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University Physics

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Fifteenth Edition in SI Units

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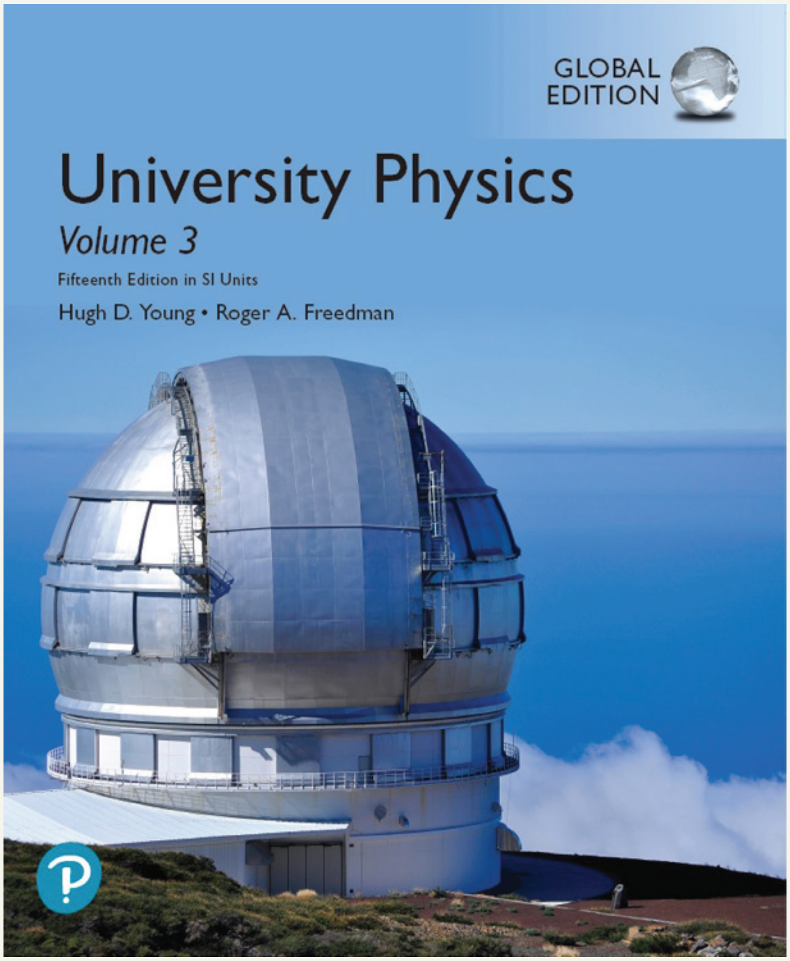
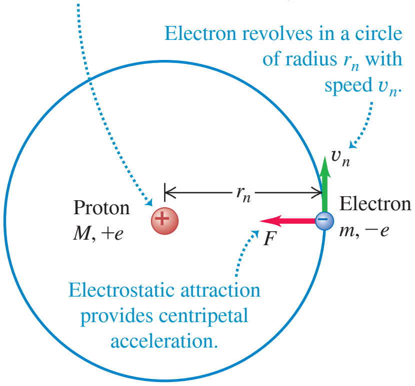


Pearson

Figure **39.23** The Bohr model of the hydrogen atom.



Hence Newton's second law states that

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n^2} = \frac{m{v_n}^2}{r_n} \qquad (39.7)$$

When we solve Eqs. (39.6) and (39.7) simultaneously for r_n and v_n, we get

Radius of *n*th orbit in the Bohr model

$$r_n = \epsilon_0 \frac{n^2h^2}{\pi m e^2} \qquad (39.8)$$

Principal quantum number ($n = 1, 2, 3, \ldots$)
Planck's constant
Magnitude of electron charge
Electric constant
Electron mass

Orbital speed in *n*th orbit in the Bohr model

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh} \qquad (39.9)$$

Magnitude of electron charge
Planck's constant
Electric constant
Principal quantum number ($n = 1, 2, 3, \ldots$)

Equation (39.8) shows that the orbital radius r_n is proportional to n^2, so the smallest orbital radius corresponds to $n = 1$. We'll denote this minimum radius, called the *Bohr radius,* as a_0:

$$a_0 = \epsilon_0 \frac{h^2}{\pi m e^2} \qquad \text{(Bohr radius)} \qquad (39.10)$$

Then we can rewrite Eq. (39.8) as

Radius of *n*th orbit in the Bohr model

$$r_n = n^2 a_0 \qquad (39.11)$$

Bohr radius
Principal quantum number ($n = 1, 2, 3, \ldots$)

The permitted orbits have radii a_0, $4a_0$, $9a_0$, and so on.

You can find the numerical values of the quantities on the right-hand side of Eq. (39.10) in Appendix G. Using these values, we find that the radius a_0 of the smallest Bohr orbit is

$$a_0 = \frac{(8.854 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(6.626 \times 10^{-34}\ \mathrm{J\cdot s})^2}{\pi(9.109 \times 10^{-31}\ \mathrm{kg})(1.602 \times 10^{-19}\ \mathrm{C})^2} = 5.29 \times 10^{-11}\ \mathrm{m}$$

This gives an atomic diameter of about $10^{-10}\ \mathrm{m} = 0.1\ \mathrm{nm}$, which is consistent with atomic dimensions estimated by other methods.

Equation (39.9) shows that the orbital speed v_n is proportional to $1/n$. Hence the greater the value of n, the larger the orbital radius of the electron and the slower its orbital speed. (We saw the same relationship between orbital radius and speed for satellite orbits in Section 13.4.) We leave it to you to calculate the speed in the $n = 1$ orbit, which is the greatest possible speed of the electron in the hydrogen atom (see Exercise 39.23); the result is $v_1 = 2.19 \times 10^6\ \mathrm{m/s}$. This is less than 1% of the speed of light, so relativistic considerations aren't significant.

Hydrogen Energy Levels in the Bohr Model

We can now use Eqs. (39.8) and (39.9) to find the kinetic and potential energies K_n and U_n when the electron is in the orbit with quantum number n:

$$K_n = \tfrac{1}{2}m{v_n}^2 = \frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2} \qquad \text{(kinetic energies in the Bohr model)} \qquad (39.12)$$

$$U_n = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n} = -\frac{1}{\epsilon_0^2}\frac{me^4}{4n^2h^2} \qquad \text{(potential energies in the Bohr model)} \qquad (39.13)$$

The electric potential energy is negative because we have taken its value to be zero when the electron is infinitely far from the nucleus. We are interested only in the *differences* in energy between orbits, so the reference position doesn't matter. The total mechanical energy E_n is the sum of the kinetic and potential energies:

$$E_n = K_n + U_n = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2} \quad \text{(total energies in the Bohr model)} \qquad (39.14)$$

Since E_n in Eq. (39.14) has a different value for each n, you can see that this equation gives the *energy levels* of the hydrogen atom in the Bohr model. Each distinct orbit corresponds to a distinct energy level.

Figure 39.24 depicts the orbits and energy levels. We label the possible energy levels of the atom by values of the quantum number n. For each value of n there are corresponding values of orbital radius r_n, speed v_n, angular momentum $L_n = nh/2\pi$, and total mechanical energy E_n. The energy of the atom is least when $n = 1$ and E_n has its most negative value. This is the *ground level* of the hydrogen atom; it is the level with the smallest orbit, of radius a_0. For $n = 2, 3, \ldots$, the absolute value of E_n is smaller and the total mechanical energy is progressively larger (less negative).

Figure 39.24 also shows some of the possible transitions from one electron orbit to an orbit of lower energy. Consider a transition from orbit n_U (for "upper") to a smaller orbit n_L (for "lower"), with $n_L < n_U$—or, equivalently, from *level* n_U to a lower *level* n_L. Then the energy hc/λ of the emitted photon of wavelength λ is equal to $E_{n_U} - E_{n_L}$. Before we use this relationship to solve for λ, it's convenient to rewrite Eq. (39.14) for the energies as

Planck's constant

Speed of light in vacuum

Electron mass

Magnitude of electron charge

Total energy for *n*th orbit in the Bohr model

$$E_n = -\frac{hcR}{n^2}, \quad \text{where} \quad R = \frac{me^4}{8\epsilon_0^2 h^3 c} \qquad (39.15)$$

Principal quantum number ($n = 1, 2, 3, \ldots$)

Rydberg constant

Electric constant

The quantity R in Eq. (39.15) is called the **Rydberg constant** (named for the Swedish physicist Johannes Rydberg, who did pioneering work on the hydrogen spectrum). When

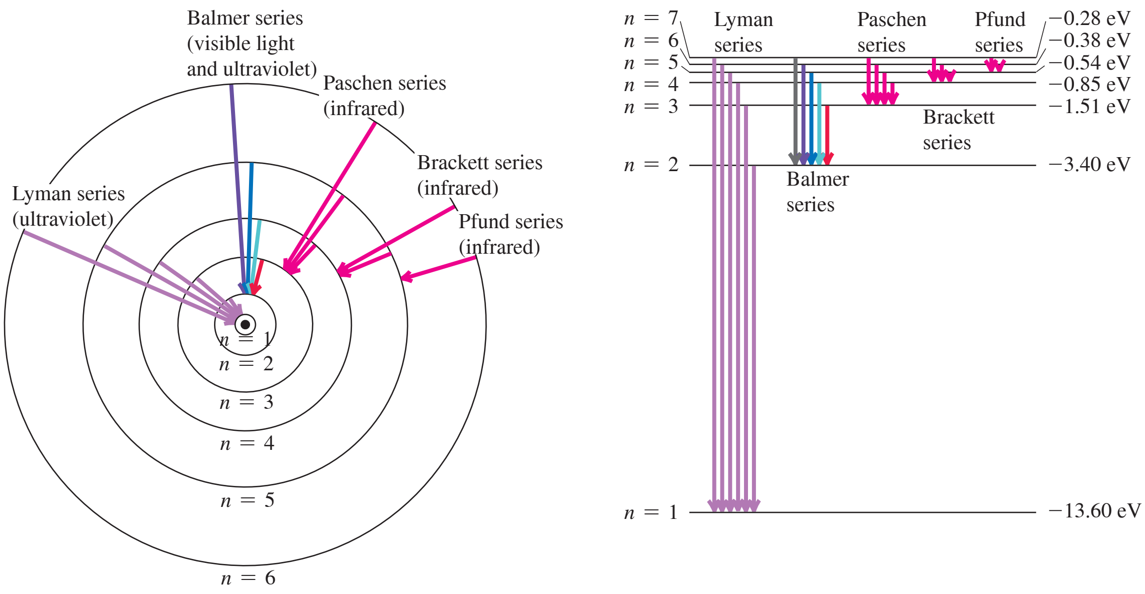


Figure **39.24** Two ways to represent the energy levels of the hydrogen atom and the transitions between them. Note that the radius of the nth permitted orbit is actually n^2 times the radius of the $n = 1$ orbit.

(a) Permitted orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series we substitute the numerical values of the fundamental physical constants m, c, e, h, and ϵ_0, all of which can be determined quite independently of the Bohr theory, we find that $R = 1.097 \times 10^7\ \text{m}^{-1}$. Now we solve for the wavelength of the photon emitted in a transition from level n_U to level n_L:

$$\frac{hc}{\lambda} = E_{n_U} - E_{n_L} = \left(-\frac{hcR}{n_U{}^2}\right) - \left(-\frac{hcR}{n_L{}^2}\right) = hcR\left(\frac{1}{n_L{}^2} - \frac{1}{n_U{}^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_L{}^2} - \frac{1}{n_U{}^2}\right) \quad \text{(hydrogen wavelengths in the Bohr model, } n_L < n_U\text{)} \tag{39.16}$$

CAUTION **"Skipping over" energy levels is allowed** An atom can make transitions between nonconsecutive energy levels, such as from $n = 5$ to $n = 3$ or from $n = 1$ to $n = 4$, by emitting or absorbing a single photon. (Figure 39.24 shows several transitions of this kind.) The atom does *not* have to move only one level at a time (e.g., emit a photon and transition from $n = 5$ to $n = 4$, then emit another photon and transition from $n = 4$ to $n = 3$).

Equation (39.16) is a *theoretical prediction* of the wavelengths found in the *emission* line spectrum of hydrogen atoms. When a hydrogen atom *absorbs* a photon, an electron makes a transition from a level n_L to a *higher* level n_U. This can happen only if the photon energy hc/λ is equal to $E_{n_U} - E_{n_L}$, which is the same condition expressed by Eq. (39.16). So this equation also predicts the wavelengths found in the *absorption* line spectrum of hydrogen.

How does this prediction compare with experiment? If $n_L = 2$, corresponding to transitions to the second energy level in Fig. 39.24, the wavelengths predicted by Eq. (39.16)—collectively called the *Balmer series* (**Fig. 39.25**)—are all in the visible and ultraviolet parts of the electromagnetic spectrum. If we let $n_L = 2$ and $n_U = 3$ in Eq. (39.16) we obtain the wavelength of the H_α line:

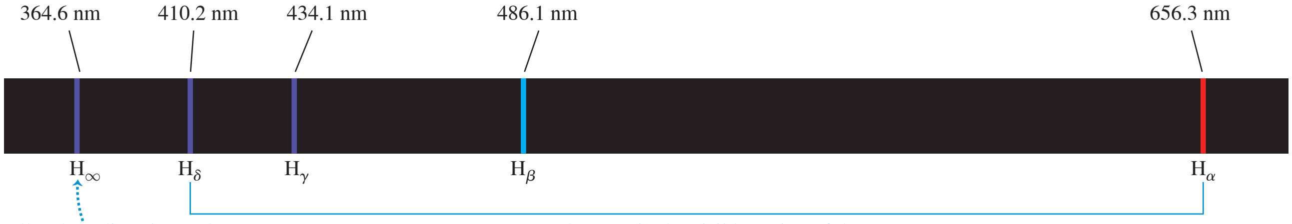
$$\frac{1}{\lambda} = (1.097 \times 10^7\ \text{m}^{-1})\left(\tfrac{1}{4} - \tfrac{1}{9}\right) \qquad \text{or} \qquad \lambda = 656.3\ \text{nm}$$

With $n_L = 2$ and $n_U = 4$ we obtain the wavelength of the H_β line, and so on. With $n_L = 2$ and $n_U = \infty$ we obtain the shortest wavelength in the series, $\lambda = 364.6$ nm. These theoretical predictions are within 0.1% of the observed hydrogen wavelengths! This close agreement provides very strong and direct confirmation of Bohr's theory.

The Bohr model also predicts many other wavelengths in the hydrogen spectrum, as Fig. 39.24 shows. The observed wavelengths of all of these series, each of which is named for its discoverer, match the predicted values with the same percent accuracy as for the Balmer series. The *Lyman series* of spectral lines is caused by transitions between the ground level and the excited levels, corresponding to $n_L = 1$ and $n_U = 2, 3, 4, \ldots$ in Eq. (39.16). The energy difference between the ground level and any of the excited levels is large, so the emitted photons have wavelengths in the ultraviolet part of the electromagnetic spectrum. Transitions among the higher energy levels involve a much smaller energy difference, so the photons emitted in these transitions have little energy and long, infrared wavelengths. That's the case for both the *Brackett series* ($n_L = 3$ and $n_U = 4, 5, 6, \ldots$, corresponding to transitions between the third and higher energy levels) and the *Pfund series* ($n_L = 4$ and $n_U = 5, 6, 7, \ldots$, with transitions between the fourth and higher energy levels).

Figure 39.24 shows only transitions in which a hydrogen atom emits a photon. But as we discussed previously, the wavelengths of those photons that an atom can *absorb* are the

Figure **39.25** The Balmer series of spectral lines for atomic hydrogen. You can see these same lines in the spectrum of *molecular* hydrogen (H_2) shown in Fig. 39.8, as well as additional lines that are present only when two hydrogen atoms are combined to make a molecule.



All Balmer lines beyond H_δ are in the ultraviolet spectrum.

H_α, H_β, H_γ, and H_δ are in the visible region of the spectrum.

same as those that it can emit. For example, a hydrogen atom in the $n = 2$ level can absorb a 656.3 nm photon and end up in the $n = 3$ level.

One additional test of the Bohr model is its predicted value of the *ionization energy* of the hydrogen atom. This is the energy required to remove the electron completely from the atom. Ionization corresponds to a transition from the ground level ($n = 1$) to an infinitely large orbital radius ($n = \infty$), so the energy that must be added to the atom is $E_\infty - E_1 = 0 - E_1 = -E_1$ (recall that E_1 is negative). Substituting the constants from Appendix G into Eq. (39.15) gives an ionization energy of 13.606 eV. The ionization energy can also be measured directly; the result is 13.60 eV. These two values agree within 0.1%.

EXAMPLE 39.6 Exploring the Bohr model

WITH VARIATION PROBLEMS

Find the kinetic, potential, and total mechanical energies of the hydrogen atom in the first excited level, and find the wavelength of the photon emitted in a transition from that level to the ground level.

IDENTIFY and SET UP This problem uses the ideas of the Bohr model. We use simplified versions of Eqs. (39.12), (39.13), and (39.14) to find the energies of the atom, and Eq. (39.16), $hc/\lambda = E_{n_\mathrm{U}} - E_{n_\mathrm{L}}$, to find the photon wavelength λ in a transition from $n_\mathrm{U} = 2$ (the first excited level) to $n_\mathrm{L} = 1$ (the ground level).

EXECUTE We could evaluate Eqs. (39.12), (39.13), and (39.14) for the nth level by substituting the values of m, e, ϵ_0, and h. But we can simplify the calculation by comparing with Eq. (39.15), which shows that the constant $me^4/8\epsilon_0^2h^2$ that appears in Eqs. (39.12), (39.13), and (39.14) is equal to hcR:

$$\begin{aligned}\frac{me^4}{8\epsilon_0^2h^2} &= hcR\\ &= (6.626 \times 10^{-34}\ \mathrm{J\cdot s})(2.998 \times 10^{8}\ \mathrm{m/s})(1.097 \times 10^{7}\ \mathrm{m^{-1}})\\ &= 2.179 \times 10^{-18}\ \mathrm{J} = 13.60\ \mathrm{eV}\end{aligned}$$

This allows us to rewrite Eqs. (39.12), (39.13), and (39.14) as

$$K_n = \frac{13.60\ \mathrm{eV}}{n^2} \qquad U_n = \frac{-27.20\ \mathrm{eV}}{n^2} \qquad E_n = \frac{-13.60\ \mathrm{eV}}{n^2}$$

For the first excited level ($n = 2$), we have $K_2 = 3.40\ \mathrm{eV}$, $U_2 = -6.80\ \mathrm{eV}$, and $E_2 = -3.40\ \mathrm{eV}$. For the ground level ($n = 1$), $E_1 = -13.60\ \mathrm{eV}$. The energy of the emitted photon is then $E_2 - E_1 = -3.40\ \mathrm{eV} - (-13.60\ \mathrm{eV}) = 10.20\ \mathrm{eV}$, and

$$\begin{aligned}\lambda = \frac{hc}{E_2 - E_1} &= \frac{(4.136 \times 10^{-15}\ \mathrm{eV\cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{10.20\ \mathrm{eV}}\\ &= 1.22 \times 10^{-7}\ \mathrm{m} = 122\ \mathrm{nm}\end{aligned}$$

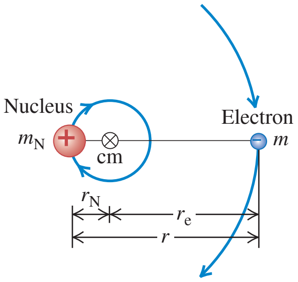
This is the wavelength of the Lyman-alpha (L_α) line, the longest-wavelength line in the Lyman series of ultraviolet lines in the hydrogen spectrum (see Fig. 39.24).

EVALUATE The total mechanical energy for any level is negative and is equal to one-half the potential energy. We found the same energy relationship for Newtonian satellite orbits in Section 13.4. The situations are similar because both the electrostatic and gravitational forces are inversely proportional to $1/r^2$.

KEYCONCEPT In the Bohr model of the hydrogen atom, the electron moves around the nucleus in a circular Newtonian orbit. However, only certain orbital radii are allowed. The nth allowed orbit has a distinct kinetic energy K_n, electric potential energy U_n, and total mechanical energy $E_n = K_n + U_n$. When the electron makes a transition between two allowed orbits, a photon is emitted or absorbed with an energy equal to the difference between the values of E_n for the two orbits.

Nuclear Motion and the Reduced Mass of an Atom

Figure **39.26** The nucleus and the electron both orbit around their common center of mass. The distance r_N has been exaggerated for clarity; for ordinary hydrogen it actually equals $r_\mathrm{e}/1836.2$.



The Bohr model is so successful that we can justifiably ask why its predictions for the wavelengths and ionization energy of hydrogen differ from the measured values by about 0.1%. The explanation is that we assumed that the nucleus (a proton) remains at rest. However, as **Fig. 39.26** shows, the proton and electron *both* orbit about their common center of mass (see Section 8.5). It turns out that we can take this motion into account by using in Bohr's equations not the electron rest mass m but a quantity called the **reduced mass** m_r of the system. For a system composed of two objects of masses m_1 and m_2, the reduced mass is

$$m_\mathrm{r} = \frac{m_1 m_2}{m_1 + m_2} \tag{39.17}$$

For ordinary hydrogen we let m_1 equal m and m_2 equal the proton mass, $m_\mathrm{p} = 1836.2m$. Thus ordinary hydrogen has a reduced mass of

$$m_\mathrm{r} = \frac{m(1836.2m)}{m + 1836.2m} = 0.99946m$$

When this value is used instead of the electron mass m in the Bohr equations, the predicted values agree very well with the measured values.

In an atom of deuterium, also called *heavy hydrogen,* the nucleus is not a single proton but a proton and a neutron bound together to form a composite object called the *deuteron*. The reduced mass of the deuterium atom turns out to be $0.99973m$. Equations (39.15) and (39.16) (with m replaced by m_r) show that all wavelengths are inversely proportional to m_r. Thus the wavelengths of the deuterium spectrum should be those of hydrogen divided by $(0.99973m)/(0.99946m) = 1.00027$. This is a small effect but well within the precision of modern spectrometers. This small wavelength shift led the American scientist Harold Urey to the discovery of deuterium in 1932, an achievement that earned him the 1934 Nobel Prize in chemistry.

Hydrogenlike Atoms

We can extend the Bohr model to other one-electron atoms, such as singly ionized helium (He^+), doubly ionized lithium (Li^{2+}), and so on. Such atoms are called *hydrogenlike* atoms. In such atoms, the nuclear charge is not e but Ze, where Z is the *atomic number,* equal to the number of protons in the nucleus. The effect in the previous analysis is to replace e^2 everywhere by Ze^2. You should verify that the orbital radii r_n given by Eq. (39.8) become smaller by a factor of Z, and the energy levels E_n given by Eq. (39.14) are multiplied by Z^2. The reduced-mass correction in these cases is even less than 0.1% because the nuclei are more massive than the single proton of ordinary hydrogen. **Figure 39.27** compares the energy levels for H and for He^+, which has $Z = 2$.

Atoms of the alkali metals (at the far left-hand side of the periodic table; see Appendix F) have one electron outside a core consisting of the nucleus and the inner electrons, with net core charge $+e$. These atoms are approximately hydrogenlike, especially in excited levels. Physicists have studied alkali atoms in which the outer electron has been excited into a very large orbit with $n = 1000$. From Eq. (39.8), the radius of such a *Rydberg atom* with $n = 1000$ is $n^2 = 10^6$ times the Bohr radius, or about 0.05 mm—about the size of a small grain of sand.

Although the Bohr model predicted the energy levels of the hydrogen atom correctly, it raised as many questions as it answered. It combined elements of classical physics with new postulates that were inconsistent with classical ideas. The model provided no insight into what happens during a transition from one orbit to another; the angular speeds of the electron motion were not in general the angular frequencies of the emitted radiation, a result that is contrary to classical electrodynamics. Attempts to extend the model to atoms with two or more electrons were not successful. An electron moving in one of Bohr's circular orbits forms a current loop and should produce a magnetic dipole moment (see Section 27.7). However, a hydrogen atom in its ground level has *no* magnetic moment due to orbital motion. In Chapters 40 and 41 we'll find that an even more radical departure from classical concepts was needed before the understanding of atomic structure could progress further.

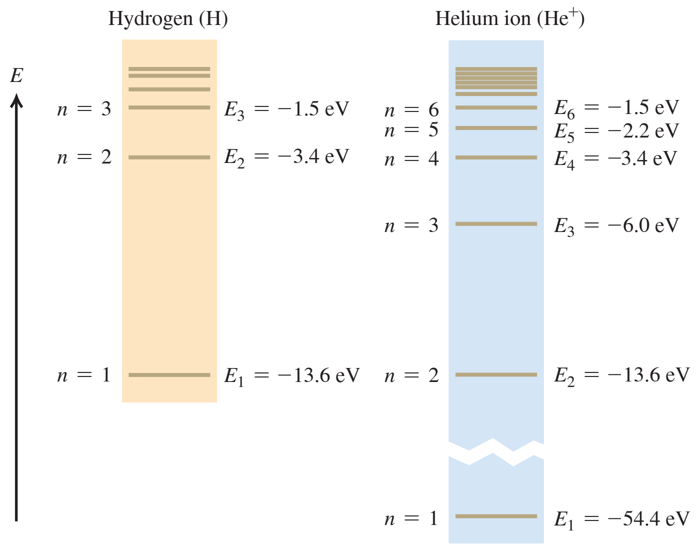


Figure **39.27** Energy levels of H and He^+. The energy expression, Eq. (39.14), is multiplied by $Z^2 = 4$ for He^+, so the energy of an He^+ ion with a given n is almost exactly four times that of an H atom with the same n. (There are small differences of the order of 0.05% because the reduced masses are slightly different.)

TEST YOUR UNDERSTANDING OF SECTION 39.3 Consider the possible transitions between energy levels in a He^+ ion. For which of these transitions in He^+ will the wavelength of the emitted photon be nearly the same as one of the wavelengths emitted by excited H atoms? (i) $n = 2$ to $n = 1$; (ii) $n = 3$ to $n = 2$; (iii) $n = 4$ to $n = 3$; (iv) $n = 4$ to $n = 2$; (v) more than one of these; (vi) none of these.

ANSWER

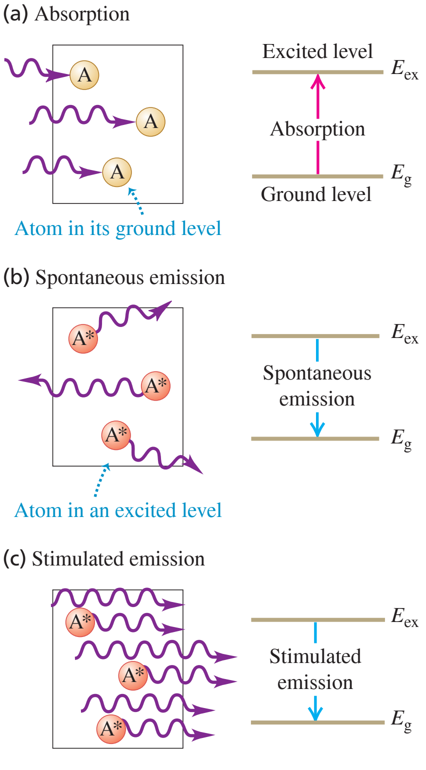
(iv) Figure 39.27 shows that many (though *not* all) of the energy levels of He^+ are the same as those of H. Hence photons emitted during transitions between corresponding pairs of levels in He^+ and H have the same energy E and the same wavelength $\lambda = hc/E$. An H atom that drops from the $n = 2$ level to the $n = 1$ level emits a photon of energy 10.20 eV and wavelength 122 nm (see Example 39.6); a He^+ ion emits a photon of the same energy and wavelength when it drops from the $n = 4$ level to the $n = 2$ level. Inspecting Fig. 39.27 will show you that every even-numbered level in He^+ matches a level in H, while none of the odd-numbered He^+ levels do. The first three He^+ transitions given in the question ($n = 2$ to $n = 1$, $n = 3$ to $n = 2$, and $n = 4$ to $n = 3$) all involve an odd-numbered level, so none of their wavelengths match a wavelength emitted by H atoms.

39.4 THE LASER

The **laser** is a light source that produces a beam of highly coherent and very nearly monochromatic light as a result of cooperative emission from many atoms. The name "laser" is an acronym for "light amplification by stimulated emission of radiation." We can understand the principles of laser operation from what we have learned about atomic energy levels and photons. To do this we'll have to introduce two new concepts: *stimulated emission* and *population inversion*.

Spontaneous and Stimulated Emission

Figure 39.28 Three processes in which atoms interact with light.



Consider a gas of atoms in a transparent container. Each atom is initially in its ground level of energy E_g and also has an excited level of energy E_{ex}. If we shine light of frequency f on the container, an atom can absorb one of the photons provided the photon energy $E = hf$ equals the energy difference $E_{ex} - E_g$ between the levels. **Figure 39.28a** shows this process, in which three atoms A each absorb a photon and go into the excited level. Some time later, the excited atoms (which we denote as A*) return to the ground level by each emitting a photon with the same frequency as the one originally absorbed (Fig. 39.28b). This process is called **spontaneous emission.** The direction and phase of each spontaneously emitted photon are random.

In **stimulated emission** (Fig. 39.28c), each incident photon encounters a previously excited atom. A kind of resonance effect induces each atom to emit a second photon with the same frequency, direction, phase, and polarization as the incident photon, which is not changed by the process. For each atom there is one photon before a stimulated emission and two photons after—thus the name *light amplification*. Because the two photons have the same phase, they emerge together as *coherent* radiation. The laser makes use of stimulated emission to produce a beam consisting of a large number of such coherent photons.

To discuss stimulated emission from atoms in excited levels, we need to know something about how many atoms are in each of the various energy levels. First, we need to make the distinction between the terms *energy level* and *state*. A system may have more than one way to attain a given energy level; each different way is a different **state.** For instance, there are two ways of putting an ideal unstretched spring in a given energy level. Remembering that the spring potential energy is $U = \frac{1}{2}kx^2$, we could compress the spring by $x = -b$ or we could stretch it by $x = +b$ to get the same $U = \frac{1}{2}kb^2$. The Bohr model had only one state in each energy level, but we'll find in Chapter 41 that the hydrogen atom (Fig. 39.24b) actually has two *ground states* in its -13.60 eV ground level, eight *excited states* in its -3.40 eV first excited level, and so on.

The Maxwell–Boltzmann distribution function (see Section 18.5) determines the number of atoms in a given state in a gas. The function tells us that when the gas is in thermal equilibrium at absolute temperature T, the number n_i of atoms in a state with energy E_i equals $Ae^{-E_i/kT}$, where k is the Boltzmann constant and A is another constant determined by the total number of atoms in the gas. (In Section 18.5, E was the kinetic energy $\frac{1}{2}mv^2$ of a