GLOBAL EDITION

College Physics

A Strategic Approach

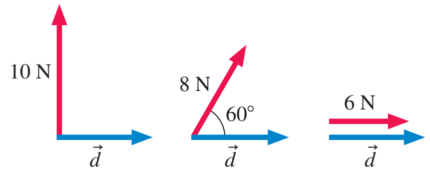
FOURTH EDITION

Knight • Jones • Field

Brief Contents

STOP TO THINK 10.2 Which force does the most work?

A. The 10 N force
B. The 8 N force
C. The 6 N force
D. They all do the same amount of work.



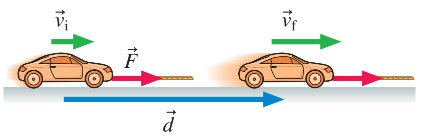
10.3 Kinetic Energy

Kinetic energy is an object's energy of motion. We can use what we've learned about work, and some simple kinematics, to find quantitative expressions for kinetic energy.

We'll start with the case of an object in motion along a line. Such an object has **translational kinetic energy.** In Chapter 7, we introduced the idea of rotational motion: Objects can be in motion even if they aren't going anywhere. An object, like the blade of a wind turbine, rotating about a fixed axis has **rotational kinetic energy,** the kinetic energy of the rotational motion.

Translational Kinetic Energy

FIGURE 10.10 The work done by the tow rope increases the car's kinetic energy.



Consider a car being pulled by a tow rope, as in **FIGURE 10.10.** The rope pulls with a constant force $\vec{F}$ while the car undergoes a displacement $\vec{d}$, so the force does work $W = Fd$ on the car. If we ignore friction and drag, the work done by $\vec{F}$ is transferred entirely into the car's energy of motion—its kinetic energy. In this case, the change in the car's kinetic energy is given by the work-energy equation, Equation 10.3, as

$$W = \Delta K = K_f - K_i \qquad (10.7)$$

Using kinematics, we can find another expression for the work done, in terms of the car's initial and final speeds. Recall from ◀◀ **SECTION 2.5** the kinematic equation

$$v_f^2 = v_i^2 + 2a\,\Delta x$$

Applied to the motion of our car, $\Delta x = d$ is the car's displacement and, from Newton's second law, the acceleration is $a = F/m$. Thus we can write

$$v_f^2 = v_i^2 + \frac{2Fd}{m} = v_i^2 + \frac{2W}{m}$$

where we have replaced Fd with the work W. If we now solve for the work, we find

$$W = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

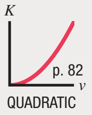
If we compare this result with Equation 10.7, we see that

$$K_f = \frac{1}{2}mv_f^2 \qquad \text{and} \qquad K_i = \frac{1}{2}mv_i^2$$

In general, then, an object of mass m moving with speed v has kinetic energy

$$K = \frac{1}{2}mv^2 \qquad (10.8)$$

Kinetic energy of an object of mass m moving with speed v



From Equation 10.8, the units of kinetic energy are those of mass times speed squared, or $\text{kg} \cdot (\text{m/s})^2$. But

$$1\ \text{kg} \cdot (\text{m/s})^2 = \underbrace{1\ \text{kg} \cdot (\text{m/s}^2)}_{1\ \text{N}} \cdot \text{m} = 1\ \text{N} \cdot \text{m} = 1\ \text{J}$$

We see that the units of kinetic energy are the same as those of work, as they must be. **TABLE 10.1** gives some approximate kinetic energies.

TABLE 10.1 Some approximate kinetic energies

Object	Kinetic energy
Ant walking	1×10^{-8} J
Coin dropped 1 m	5×10^{-3} J
Person walking	70 J
Fastball, 100 mph	150 J
Bullet	5000 J
Car, 60 mph	5×10^{5} J
Supertanker, 20 mph	2×10^{10} J

EXAMPLE 10.4 Finding the work to set a boat in motion

At a history center, an old canal boat is pulled by two draft horses. It doesn't take much force to keep the boat moving; the drag force is quite small. But it takes some work to get the 55,000 kg boat up to speed! The horses can pull with a steady force and put a 1400 N tension in the rope that connects to the boat. The rope is straight and level. The boat starts from rest, and the horses pull steadily as they begin their walk down the tow-path. How much distance do the horses cover as they bring the boat up to its final speed of 0.70 m/s?

STRATEGIZE Let's take the system to be the boat. We could include the water, but since we can ignore the drag force (we're told that it's small), it's not important to do so. The rope is not part of the system, so the tension force does work on the boat. It's this work, which comes from energy provided by the horses, that increases the kinetic energy, and thus the speed, of the boat. We'll consider the initial state to be the boat at rest, the final state to be the boat in motion at its final speed.

PREPARE **FIGURE 10.11** is a before-and-after visual overview of the situation. The work that is done by the rope will change the energy of the system, so we can use Equation 10.3, the work-energy equation. Because the only thing that changes is the speed, the only form of energy that changes is the kinetic energy, so we can simplify the equation to

$$\Delta K = W$$

This makes sense—the work done changes the kinetic energy of the boat. The tension force is in the direction of the motion, so the work done is $W = Td$. The boat starts at rest, with kinetic energy equal to zero, so the change in kinetic energy is just the final kinetic energy: $\Delta K = \frac{1}{2}mv_f^2$.

SOLVE With the details noted, the work-energy equation reduces to

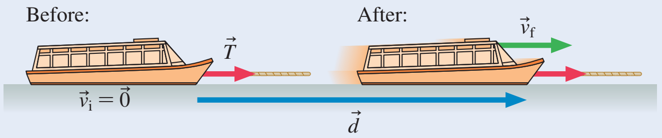
$$\frac{1}{2}mv_f^2 = Td$$

We are looking for the distance the horses pull the boat:

$$d = \frac{mv_f^2}{2T} = \frac{(55{,}000\ \text{kg})(0.70\ \text{m/s})^2}{2(1400\ \text{N})} = 9.6\ \text{m}$$

ASSESS This distance is about 30 feet. This seems a reasonable distance; the horses would be pulling for several strides as they get the boat up to speed.

FIGURE 10.11 Getting the canal boat up to speed.



Known
$m = 55{,}000$ kg $\quad v_i = 0$ m/s
$T = 1400$ N $\quad v_f = 0.70$ m/s

Find: d

STOP TO THINK 10.3 Rank in order, from greatest to least, the kinetic energies of the sliding pucks.

Rotational Kinetic Energy

FIGURE 10.12 Rotational kinetic energy of a spinning wind turbine.

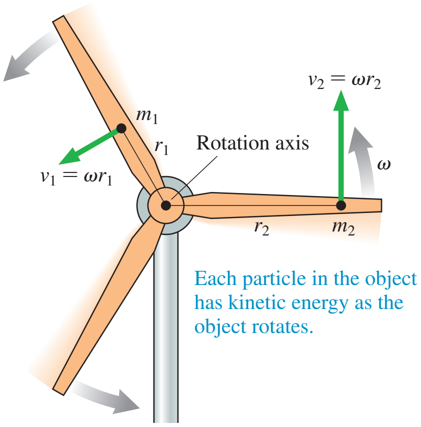


FIGURE 10.12 shows the rotating blades of a wind turbine. Although the blades have no overall translational motion, each particle in the blades is moving and hence has kinetic energy. Adding up the kinetic energy of all the particles that make up the blades, we find that the blades have rotational kinetic energy, the kinetic energy due to rotation.

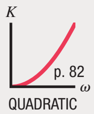
In Figure 10.12, we focus on the motion of two particles in the wind turbine blades. The blade assembly rotates with angular velocity ω. Recall from ◀◀ SECTION 7.1 that a particle moving with angular velocity ω in a circle of radius r has a speed $v = \omega r$. Thus particle 1, which rotates in a circle of radius r_1, moves with speed $v_1 = r_1\omega$ and so has kinetic energy $\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 r_1^2\omega^2$. Similarly, particle 2, which rotates in a circle with a larger radius r_2, has kinetic energy $\frac{1}{2}m_2 r_2^2\omega^2$. The object's rotational kinetic energy is the sum of the kinetic energies of *all* the particles:

$$K_{rot} = \frac{1}{2}m_1 r_1^2\omega^2 + \frac{1}{2}m_2 r_2^2\omega^2 + \cdots = \frac{1}{2}\left(\sum mr^2\right)\omega^2$$

You will recognize the term in parentheses as our old friend, the moment of inertia I. Thus the rotational kinetic energy is

$$K_{rot} = \frac{1}{2}I\omega^2 \qquad (10.9)$$

Rotational kinetic energy of an object with moment of inertia I and angular velocity ω



NOTE ▶ Rotational kinetic energy is *not* a new form of energy. It is the ordinary kinetic energy of motion, only now expressed in a form that is especially convenient for rotational motion. Comparison with the familiar $\frac{1}{2}mv^2$ shows again that the moment of inertia I is the rotational equivalent of mass. ◀

A rolling object, such as a wheel, is undergoing both rotational *and* translational motions. Consequently, its total kinetic energy is the sum of its rotational and translational kinetic energies:

$$K = K_{trans} + K_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad (10.10)$$

This illustrates an important fact: **The kinetic energy of a rolling object is always greater than that of a nonrotating object moving at the same speed.**

◀ **Rotational recharge** A promising new technology would replace spacecraft batteries that need periodic and costly replacement with a *flywheel*—a cylinder rotating at a very high angular speed. Energy from solar panels is used to speed up the flywheel, which stores energy as rotational kinetic energy that can then be converted back into electric energy as needed.

EXAMPLE 10.5 Where should you trim the weight?

Any time a cyclist stops, it will take energy to get moving again. Using less energy to get going means more energy is available to go farther or go faster, so racing cyclists want their bikes to be as light as possible. It's particularly important to have lightweight wheels, as this example will show. Consider two bikes that have the same total mass but different mass wheels. Bike 1 has a 10.0 kg frame and two 1.00 kg wheels; bike 2 has a 9.00 kg frame and two 1.50 kg wheels. Both bikes thus have the same 12.0 kg total mass. What is the kinetic energy of each bike when they are moving at 12.0 m/s? Most of the weight of the tire and wheel is at the rim, so we can model each wheel as a hoop.

STRATEGIZE As the bike moves, the wheels rotate. The bike has translational kinetic energy, but the wheels have both translational and rotational kinetic energy. If the bike is moving at speed v, we know from Chapter 7 that the wheels rotate at $\omega = v/R$, where R is the radius of a wheel.

PREPARE Each bike's frame has only translational kinetic energy $K_{frame} = \frac{1}{2}mv^2$, where m is the mass of the frame. The kinetic energy of each rolling wheel is given by Equation 10.10. From Table 7.1, we find that I for a hoop is MR^2, where M is the mass of one wheel.

SOLVE From Equation 10.10 the kinetic energy of each rolling wheel is

$$K_{wheel} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\underbrace{(MR^2)}_{I}\underbrace{\left(\frac{v}{R}\right)^2}_{\omega^2} = Mv^2$$

Then the total kinetic energy of a bike is

$$K = K_{frame} + 2K_{wheel} = \frac{1}{2}mv^2 + 2Mv^2$$

The factor of 2 in the second term occurs because each bike has two wheels. Thus the kinetic energies of the two bikes are

$$K_1 = \frac{1}{2}(10.0\text{ kg})(12.0\text{ m/s})^2 + 2(1.00\text{ kg})(12.0\text{ m/s})^2$$
$$= 1010\text{ J}$$
$$K_2 = \frac{1}{2}(9.00\text{ kg})(12.0\text{ m/s})^2 + 2(1.50\text{ kg})(12.0\text{ m/s})^2$$
$$= 1080\text{ J}$$

The kinetic energy of bike 2 is about 7% higher than that of bike 1. Note that the radius of the wheels was not needed in this calculation.

ASSESS We were told that it's particularly important for cyclists to have lightweight wheels, so this result makes sense. Both of the bikes in the example have the same total mass, but the one with lighter wheels takes less energy to get moving. Shaving a little extra weight off your bike's wheels is more useful than taking that same weight off the bike's frame.

10.4 Potential Energy

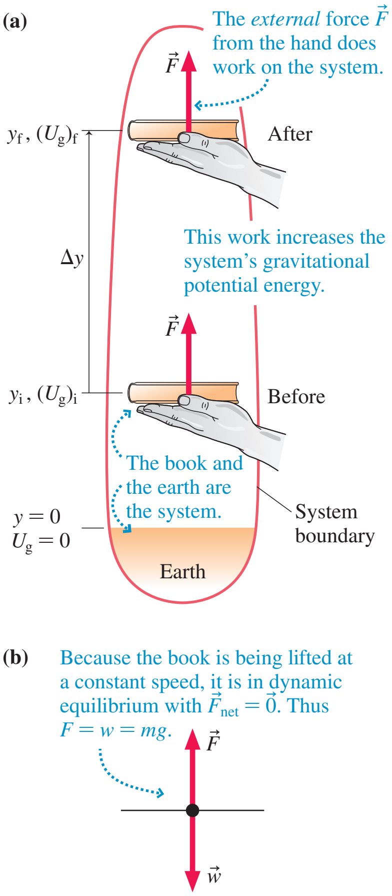
When two or more objects in a system interact, it is sometimes possible to *store* energy in the system in a way that the energy can be easily recovered. For instance, the earth and a ball interact by the gravitational force between them. If the ball is lifted up into the air, energy is stored in the ball + earth system, energy that can later be recovered as kinetic energy when the ball is released and falls. Similarly, a spring is a system made up of countless atoms that interact via their atomic "springs." If we push a box against a spring, energy is stored that can be recovered when the spring later pushes the box across the table. This sort of stored energy is called **potential energy,** since it has the *potential* to be converted into other forms of energy, such as kinetic or thermal energy.

NOTE ▸ Potential energy is really a property of a *system*, but we often speak informally of the potential energy of an *object*. We might say, for instance, that raising a ball increases its potential energy. This is fine as long as we remember that this energy is really stored in the ball + earth system. ◂

The forces due to gravity and springs are special in that they allow for the storage of energy. Other interaction forces do not. When a dog pulls a sled, the sled interacts with the ground via the force of friction, and the work that the dog does on the sled is converted into thermal energy. The energy is *not* stored up for later recovery—it slowly diffuses into the environment and cannot be recovered.

Gravitational Potential Energy

FIGURE 10.13 Lifting a book increases the system's gravitational potential energy.



To find an expression for **gravitational potential energy U_g,** let's consider the system of the book and the earth shown in FIGURE 10.13a. The book is lifted at a constant speed from its initial position at y_i to a final height y_f. The lifting force of the hand is external to the system and so does work W on the system, increasing its energy. The book is lifted at a constant speed, so its kinetic energy doesn't change. Because there's no friction, the book's thermal energy doesn't change either. Thus the work done goes entirely into increasing the gravitational potential energy of the system. According to Equation 10.3, the work-energy equation, this can be written as $\Delta U_g = W$. Because $\Delta U_g = (U_g)_f - (U_g)_i$, Equation 10.3 can be written

$$(U_g)_f = (U_g)_i + W \tag{10.11}$$

The work done is $W = Fd$, where $d = \Delta y = y_f - y_i$ is the vertical distance that the book is lifted. From the free-body diagram of FIGURE 10.13b, we see that $F = mg$. Thus $W = mg\,\Delta y$, and so

$$(U_g)_f = (U_g)_i + mg\Delta y \tag{10.12}$$

Because our final height was greater than our initial height, Δy is positive and $(U_g)_f > (U_g)_i$. **The higher the object is lifted, the greater the gravitational potential energy in the object + earth system.**

We can express Equation 10.12 in terms of the change in potential energy, $\Delta U_g = (U_g)_f - (U_g)_i$:

$$\Delta U_g = mg\Delta y \tag{10.13}$$

If we lift a 1.5 kg book up by $\Delta y = 2.0$ m, we increase the system's gravitational potential energy by $\Delta U_g = (1.5\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m}) = 29.4$ J. This increase is *independent* of the book's starting height: The gravitational potential energy increases by 29.4 J whether we lift the book 2.0 m starting at sea level or starting at the top of the Washington Monument. This illustrates an important general fact about *every* form of potential energy: **Only *changes* in potential energy are significant.**

Because of this fact, we are free to choose a *reference level* where we define U_g to be zero. Our expression for U_g is particularly simple if we choose this reference level to be at $y = 0$. We then have

$$U_g = mgy \tag{10.14}$$

Gravitational potential energy of an object of mass m at height y
(assuming $U_g = 0$ when the object is at $y = 0$)

EXAMPLE 10.6 Racing up a skyscraper

In the Empire State Building Run-Up, competitors race up the 1576 steps of the Empire State Building, climbing a total vertical distance of 320 m. How much gravitational potential energy does a 70 kg racer gain during this race?

Racers head up the staircase in the Empire State Building Run-Up.

STRATEGIZE We'll take the system to be the racer + earth so that we can consider gravitational potential energy.

PREPARE We are asked for the change in gravitational potential energy as the racer goes up the stairs, so we need only consider the change in height, which is given. We can use Equation 10.13 to compute the change in potential energy during the run.

SOLVE As the racer goes up the stairs, her change in gravitational potential energy is

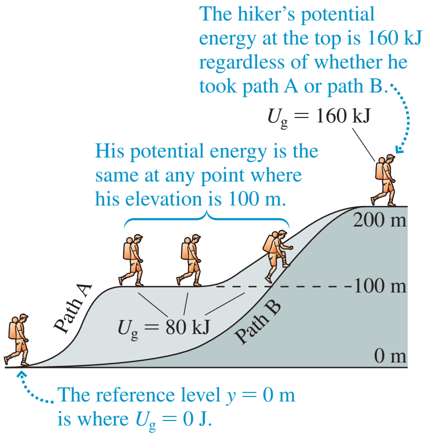
$$\Delta U_g = mg\Delta y = (70\text{ kg})(9.8\text{ m/s}^2)(320\text{ m}) = 2.2 \times 10^5\text{ J}$$

ASSESS This is a lot of energy. According to Table 10.1, it's comparable to the energy of a speeding car. But the difference in height is pretty great, so this seems reasonable. In Chapter 11, we'll consider how much food energy you'd need to consume to fuel this climb.

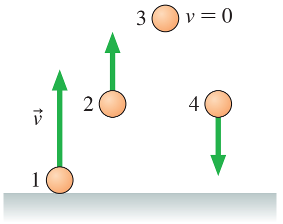
An important conclusion from Equation 10.14 is that gravitational potential energy depends only on the height of the object above the reference level $y = 0$, not on the object's horizontal position. To understand why, consider carrying a briefcase while walking on level ground at a constant speed. As shown in the table on page 349, the vertical force of your hand on the briefcase is *perpendicular* to the displacement. No work is done on the briefcase, so its gravitational potential energy remains constant as long as its height above the ground doesn't change.

This idea can be applied to more complicated cases, such as the 82 kg hiker in FIGURE 10.14. His gravitational potential energy depends *only* on his height y above the reference level. Along path A, it's the same value $U_g = mgy = 80$ kJ at any point where he is at height $y = 100$ m above the reference level. If he had instead taken path B, his gravitational potential energy at $y = 100$ m would be the same 80 kJ. It doesn't matter *how* he gets to the 100 m elevation; his potential energy at that height is always the same. **Gravitational potential energy depends only on the *height* of an object and not on the path the object took to get to that position.** This fact will allow us to use the law of conservation of energy to easily solve a variety of problems that would be very difficult to solve using Newton's laws alone.

FIGURE 10.14 The hiker's gravitational potential energy depends only on his height above the $y = 0$ m reference level.



STOP TO THINK 10.4 Rank in order, from largest to smallest, the gravitational potential energies of identical balls 1 through 4.



Elastic Potential Energy

Energy can also be stored in a compressed or extended spring as **elastic** (or **spring**) **potential energy** U_s. We can find out how much energy is stored in a spring by using an external force to slowly compress the spring. This external force does work on the spring, transferring energy to the spring. Since only the elastic potential energy of the spring is changing, Equation 10.3 becomes

$$\Delta U_s = W \qquad (10.15)$$

That is, we can find out how much elastic potential energy is stored in the spring by calculating the amount of work needed to compress the spring.

FIGURE 10.15 shows a spring being compressed by a hand. In ◀◀ SECTION 8.3 we found that the force the spring exerts on the hand is $F_s = -k\,\Delta x$ (Hooke's law), where Δx is the displacement of the end of the spring from its equilibrium position and k is the spring constant. In Figure 10.15 we have set the origin of our coordinate system at the equilibrium position. The displacement from equilibrium Δx is therefore equal to x, and the spring force is then $-kx$. By Newton's third law, the force that the hand exerts on the spring is thus $F = +kx$.

As the hand pushes the end of the spring from its equilibrium position to a final position x, the applied force increases from 0 to kx. This is not a constant force, so we can't use Equation 10.5, $W = Fd$, to find the work done. However, it seems reasonable to calculate the work by using the *average* force in Equation 10.5. Because the force varies from $F_i = 0$ to $F_f = kx$, the average force used to compress the spring is $F_{avg} = \frac{1}{2}kx$. Thus the work done by the hand is

$$W = F_{avg}d = F_{avg}x = \left(\frac{1}{2}kx\right)x = \frac{1}{2}kx^2$$

This work is stored as potential energy in the spring, so we can use Equation 10.15 to find that as the spring is compressed, the elastic potential energy increases by

$$\Delta U_s = \frac{1}{2}kx^2$$

FIGURE 10.15 The force required to compress a spring is not constant.