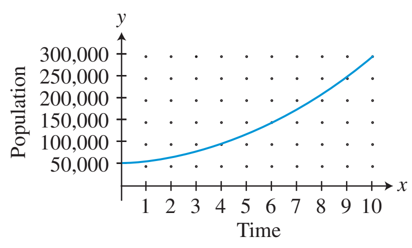
(a) Express the amount of substance remaining as a function of time $t$.

(b) When will there be less than 1 g remaining?

**35. Writing to Learn** Without using formulas or graphs, compare and contrast exponential functions and linear functions.

**36. Writing to Learn** Without using formulas or graphs, compare and contrast exponential functions and logistic functions.

**37. Writing to Learn** Using the population model that is graphed in the figure, explain why the time it takes the population to double (doubling time) is independent of the population size.



**38. Writing to Learn** Explain why the half-life of a radioactive substance is independent of the initial amount of the substance that is present.

**39. Bacterial Growth** The number $B$ of bacteria in a petri dish culture after $t$ hours is given by

$$B = 100e^{0.693t}.$$

When will the number of bacteria be 200? Estimate the doubling time of the bacteria.

**40. Radiocarbon Dating** The amount $C$ in grams of carbon-14 present in a certain substance after $t$ years is given by

$$C = 20e^{-0.0001216t}.$$

Estimate the half-life of carbon-14.

**41. Atmospheric Pressure** Determine the atmospheric pressure outside an aircraft flying at 52,800 ft (10 mi above sea level).

**42. Atmospheric Pressure** Find the altitude above sea level at which the atmospheric pressure is 2.5 lb/in.$^2$.

**43. Population Modeling** Use the 1950–2020 data in Table 3.12 and exponential regression to predict Santiago's population for 2030. Would logistic regression be a more appropriate model? Explain. (*Hint:* Let 1900 be $t = 0$.)

**44. Population Modeling** Use the 1950–2020 data in Table 3.12 and exponential regression to predict Kuala Lumpur's population for 2030. Would logistic regression be a more appropriate model? Explain. (*Hint:* Let 1900 be $t = 0$.)

**Table 3.12** Populations of Two World Cities (in thousands)

| Year | Kuala Lumpur (Malaysia) | Santiago (Chile) |
|---|---|---|
| 1950 | 262 | 1322 |
| 1960 | 344 | 1980 |
| 1970 | 451 | 2647 |
| 1980 | 971 | 3721 |
| 1990 | 2098 | 4616 |
| 2000 | 4176 | 5658 |
| 2010 | 5810 | 6269 |
| 2020 | 7997 | 6767 |

*Source: https://www.macrotrends.net.*

**45. Spread of Flu** The number of students infected with flu at Springfield High School after $t$ days is modeled by the function

$$P(t) = \frac{800}{1 + 49e^{-0.2t}}.$$

(a) What was the initial number of infected students?

(b) When will the number of infected students be 200?

(c) The school will close when 300 of the 800-student body are infected. When will the school close?

**46. Population of Deer** The population of deer after $t$ years in Cedar State Park is modeled by the function

$$P(t) = \frac{1001}{1 + 90e^{-0.2t}}.$$

(a) What was the initial population of deer?

(b) When will the number of deer be 600?

(c) What is the maximum number of deer possible in the park?

**47. Population Growth** Using all of the data in Table 3.9, compute a logistic regression model, and use it to predict the U.S. population in 2020.

**48. Population Growth** Using the data in Table 3.13, confirm the following model:

$$P(t) = \frac{18871.4}{1 + 13.3404e^{-0.033289t}}$$

**Table 3.13** Population of Buenos Aires, Argentina (in thousands)

| Year | Population |
|---|---|
| 1904 | 951 |
| 1950 | 5166 |
| 1960 | 6762 |
| 1970 | 8416 |
| 1980 | 9920 |
| 1990 | 11,148 |
| 2000 | 12,504 |
| 2010 | 14,246 |
| 2020 | 15,154 |

*Source: https://www.macrotrends.net.*

**49. Population Growth** Using the data in Table 3.14, confirm the model used in Exercise 56 of Section 3.1.

**Table 3.14** Populations of Two Countries (in millions)

| Year | South Korea | Japan |
|---|---|---|
| 1900 | 9.93 | 44.30 |
| 1910 | 10.20 | 49.60 |
| 1920 | 11.80 | 56.00 |
| 1930 | 13.90 | 64.30 |
| 1940 | 15.70 | 73.20 |
| 1950 | 19.21 | 82.80 |
| 1960 | 25.33 | 93.67 |
| 1970 | 32.20 | 104.93 |
| 1980 | 38.05 | 117.82 |
| 1990 | 42.92 | 124.51 |
| 2000 | 47.38 | 127.52 |
| 2010 | 49.55 | 128.54 |
| 2020 | 51.27 | 126.48 |

*Source: https://www.statista.com.*

**50. Population Growth** Using the data in Table 3.14, compute a logistic regression model for South Korea's population for $t$ years since 1900. Based on your model and Japan's population model from Exercise 56 of Section 3.1, will the population of South Korea ever surpass that of Japan? If so, when?

## Standardized Test Questions

**51. True or False** Exponential population growth is constrained with a maximum sustainable population. Justify your answer.

**52. True or False** If the constant percentage rate of an exponential function is negative, then the base of the function is negative. Justify your answer.

In Exercises 53–56, you may use a graphing calculator to solve the problem.

**53. Multiple Choice** What is the constant percentage growth rate of $P(t) = 1.23 \cdot 1.049^t$?

(A) 49% (B) 23% (C) 4.9%

(D) 2.3% (E) 1.23%

**54. Multiple Choice** What is the constant percentage decay rate of $P(t) = 22.7 \cdot 0.834^t$?

(A) 22.7% (B) 16.6% (C) 8.34%

(D) 2.27% (E) 0.834%

**55. Multiple Choice** A single-cell amoeba divides into two every 4 days. About how long will it take one amoeba to produce a population of 1000?

(A) 10 days (B) 20 days (C) 30 days

(D) 40 days (E) 50 days

**56. Multiple Choice** A rumor spreads logistically so that $S(t) = 789/(1 + 16 \cdot e^{-0.8t})$ models the number of persons who have heard the rumor by the end of $t$ days. Based on this model, which of the following is **true**?

(A) After 0 days, 16 persons have heard the rumor.

(B) After 2 days, 439 persons have heard the rumor.

(C) After 4 days, 590 persons have heard the rumor.

(D) After 6 days, 612 persons have heard the rumor.

(E) After 8 days, 769 persons have heard the rumor.

## Explorations

**57. Population Growth** (a) Use the 1900–2010 data in Table 3.9 and *logistic* regression to predict the U.S. population for 2016.

(b) **Writing to Learn** Compare the prediction with the value listed in the table for 2016.

(c) Noting the results of Example 6, which model—exponential or logistic—makes the better prediction in this case?

**58. Population Growth** Use all of the data in Tables 3.9 and 3.15.

(a) Based on exponential growth models, will Mexico's population surpass that of the United States, and if so, when?

(b) Based on logistic growth models, will Mexico's population surpass that of the United States, and if so, when?

(c) What are the maximum sustainable populations for the two countries?

(d) **Writing to Learn** Which model—exponential or logistic—is more valid in this case? Justify your choice.

**Table 3.15** Population of Mexico (in millions)

| Year | Population |
|---|---|
| 1900 | 13.6 |
| 1950 | 25.8 |
| 1960 | 34.9 |
| 1970 | 48.2 |
| 1980 | 66.8 |
| 1990 | 88.1 |
| 2001 | 101.9 |
| 2011 | 115.0 |
| 2016 | 123.2 |

*Sources: Statesman's Yearbook, and World Almanac and Book of Facts.*

## Extending the Ideas

**59.** The **hyperbolic sine function** is defined by $\sinh(x) = (e^x - e^{-x})/2$. Prove that sinh is an odd function.

**60.** The **hyperbolic cosine function** is defined by $\cosh(x) = (e^x + e^{-x})/2$. Prove that cosh is an even function.

**61.** The **hyperbolic tangent function** is defined by $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$.

(a) Prove that $\tanh(x) = \sinh(x)/\cosh(x)$.

(b) Prove that tanh is an odd function.

(c) Prove that $f(x) = 1 + \tanh(x)$ is a logistic function.

# 3.3 Logarithmic Functions and Their Graphs

### What you'll learn about

- Inverses of Exponential Functions
- Common Logarithms—Base 10
- Natural Logarithms—Base $e$
- Graphs of Logarithmic Functions
- Measuring Sound Using Decibels

### ... and why

Logarithmic functions are used in many applications, including the measurement of the relative intensity of sounds.

## Inverses of Exponential Functions

If a function passes the *horizontal line test*, then the inverse of the function is also a function. Figure 3.19 shows that an exponential function $f(x) = b^x$ would pass the horizontal line test. So it has an inverse that is a function. This inverse is the **logarithmic function with base *b***, denoted $\log_b(x)$, or more simply as $\log_b x$. That is, if $f(x) = b^x$ with $b > 0$ and $b \neq 1$, then $f^{-1}(x) = \log_b x$. See Figure 3.20.

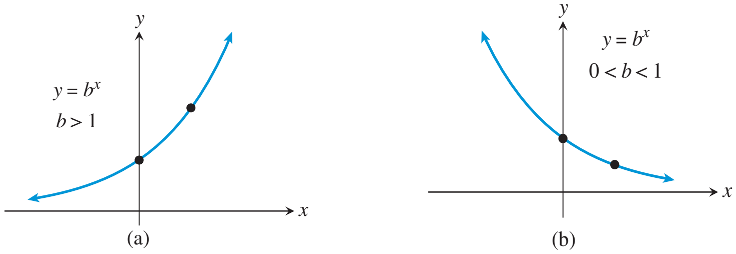



**Figure 3.19** Exponential functions are either (a) increasing or (b) decreasing.

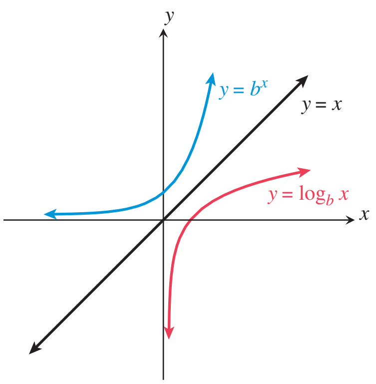



**Figure 3.20** Because logarithmic functions are inverses of exponential functions, we can obtain the graph of a logarithmic function by the mirror or rotational methods discussed in Section 1.5.

### A Bit of History

Logarithmic functions were developed around 1614 as computational tools by Scottish mathematician John Napier (1550–1617). He originally called them "artificial numbers," but changed the name to logarithms, which means "reckoning numbers."

An immediate and useful consequence of this definition is the link between an exponential equation and its logarithmic counterpart.

### Changing Between Logarithmic and Exponential Form

If $x > 0$ and $0 < b \neq 1$, then

$$y = \log_b(x) \quad \text{if and only if} \quad b^y = x.$$

This linking statement says that *a logarithm is an exponent*. Because logarithms are exponents, we can evaluate simple logarithmic expressions using our understanding of exponents.

### EXAMPLE 1 Evaluating Logarithms

**(a)** $\log_2 8 = 3$ because $2^3 = 8$.

**(b)** $\log_3 \sqrt{3} = 1/2$ because $3^{1/2} = \sqrt{3}$.

**(c)** $\log_5 \dfrac{1}{25} = -2$ because $5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$.

**(d)** $\log_4 1 = 0$ because $4^0 = 1$.

**(e)** $\log_7 7 = 1$ because $7^1 = 7$.

**Now try Exercise 1.**

We can generalize the relationships observed in Example 1.

### Generally $b > 1$

In practice, logarithmic bases are almost always greater than 1.

### Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number $y$,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

These properties give us efficient ways to evaluate simple logarithms and some exponential expressions. The first two parts of Example 2 are the same as the first two parts of Example 1.

### EXAMPLE 2 Evaluating Logarithmic and Exponential Expressions

**(a)** $\log_2 8 = \log_2 2^3 = 3.$

**(b)** $\log_3 \sqrt{3} = \log_3 3^{1/2} = 1/2.$

**(c)** $6^{\log_6 11} = 11.$

**Now try Exercise 5.**

Logarithmic functions are inverses of exponential functions. So the inputs and outputs are switched. Table 3.16 illustrates this relationship for $f(x) = 2^x$ and $f^{-1}(x) = \log_2 x$.

**Table 3.16** An Exponential Function and Its Inverse

| $x$ | $f(x) = 2^x$ | $x$ | $f^{-1}(x) = \log_2 x$ |
|---|---|---|---|
| $-3$ | $1/8$ | $1/8$ | $-3$ |
| $-2$ | $1/4$ | $1/4$ | $-2$ |
| $-1$ | $1/2$ | $1/2$ | $-1$ |
| 0 | 1 | 1 | 0 |
| 1 | 2 | 2 | 1 |
| 2 | 4 | 4 | 2 |
| 3 | 8 | 8 | 3 |

This relationship can be used to produce both tables and graphs for logarithmic functions, as you will discover in Exploration 1.

### EXPLORATION 1 Comparing Exponential and Logarithmic Functions

1. Set your grapher to Parametric mode and Simultaneous graphing mode.

   Set X1T = T and Y1T = 2^T.

   Set X2T = 2^T and Y2T = T.

   *Creating Tables*. Set TblStart = $-3$ and $\Delta$Tbl = 1. Use the Table feature of your grapher to obtain the decimal form of both parts of Table 3.16. Be sure to scroll to the right to see X2T and Y2T.

   *Drawing Graphs*. Set Tmin = $-6$, Tmax = 6, and Tstep = 0.5. Set the $(x, y)$ window to $[-6, 6]$ by $[-4, 4]$. Use the Graph feature to obtain the simultaneous graphs of $f(x) = 2^x$ and $f^{-1}(x) = \log_2 x$. Use the Trace feature to explore the numerical relationships within the graphs.

2. *Graphing in Function mode*. Graph $y = 2^x$ in the same window. Then use the "draw inverse" command to draw the graph of $y = \log_2 x$.

## Common Logarithms—Base 10

Because of their connection to our base-ten number system, the metric system, and scientific notation, logarithms with base 10 are especially useful; such logarithms are **common logarithms**. We often drop the subscript of 10 for the base when using common logarithms. The common logarithmic function $\log_{10} x = \log x$ is the inverse of the exponential function $f(x) = 10^x$. Therefore,

$$y = \log x \quad \text{if and only if} \quad 10^y = x.$$

Applying this relationship, we can obtain other relationships for logarithms with base 10.