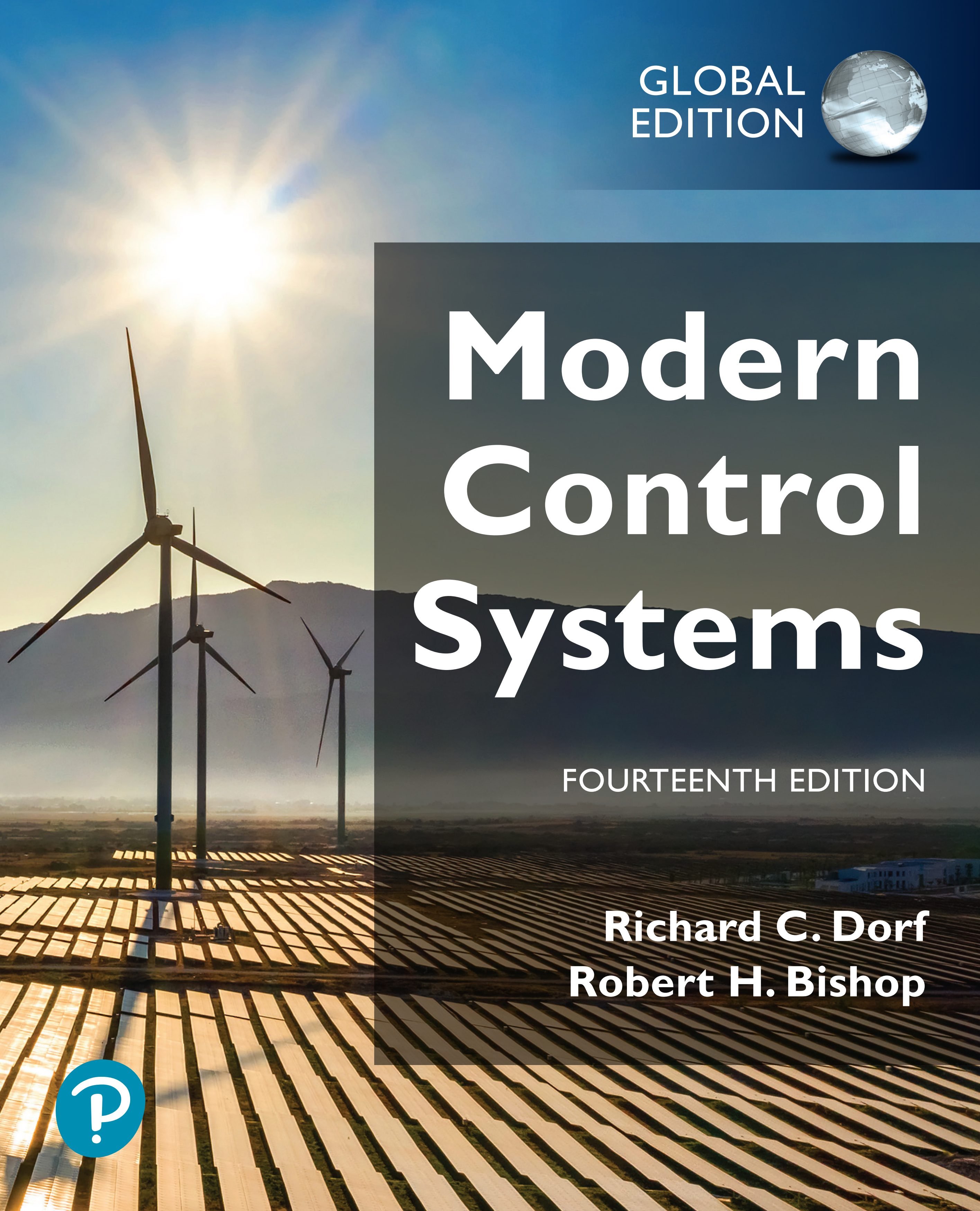


GLOBAL EDITION

Modern Control Systems

FOURTEENTH EDITION

Richard C. Dorf
Robert H. Bishop

Modern Control Systems

where $\zeta = 0.707$ and $\omega_n = 10$. The transfer function of the power amplifier is approximately

$$G_1(s) = \frac{k_a}{\tau s + 1},$$

where $\tau = 0.2$ s. (a) Determine the sensitivity of the system to a change of the parameter k_a. (b) The system is subjected to a disturbance $T_d(s) = 1/s$. Determine the required magnitude of k_a in order to maintain the steady-state error of the system less than 20° when the input $R(s)$ is zero. (c) Determine the error of the system when subjected to a disturbance $T_d(s) = 10/s$ when it is operating as an open-loop system ($k_s = 0$) with $R(s) = 0$.

P4.6 An automatic speed control system will be necessary for passenger cars traveling on the automatic highways of the future. A model of a feedback speed control system for a standard vehicle is shown in Figure P4.6. The load disturbance due to a percent grade $\Delta T_d(s)$ is also shown. The engine gain K_e varies within the range of 10 to 1000 for various models of automobiles. The engine time constant τ_e is 20 seconds. (a) Determine the sensitivity of the system to changes in the engine gain K_e. (b) Determine the effect of the load torque on the speed. (c) Determine the constant percent grade $\Delta T_d(s) = \Delta d/s$ for which the vehicle stalls (velocity $V(s) = 0$) in terms of the gain factors. Note that since the grade is constant, the steady-state solution is sufficient. Assume that $R(s) = 30/s$ km/hr and that $K_e K_1 \gg 1$. When $K_g/K_1 = 2$, what percent grade Δd would cause the automobile to stall?

P4.7 A robot uses feedback to control the orientation of each joint axis. The load effect varies due to varying load objects and the extended position of the arm. The system will be deflected by the load carried in the gripper. Thus, the system may be represented by Figure P4.7, where the load torque is $T_d(s) = D/s$. Assume $R(s) = 0$ at the index position. (a) What is the effect of $T_d(s)$ on $Y(s)$? (b) Determine the sensitivity of the closed loop to k_2. (c) What is the steady-state error when $R(s) = 1/s$ and $T_d(s) = 0$?

P4.8 Extreme temperature changes result in many failures of electronic circuits [1]. Temperature control feedback systems reduce the change of temperature by using a heater to overcome outdoor low temperatures. A block diagram of one system is shown in Figure P4.8. The effect of a drop in environmental temperature is a step decrease in $T_d(s)$. The actual temperature of the electronic circuit is $Y(s)$. The dynamics of the electronic circuit temperature change are represented by the transfer function.

$$G(s) = \frac{100}{s^2 + 25s + 100}.$$

(a) Determine the sensitivity of the system to K.
(b) Obtain the effect of the disturbance $T_d(s)$ on the output $Y(s)$.
(c) Find the range of K such that the output $Y(s)$ is less than 10% of the step disturbance input with magnitude A (that is, $T_d(s) = A/s$).

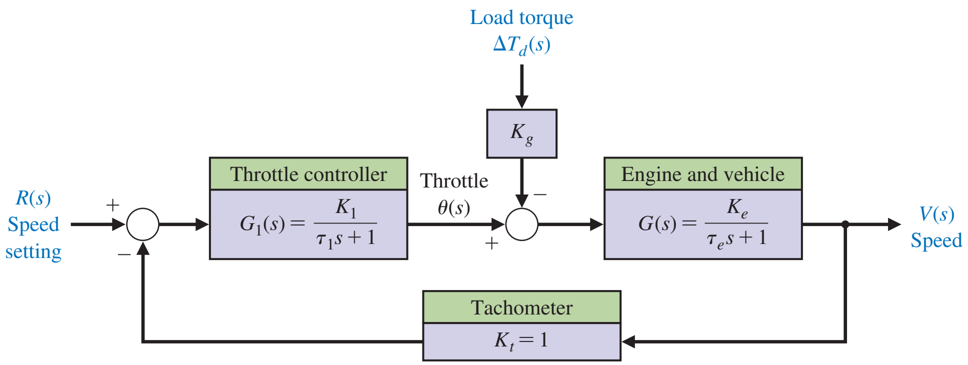


FIGURE P4.6 Automobile speed control.

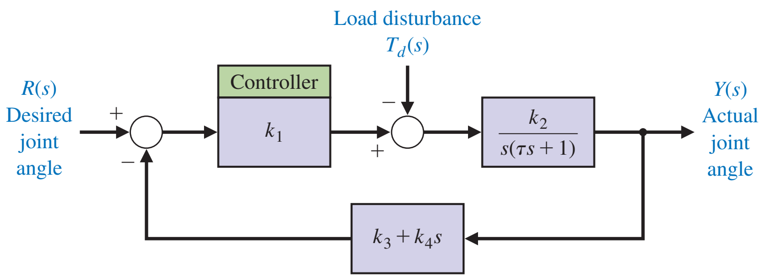


FIGURE P4.7 Robot control system.

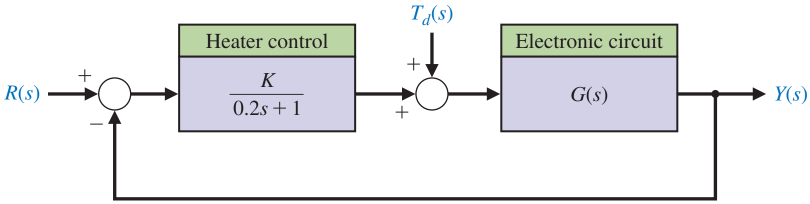


FIGURE P4.8 Temperature control system.

P4.9 A useful unidirectional sensing device is the photoemitter sensor [15]. A light source is sensitive to the emitter current flowing and alters the resistance of the photosensor. Both the light source and the photoconductor are packaged in a single four-terminal device. This device provides a large gain and total isolation. A feedback circuit utilizing this device is shown in Figure P4.9(a), and a typical nonlinear resistance–current characteristic is shown in Figure P4.9(b). The resistance curve can be represented by the equation

$$\log_{10} R = \frac{0.175}{(i - 0.005)^{1/2}},$$

where i is the lamp current. The normal operating point is obtained when $v_{in} = 2.0$ V, and $v_o = 35$ V. (a) Determine the closed-loop transfer function of the system. (b) Determine the sensitivity of the system to changes in the gain, K.

P4.10 For a paper processing plant, it is important to maintain a constant tension on the continuous sheet of paper between the wind-off and wind-up rolls. The tension varies as the widths of the rolls change, and an adjustment in the take-up motor speed is necessary, as shown in Figure P4.10. If the wind-up motor speed is uncontrolled, as the paper transfers from the wind-off roll to the wind-up roll, the velocity $v_0(t)$ decreases and the tension of the paper drops [10, 14]. The three-roller and spring combination provides a measure of the tension of the paper. The spring force is equal to $k_1 Y(s)$, and the linear differential transformer, rectifier, and amplifier may be represented by $E_0(s) = -k_2 Y(s)$. Therefore, the

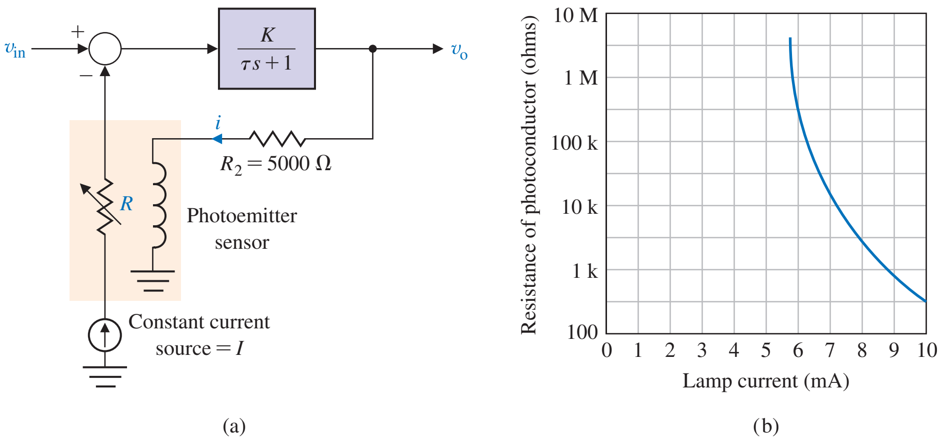


FIGURE P4.9 Photosensor system.

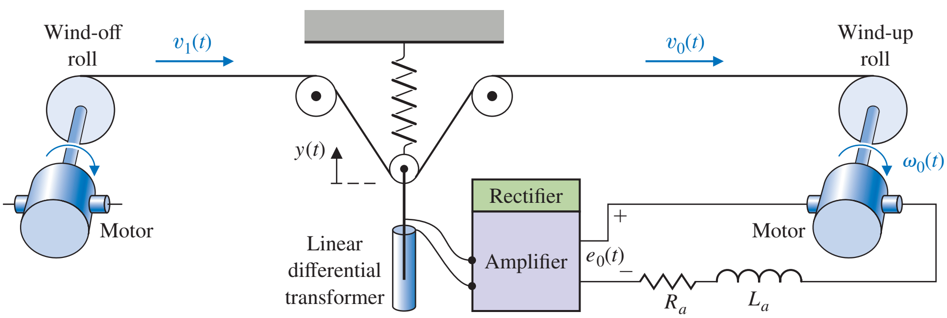


FIGURE P4.10 Paper tension control.

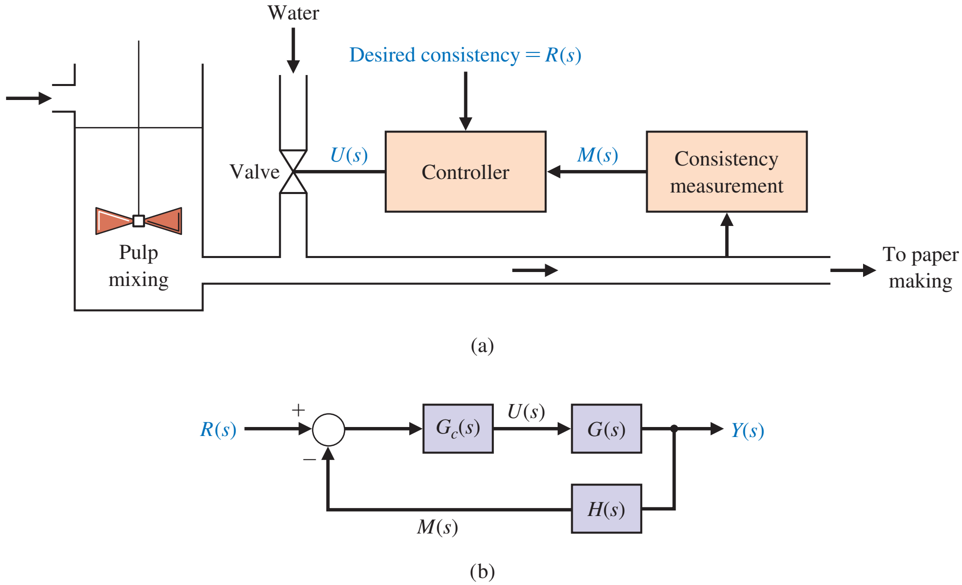


FIGURE P4.11 Paper-making control.

measure of the tension is described by the relation $2T(s) = k_1 Y(s)$, where $Y(s)$ is the deviation from the equilibrium condition, and $T(s)$ is the vertical component of the deviation in tension from the equilibrium condition. The time constant of the motor is $\tau = L_a/R_a$, and the linear velocity of the wind-up roll is twice the angular velocity of the motor, that is, $V_0(s) = 2\omega_0(s)$. The equation of the motor is then

$$E_0(s) = \frac{1}{K_m}[\tau s\omega_0(s) + \omega_0(s)] + k_3\,\Delta T(s),$$

where $\Delta T(s) =$ a tension disturbance. (a) Draw the closed-loop block diagram for the system, including the disturbance $\Delta T(s)$. (b) Add the effect of a disturbance in the wind-off roll velocity $\Delta V_1(s)$ to the block diagram. (c) Determine the sensitivity of the system to the motor constant K_m. (d) Determine the steady-state error in the tension when a step disturbance in the input velocity, $\Delta V_1(s) = A/s$, occurs.

P4.11 One important objective of the paper-making process is to maintain uniform consistency of the stock output as it progresses to drying and rolling. A diagram of the thick stock consistency dilution control system is shown in Figure P4.11(a). The amount of water added determines the consistency. The block diagram of the system is shown in Figure P4.11(b). Let $H(s) = 1$ and

$$G_c(s) = \frac{K}{8s+1}, \qquad G(s) = \frac{1}{3s+1}.$$

Determine (a) the closed-loop transfer function $T(s) = Y(s)/R(s)$, (b) the sensitivity S_K^T, and (c) the steady-state error for a step change in the desired consistency $R(s) = A/s$. (d) Calculate the value of K required for an allowable steady-state error of 2%.

P4.12 Two feedback systems are shown in Figures P4.12(a) and (b). (a) Evaluate the closed-loop transfer functions T_1 and T_2 for each system. (b) Compare the sensitivities of the two systems with respect to the parameter K_1 for the nominal values of $K_1 = K_2 = 1$.

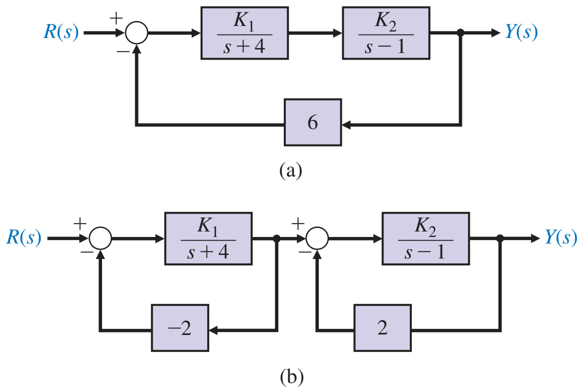


FIGURE P4.12 Two feedback systems.

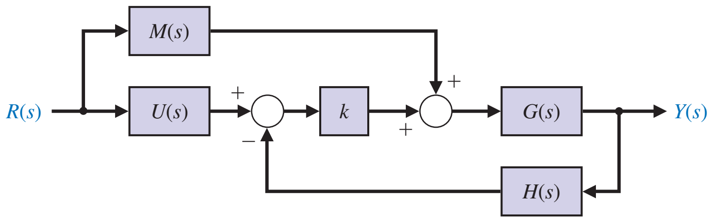


FIGURE P4.13 Closed-loop system.

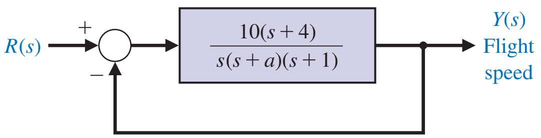


FIGURE P4.14 Hypersonic airplane speed control.

P4.13 One form of a closed-loop transfer function is

$$T(s) = \frac{G_1(s) + kG_2(s)}{G_3(s) + kG_4(s)}.$$

(a) Show that

$$S_k^T = \frac{k(G_2G_3 - G_1G_4)}{(G_3 + kG_4)(G_1 + kG_2)}.$$

(b) Determine the sensitivity of the system shown in Figure P4.13, using the equation verified in part (a).

P4.14 A proposed hypersonic plane would climb to 80,000 feet, fly 3800 miles per hour, and cross the Pacific in 2 hours. Control of the aircraft speed could be represented by the model in Figure P4.14. (a) Find the sensitivity of the closed-loop transfer function $T(s)$ to a small change in the parameter a. (b) What is the range of the parameter a for a stable closed-loop system?

P4.15 Figure P4.15 shows the model of a two-tank system containing a heated liquid, where $T_0(s)$ is the temperature of the fluid flowing into the first tank and $T_2(s)$ is the temperature of the liquid flowing out of the second tank. The system of two tanks has a heater in the first tank with a controllable heat input Q. The time constants are $\tau_1 = 10$ s and $\tau_2 = 50$ s. (a) Determine $T_2(s)$ in terms of $T_0(s)$ and $T_{2d}(s)$. (b) If $T_{2d}(s)$, the desired output temperature, is changed instantaneously from $T_{2d}(s) = A/s$ to $T_{2d}(s) = 2A/s$, where $T_0(s) = A/s$, determine the transient response of $T_2(s)$ when $G_c(s) = K = 500$. (c) Find the steady-state error e_{ss} for the system of part (b), where $E(s) = T_{2d}(s) - T_2(s)$.

P4.16 The steering control of a modern ship may be represented by the system shown in Figure P4.16 [16, 20]. (a) Find the steady-state effect of a constant wind force represented by $T_d(s) = 1/s$ for $K = 10$ and $K = 25$. Assume that the rudder input $R(s)$

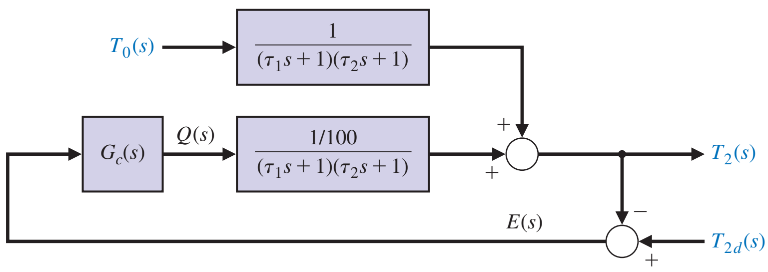


FIGURE P4.15 Two-tank temperature control.

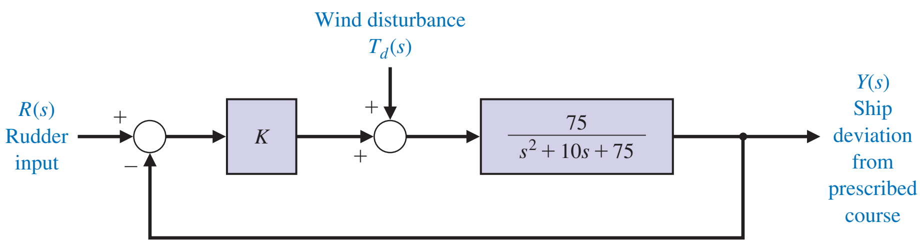
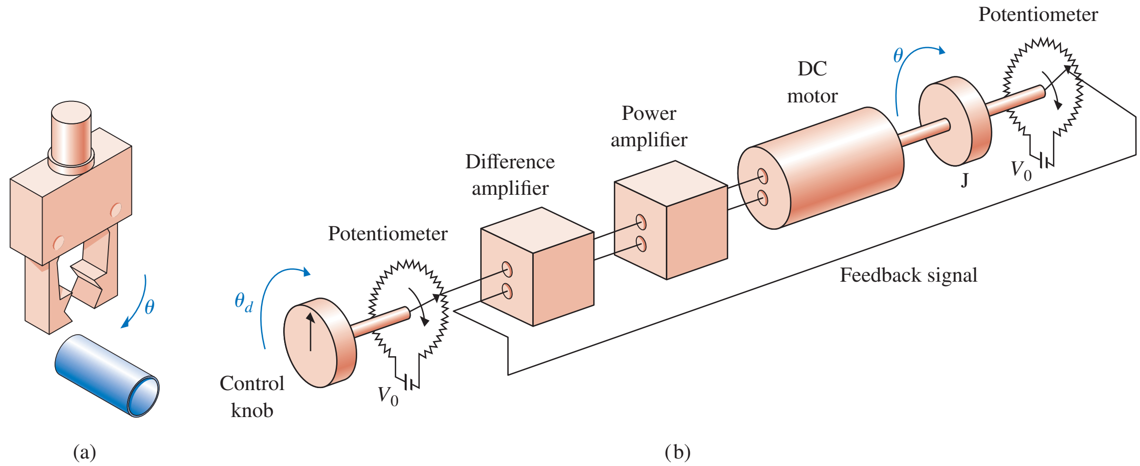


FIGURE P4.16 Ship steering control.

is zero, without any disturbance, and has not been adjusted. (b) Show that the rudder can then be used to bring the ship deviation back to zero.

P4.17 A robot gripper, shown in part (a) of Figure P4.17, is to be controlled so that it closes to an angle θ by using a DC motor control system, as shown in part (b). The model of the control system is shown in part (c), where $K_m = 30$, $R_f = 1\ \Omega$, $K_f = K_i = 1$, $J = 0.1$, and $b = 1$. (a) Determine the response $\theta(t)$ of the system to a step change in $\theta_d(t)$ when $K = 20$. (b) Assuming $\theta_d(t) = 0$, find the effect of a load disturbance $T_d(s) = A/s$. (c) Determine the steady-state error e_{ss} when the input is $r(t) = t$, $t > 0$. (Assume that $T_d(s) = 0$.)



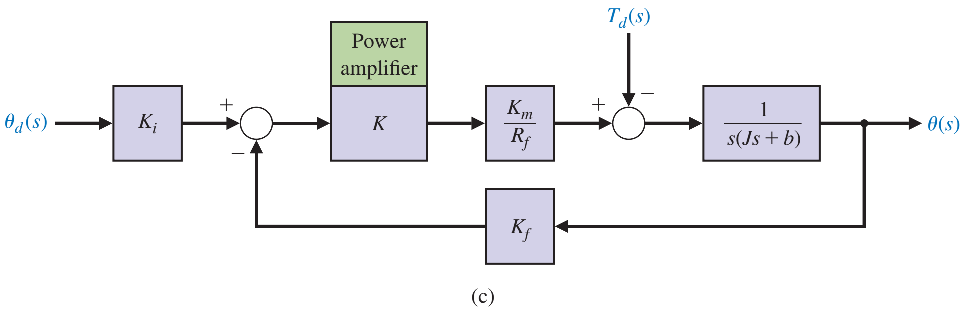


FIGURE P4.17 Robot gripper control.

ADVANCED PROBLEMS

AP4.1 A tank level regulator control is shown in Figure AP4.1(a). It is desired to regulate the level $H(s)$ in response to a disturbance change $Q_3(s)$. The block diagram shows small variable changes about the equilibrium conditions so that the desired $H_d(s) = 0$. Determine the equation for the error $E(s)$, and determine the steady-state error for a unit step disturbance when (a) $G(s) = K$ and (b) $G(s) = K/s$.

AP4.2 The shoulder joint of a robotic arm uses a DC motor with armature control and a set of gears on the output shaft. The model of the system is shown in Figure AP4.2 with a disturbance torque $T_d(s)$ which represents the effect of the load. Determine the steady-state error when the desired angle input is a step so that $\theta_d(s) = A/s$, $G_c(s) = K$, and the disturbance input is zero. When $\theta_d(s) = 0$ and the load

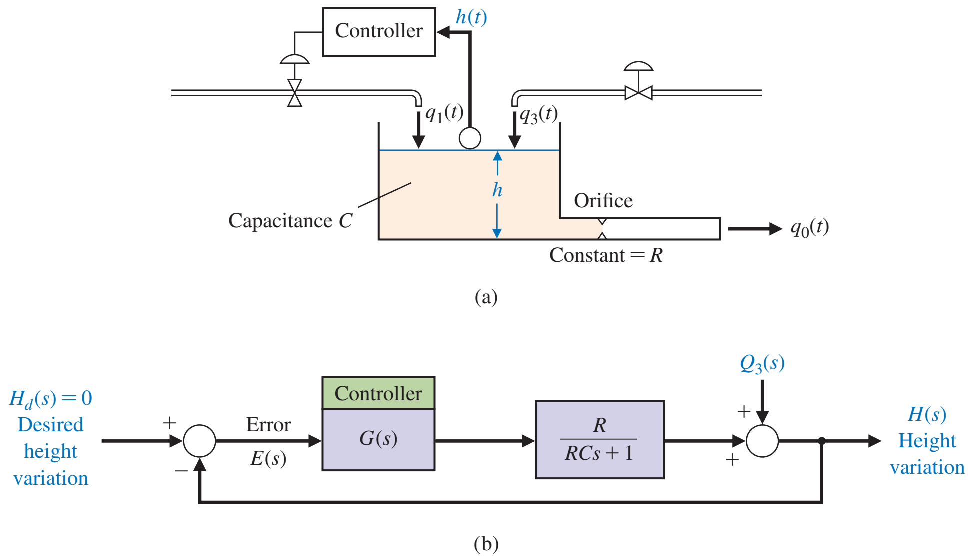


FIGURE AP4.1
A tank level regulator.

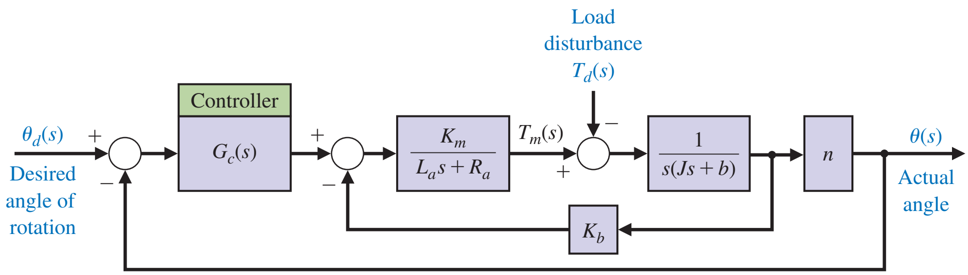


FIGURE AP4.2
Robot joint control.

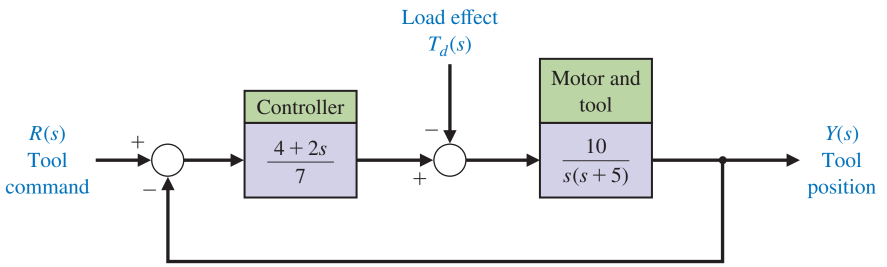


FIGURE AP4.3
Machine tool feedback.

effect is $T_d(s) = M/s$, determine the steady-state error when (a) $G_c(s) = K$ and (b) $G_c(s) = K/s$.

AP4.3 A machine tool is designed to follow a desired path so that

$$r(t) = (1 - t)u(t),$$

where $u(t)$ is the unit step function. The machine tool control system is shown in Figure AP4.3.

(a) Determine the steady-state error when $R(s)$ is the desired path as given and $T_d(s) = 0$.
(b) Plot the error $e(t)$ for the desired path for part (a) for $0 < t \leq 10$ s.
(c) If $R(s) = 0$, find the steady-state error when $T_d(s) = 1/s$.
(d) Plot the error $e(t)$ for part (c) for $0 < t \leq 10$ s.