GLOBAL EDITION

Essential University Physics

Volume 1

FOURTH EDITION

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Chapters 1–19

Essential University Physics

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Conservation of Energy

Learning Outcomes

After finishing this chapter you should be able to:

LO 7.1 Distinguish conservative from nonconservative forces.

LO 7.2 Calculate potential energy, especially with gravity and springs.

LO 7.3 Use conservation of mechanical energy to solve problems that would be difficult using Newton's second law.

LO 7.4 Evaluate situations where nonconservative forces result in loss of mechanical energy.

LO 7.5 Distinguish internal energy from mechanical energy.

LO 7.6 Work with potential-energy curves for a wide variety of systems.

Skills & Knowledge You'll Need

- The concept of work (Section 6.1)
- The concept of kinetic energy (Section 6.4)
- The work–kinetic energy theorem (Section 6.4)

The rock climber of Fig. 7.1*a* does work as she ascends the vertical cliff. So does the mover of Fig. 7.1*b* as he pushes a heavy chest across the floor. But there's a difference. If the rock climber lets go, down she goes, gaining kinetic energy as she falls. If the mover lets go of the chest, though, he and the chest stay right where they are.

This contrast highlights a distinction between two types of forces, called *conservative* and *nonconservative*. That distinction will help us develop one of the most important principles in physics: **conservation of energy**. The introduction to Chapter 6 briefly mentioned three forms of energy: kinetic energy, potential energy, and internal energy—although there we worked quantitatively only with kinetic energy. Here we'll develop the concept of potential energy and show how it's associated with conservative forces. Nonconservative forces, in contrast, are associated with irreversible transformations of mechanical energy into internal energy. We'll take a brief look at such transformations here and formulate a broad statement of energy conservation. In Chapters 16–19 we'll elaborate on internal energy and see how it's related to temperature, and we'll expand our statement of energy conservation to include not only work but also heat as modes of energy transfer.

How many different energy conversions take place as the Yellowstone River plunges over Yellowstone Falls?

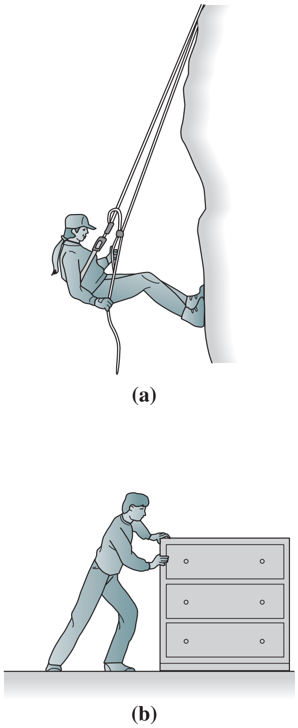


FIGURE 7.1 Both the rock climber and the mover do work, but only the climber can recover that work as kinetic energy.

7.1 Conservative and Nonconservative Forces

LO 7.1 *Distinguish conservative from nonconservative forces.*

Both the climber and the mover in Fig. 7.1 are doing work against forces—gravity for the climber and friction for the mover. The difference is this: If the climber lets go, the gravitational force "gives back" the energy she supplied by doing work, which then manifests itself as the kinetic energy of her fall. But the frictional force doesn't "give back" the energy supplied by the mover, in the sense that this energy can't be recovered as kinetic energy.

A **conservative force** is a force like gravity or a spring that "gives back" energy that was transferred by doing work. A more precise description of what it means for a force to be conservative follows from considering the work involved as an object moves over a closed path—one that ends where it started. Suppose our rock climber ascends a cliff of height h and then descends to her starting point. As she climbs, the gravitational force is directed opposite to her motion, so gravity does negative work $-mgh$ (recall Fig. 6.4). When she descends, the gravitational force is in the same direction as her motion, so the gravitational work is $+mgh$. The total work that gravity does on the climber as she traverses the closed path up and down the cliff is therefore zero.

Now consider the mover in Fig. 7.1*b*. Suppose he pushes the chest across a room, discovers it's the wrong room, and pushes it back to the door. Like the climber, the mover and chest describe a closed path. But the frictional force always acts to oppose the chest's motion. The mover needs to apply a force to oppose friction, so he ends up doing positive work as he crosses the room in both directions. Therefore, the total work he does is positive even when he moves the chest over a closed path. That's the nature of the frictional force, and, in contrast to the conservative gravitational force the climber had to deal with, this makes friction a **nonconservative force**.

Our two examples clearly distinguish between conservative and nonconservative forces: Only for *conservative* forces is the work done in moving around a closed path equal to zero. This fact provides a precise definition of a conservative force:

> When the total work done by a force $\vec{F}$ acting as an object moves over any closed path is zero, then the force is conservative.

This definition suggests a related property of conservative forces. Suppose a conservative force acts on an object in the region shown in Fig. 7.2. Move the object along the straight path from point A to point B, and designate the work done by the conservative force as W_{AB}. Since the work done over any closed path is zero, the work W_{BA} done in moving back from B to A must be $-W_{AB}$, whether we return along the straight path, the curved path, or any other path. So, going from A to B involves work W_{AB}, regardless of the path taken. In other words:

> The work done by a conservative force in moving between two points is independent of the path taken.

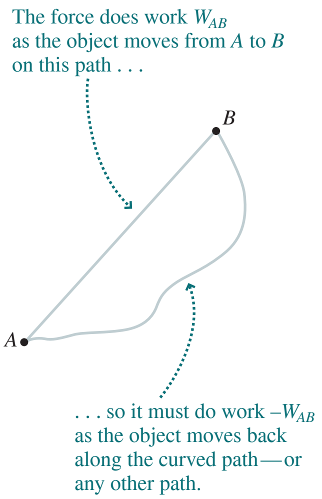


FIGURE 7.2 The work done by a conservative force is independent of path.

Important examples of conservative forces include gravity and the static electric force. The force of an ideal spring—fundamentally an electric force—is also conservative. Nonconservative forces include friction, drag forces, and the electric force in the presence of time-varying magnetic effects, which we'll encounter in Chapter 27.

GOT IT?

7.1 Suppose it takes the same amount of work to push a trunk straight across a rough floor as it does to lift a weight the same distance straight upward. If both trunk and weight are moved instead on identically shaped curved paths between the same two points as before, is the work (a) still the same for both, (b) greater for the weight, or (c) greater for the trunk?

Equation 6.11 introduced a general expression for the work done when an object moves along an arbitrary path, subject to a force that might vary with position: $W = \int_A^B \vec{F} \cdot d\vec{r}$, where A and B are the endpoints of the path. For a closed path, the two endpoints are the same, which we designate by putting a circle on the integral sign. Thus, our definition of a conservative force can be written mathematically as

$$\oint \vec{F} \cdot d\vec{r} = 0 \qquad \text{(conservative force)} \tag{7.1}$$

Following Fig. 7.2, we can equally well describe a conservative force with the statement that $\int_A^B \vec{F} \cdot d\vec{r}$ is independent of the path taken between the endpoints A and B.

7.2 Potential Energy

LO 7.2 *Calculate potential energy, especially with gravity and springs.*

The climber in Fig. 7.1*a* did work ascending the cliff, and the energy transferred as she did that work was somehow stored, in that she could get it back in the form of kinetic energy. She's acutely aware of that stored energy, since it gives her the potential for a dangerous fall. *Potential* is an appropriate word here: The stored energy is **potential energy**, in the sense that it has the potential to be converted into kinetic energy.

We'll give potential energy the symbol U, and we begin by defining *changes* in potential energy. Specifically:

The change ΔU_{AB} in potential energy associated with a conservative force is the negative of the work done by that force as it acts over any path from point A to point B:

ΔU_{AB} is the change in an object's potential energy as it moves from point A to point B under the influence of a conservative force $\vec{F}$.

$\vec{F}$ is the conservative force.

$d\vec{r}$ is an infinitesimal displacement.

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r} \qquad \text{(potential energy)} \tag{7.2}$$

The minus sign arises because an *increase* in potential energy results when the object moves *against* the conservative force, in which case the force does negative work.

Equation 6.11 showed that this integral is the work done *by* the conservative force.

The annotations on Equation 7.2 explain the minus sign. But another way to think about this is to consider the work *you* would have to do in order to counter a conservative force like gravity. If $\vec{F}$ is the conservative force (e.g., gravity, pointing down), then you'd have to apply a force $-\vec{F}$ (e.g., upward), and the work you do would be $\int_A^B (-\vec{F}) \cdot d\vec{r}$ or $-\int_A^B \vec{F} \cdot d\vec{r}$, which is the right-hand side of Equation 7.2. Your work represents a transfer of energy, which here ends up stored as potential energy. So another way of interpreting Equation 7.2 is to say that the change in potential energy is equal to the work an external agent would have to do in just countering a conservative force.

Changes in potential energy are all that ever matter physically; the actual value of potential energy is meaningless. Often, though, it's convenient to establish a reference point

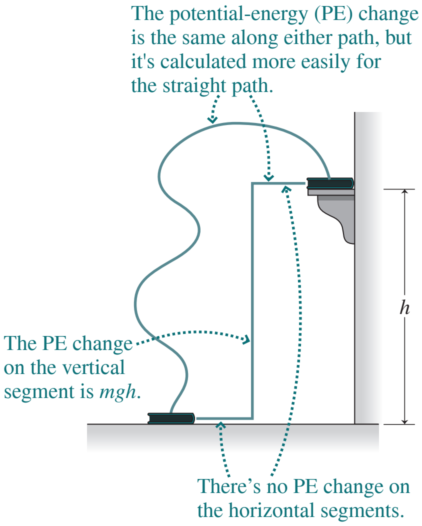


FIGURE 7.3 A good choice of path makes it easier to calculate the potential-energy change.

at which the potential energy is defined to be zero. When we say "the potential energy U," we really mean the potential-energy difference ΔU between that reference point and whatever other point we're considering. Our rock climber, for example, might find it convenient to take the zero of potential energy at the base of the cliff. But the choice is purely for convenience; only potential-energy *differences* really matter. We'll often drop the subscript AB and write simply ΔU for a potential-energy difference. Keeping the subscript is important, though, when we need to be clear about whether we're going from A to B or from B to A.

Equation 7.2 is a completely general definition of potential energy, applicable in all circumstances. Often, though, we can consider a path where force and displacement are parallel (or antiparallel). Then Equation 7.2 simplifies to

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx \tag{7.2a}$$

where x_1 and x_2 are the starting and ending points on the x-axis, taken to coincide with the path. When the force is constant, this equation simplifies further to

$$\Delta U = -F(x_2 - x_1) \tag{7.2b}$$

TIP **UNDERSTAND YOUR EQUATIONS** Equation 7.2b provides a very simple expression for potential-energy changes, but it applies *only* when the force is constant. Equation 7.2b is a special case of Equation 7.2a that follows because a constant force can be taken outside the integral.

Gravitational Potential Energy

We're frequently moving things up and down, causing changes in potential energy. Figure 7.3 shows two possible paths for a book that's lifted from the floor to a shelf of height h. Since the gravitational force is conservative, we can use either path to calculate the potential-energy change. It's easiest to use the path consisting of straight segments. No work or potential-energy change occurs on the horizontal segments since the gravitational force is perpendicular to the motion. For the vertical lift, the force of gravity is constant and Equation 7.2b immediately gives $\Delta U = mgh$, where the minus sign in Equation 7.2b cancels with the minus sign associated with the *downward* direction of gravity. This result is quite general: When a mass m undergoes a vertical displacement Δy near Earth's surface, gravitational potential energy changes by

$$\Delta U = mg\,\Delta y \qquad \text{(gravitational potential energy)} \tag{7.3}$$

The quantity Δy can be positive or negative, depending on whether the object moves up or down; correspondingly, the potential energy can either increase or decrease. We emphasize that Equation 7.3 applies *near Earth's surface*—that is, for distances small compared with Earth's radius. That assumption allows us to treat the gravitational force as constant over the path. We'll explore the more general case in Chapter 8.

We've found the *change* in potential energy associated with raising the book, but what about the potential energy itself? That depends on where we define the zero of potential energy. If we choose $U = 0$ at the floor, then $U = mgh$ on the shelf. But we could just as well take $U = 0$ at the shelf; then potential energy when the book is on the floor would be $-mgh$. Negative potential energies arise frequently, and that's OK because only *differences* in potential energy really matter. Figure 7.4 shows a plot of potential energy versus height with $U = 0$ taken at the floor. The *linear* increase in potential energy with height reflects the *constant* gravitational force.

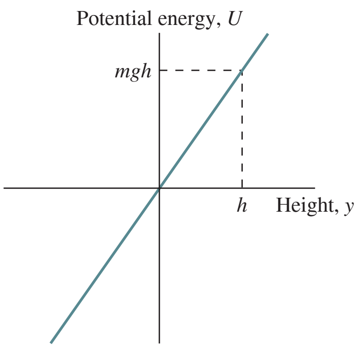


FIGURE 7.4 Gravitational force is constant, so potential energy increases linearly with height.

Elastic Potential Energy

When you stretch or compress a spring or other elastic object, you do work against the spring force, and that work ends up stored as **elastic potential energy**. For an ideal spring, the force is $F = -kx$, where x is the distance the spring is stretched from equilibrium, and

EXAMPLE 7.1 Gravitational Potential Energy: Riding the Elevator

A 55-kg engineer leaves her office on the 33rd floor of a skyscraper and takes an elevator up to the 59th floor. Later she descends to street level. If the engineer chooses the zero of potential energy at her office and if the distance from one floor to the next is 3.5 m, what's the potential energy when the engineer is (a) in her office, (b) on the 59th floor, and (c) at street level?

INTERPRET This is a problem about gravitational potential energy relative to a specified point of zero energy—namely, the engineer's office.

DEVELOP Equation 7.3, $\Delta U = mg\,\Delta y$, gives the change in gravitational energy associated with a change Δy in vertical position. We're given positions in floors, not meters, so we need to convert using the given factor 3.5 m per floor.

EVALUATE (a) When the engineer is in her office, the potential energy is zero, since she defined it that way. (b) The 59th floor is $59 - 33 = 26$ floors higher, so the potential energy when she's there is

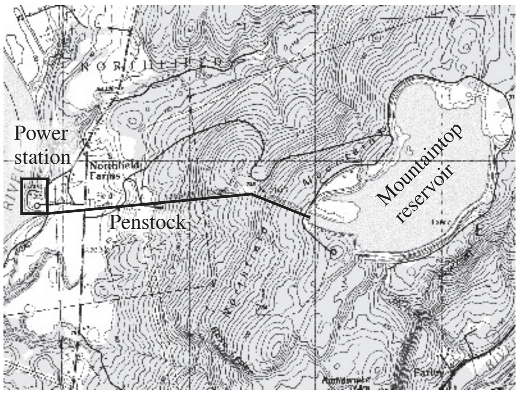
$$U_{59} = mg\,\Delta y = (55\text{ kg})(9.8\text{ m/s}^2)(26\text{ floors})(3.5\text{ m/floor}) = 49\text{ kJ}$$

Here we can write U rather than ΔU because we're calculating the potential-energy *change* from the place where $U = 0$. (c) The street level is 32 floors *below* the engineer's office, so

$$U_{\text{street}} = mg\,\Delta y = (55\text{ kg})(9.8\text{ m/s}^2)(-32\text{ floors})(3.5\text{ m/floor}) = -60\text{ kJ}$$

ASSESS Makes sense: When the engineer goes *up*, the potential energy relative to her office is positive; when she goes *down*, it's negative. And the distance down is a bit farther, so the magnitude of the change is greater going down.

APPLICATION Pumped Storage



Electricity is a wonderfully versatile form of energy, but it's not easy to store. Large electric power plants are most efficient when operated continuously, yet the demand for power fluctuates. Renewable energy sources like wind and solar vary, not necessarily with demand. Energy storage can help in both cases. Today, the only practical way to store large amounts of excess electrical energy is to convert it to gravitational potential energy. In so-called pumped-storage facilities, surplus electric power pumps water from a lower reservoir to a higher one, thereby increasing gravitational potential energy. When power demand is high, water runs back down, turning the pump motors into generators that produce electricity. The map here shows the Northfield Mountain Pumped Storage Project in Massachusetts, including the mountaintop reservoir, the location of the power station 214 m below on the Deerfield River, and the *penstock*, the pipe that conveys water in both directions between the power station and the reservoir. You can explore this facility quantitatively in Problem 35.

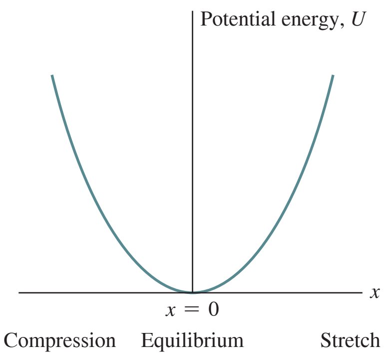


FIGURE 7.5 The potential-energy curve for a spring is a parabola.

the minus sign shows that the force opposes the stretching or compression. Since the force varies with position, we use Equation 7.2a to evaluate the potential energy:

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx = -\int_{x_1}^{x_2} (-kx)\,dx = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$

where x_1 and x_2 are the initial and final values of the stretch. If we take $U = 0$ when $x = 0$ (that is, when the spring is neither stretched nor compressed) then we can use this result to write the potential energy at an arbitrary stretch (or compression) x as

$$U = \tfrac{1}{2}kx^2 \qquad \text{(elastic potential energy)} \qquad (7.4)$$

Comparison with Equation 6.10, $W = \frac{1}{2}kx^2$, shows that this is equal to the work done in stretching the spring. Thus the energy transferred by doing work gets stored as potential energy. Figure 7.5 shows potential energy as a function of the stretch or compression of a spring. The *parabolic* shape of the potential-energy curve reflects the *linear* change of the spring force with stretch or compression.

EXAMPLE 7.2 Energy Storage: Springs versus Gasoline

A car's suspension consists of springs with an overall effective spring constant of 120 kN/m. How much would you have to compress the springs to store the same amount of energy as in 1 g of gasoline?

INTERPRET This problem is about the energy stored in a spring, as compared with the chemical energy of gasoline.

DEVELOP Equation 7.4, $U = \frac{1}{2}kx^2$, gives a spring's stored energy when it's been compressed a distance x. Here we want that energy to equal the energy in 1 g of gasoline. We can get that value from the "Energy Content of Fuels" table in Appendix C, which lists 44 MJ/kg for gasoline.

EVALUATE At 44 MJ/kg, the energy in 1 g of gasoline is 44 kJ. Setting this equal to the spring energy $\frac{1}{2}kx^2$ and solving for x, we get

$$x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{(2)(44\text{ kJ})}{120\text{ kN/m}}} = 86\text{ cm}$$

ASSESS This answer is absurd. A car's springs couldn't compress anywhere near that far before the underside of the car hit the ground. And 1 g isn't much gasoline. This example shows that springs, though useful energy-storage devices, can't possibly compete with chemical fuels.

EXAMPLE 7.3 Elastic Potential Energy: A Climbing Rope

Worked Example with Variation Problems

Ropes used in rock climbing are "springy" so that they cushion a fall. A particular rope exerts a force $F = -kx + bx^2$, where $k = 223$ N/m, $b = 4.10$ N/m^2, and x is the stretch. Find the potential energy stored in this rope when it's been stretched 2.62 m, taking $U = 0$ at $x = 0$.

INTERPRET Like Example 7.2, this one is about elastic potential energy. But this one isn't so easy because the rope isn't a simple $F = -kx$ spring for which we already have a potential-energy formula.

DEVELOP Because the rope force varies with stretch, we'll have to integrate. Since force and displacement are in the same direction, we can use Equation 7.2a, $\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$. But that's not so much a formula as a strategy for deriving one.

EVALUATE Applying Equation 7.2 to this particular rope, we have

$$U = -\int_{x_1}^{x_2} F(x)\,dx = -\int_0^x (-kx + bx^2)\,dx = \tfrac{1}{2}kx^2 - \tfrac{1}{3}bx^3\Big|_0^x$$
$$= \tfrac{1}{2}kx^2 - \tfrac{1}{3}bx^3$$
$$= (\tfrac{1}{2})(223\text{ N/m})(2.62\text{ m})^2 - (\tfrac{1}{3})(4.1\text{ N/m}^2)(2.62\text{ m})^3$$
$$= 741\text{ J}$$

ASSESS This result is about 3% less than the potential energy $U = \frac{1}{2}kx^2$ of an ideal spring with the same spring constant. This shows the effect of the extra term $+bx^2$, whose positive sign reduces the restoring force and thus the work needed to stretch the spring.

GOT IT? **7.2** Gravitational force actually decreases with height, but that decrease is negligible near Earth's surface. To account for the decrease, would the exact value for the potential-energy change associated with a height change h be (a) greater than, (b) less than, or (c) equal to mgh, where g is the gravitational acceleration at Earth's surface?

Where's the Stored Energy and What's the System?

In discussing the climber of Fig. 7.1*a*, the book of Fig. 7.3, and the engineer of Example 7.1, we were careful not to use phrases like "the climber's potential energy," "the potential energy of the book," or "the engineer's potential energy." After all, the climber herself hasn't changed in going from the bottom to the top of the cliff; nor is the book any different after you've returned it to the shelf. So it doesn't make a lot of sense to say that potential energy is somehow a property of these objects. Indeed, the idea of potential energy requires that two (or more) objects interact via a force. In the examples of the climber, the book, and the engineer, that force is gravity—and the pairs of interacting objects are, correspondingly, the climber and Earth, the book and Earth, and the engineer and Earth. So to characterize potential energy, we need in each case to consider a system consisting of at least two objects. In each example the *configuration* of that system changes, because the relative positions of the objects making up the system are altered. In each case, one member of the system—climber, book, or engineer—has moved relative to Earth. So