GLOBAL EDITION

Feedback Control of Dynamic Systems

EIGHTH EDITION

Franklin • Powell • Emami-Naeini

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

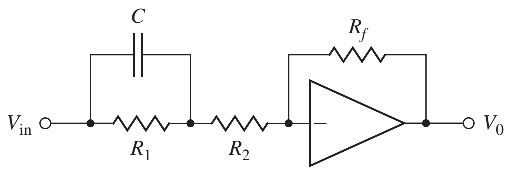

Figure 5.31
Possible circuit of a lead compensation

When considering notch or phase stabilization, it is important to understand that its success depends on maintaining the correct phase at the frequency of the resonance. If that frequency is subject to significant change, which is common in many cases, then the notch needs to be removed far enough from the nominal frequency in order to work for all cases. The result may be interference of the notch with the rest of the dynamics and poor performance. As a general rule, gain stabilization is substantially more robust to plant changes than is phase stabilization.

△ 5.4.4 Analog and Digital Implementations

Compensation can be physically realized in various ways. Most compensation can be implemented using analog electronics similar to that described in Section 2.2. However, it is very common today to implement compensation using digital devices.

As an example of an analog realization, a circuit diagram for lead compensation using an operational amplifier is shown in Fig. 5.31. The transfer function of the circuit in Fig. 5.31 is readily found by the methods from Chapter 2 to be

$$D_{lead}(s) = -a\frac{s+z}{s+p}, \qquad (5.77)$$

where

$$a = \frac{p}{z}, \quad \text{if} \quad R_f = R_1 + R_2,$$

$$z = \frac{1}{R_1 C},$$

$$p = \frac{R_1 + R_2}{R_2} \cdot \frac{1}{R_1 C}.$$

A short section describing the implementation of a lead compensation using a digital device and a comparison of the results with an analog implementation is contained in online Appendix W5.4.4. (See www.pearsonglobaleditions.com)

5.5 Design Examples Using the Root Locus

EXAMPLE 5.12 *Control of a Quadrotor Drone Pitch Axis*

For the quadrotor shown in Fig. 2.13, the transfer function between a pitch control input, T_{lon}, and the pitch angle, θ, is

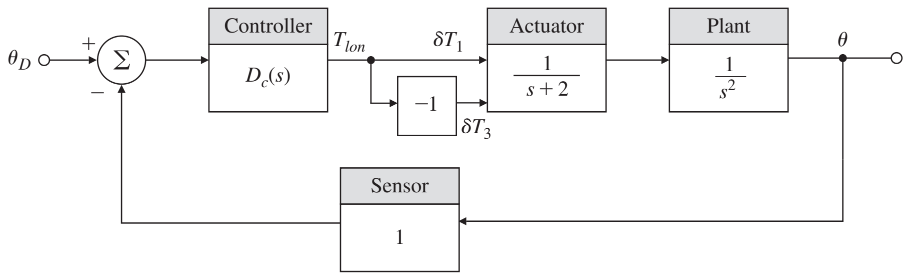

Figure 5.32
Block diagram for the quadrotor design Example 5.12

$$\frac{\theta(s)}{T_{lon}(s)} = G_1(s) = \frac{1}{s^2(s+2)}.$$

This is similar to the transfer function obtained in Eq. (2.15) in Chapter 2; however, an extra term has been added to account for the lag associated with the rotor coming up to the newly commanded thrust and speed. The lag term selected, $(s+2)$, is for a fairly large quadrotor of perhaps 2 meters in diameter. The more detailed drone example in Chapter 10 (see Example 10.5) will include this term along with some of the aerodynamic terms. However, for purposes of understanding the essential control features, this simplified example should suffice. The block diagram of the control system is shown in Fig. 5.32. It shows the quadrotor dynamics given by $\theta(s)/T_{lon}(s)$ and shows the compensator, $D_c(s)$, to be designed via the root locus method. The desired specifications for this system are:

$$\omega_n \geq 1 \text{ rad/sec},$$
$$\zeta \geq 0.44.$$

Using lead compensation, find a set of parameters for $D_c(s)$ that meet the required specifications.

Solution. Knowing the desired ω_n and ζ values is the **first step** in the Lead Compensation Design Procedure. The **second step** in the process is to determine a root locus for the uncompensated system. The ensuing Matlab commands will generate such a locus:

```
s = tf('s');
sysG1=1/((s^2)*(s + 2));
rlocus(sysG1)
axis([-8 4 -6 6])
grid on
```

Note use of the `grid` command places the ω_n and ζ values on the root locus plot as an aid in the determination of whether the specifications are met. The result is shown in Fig. 5.33.

The uncompensated system exhibits increasing instability as the gain, K, is increased; therefore, it is likely that significant more lead will be required compared to Example 5.11 where the uncompensated

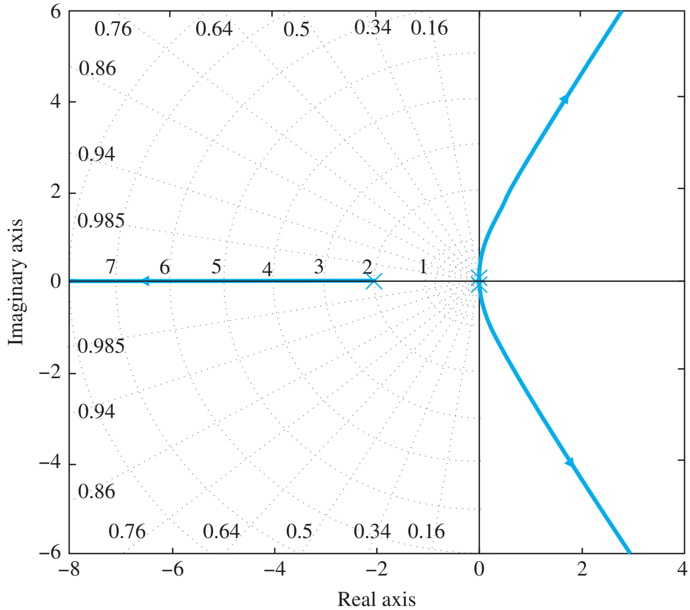

Figure 5.33
Uncompensated system, i.e., with $D_c(s) = K$.

system was always stable, as was shown in Fig. 5.22. For the **third step** we select $z = 1$ and $p = 10$ in Eq. (5.70) so

$$D_c(s) = K\frac{s+1}{s+10}.$$

This compensation is implemented into the quadrotor control system by the Matlab commands

```
s = tf('s');
sysG1=1/((s^2 )*(s + 2));
sysD=(s+1)/(s+10);
rlocus(sysG1*sysD)
axis([-3 1 -2 2])
grid on
```

which produce the root locus in Fig. 5.34. It shows that no value of K will produce the level of damping required, that is, $\zeta \geq 0.44$.

Clearly, significantly more damping from the compensator is required so we move on to **step 4** in the procedure. For our next attempt, let's choose a value of $z = 0.5$ instead of 1. However, it will show that it is still not possible to meet both specifications. Therefore, let's also increase p to 15 and examine whether that will create a locus with $\zeta \geq 0.44$. Therefore the compensation is now

$$D_c(s) = K\frac{s+0.5}{s+15}.$$

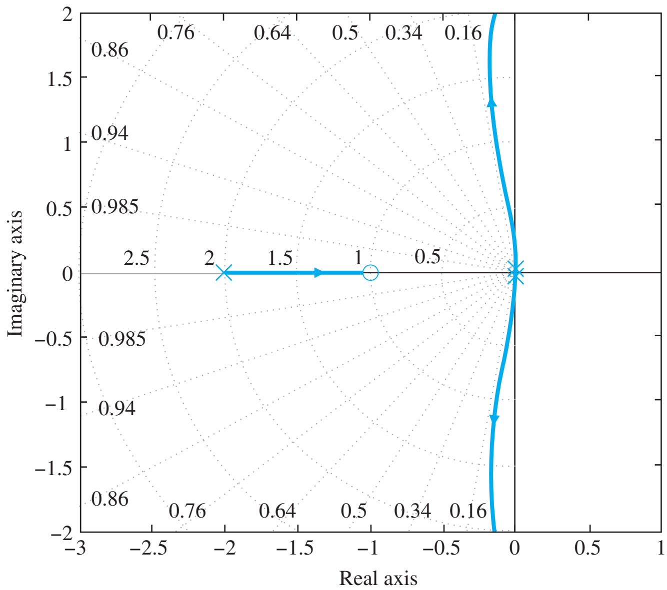

Figure 5.34
Compensated system with $D_c(s) = K\frac{s+1}{s+10}$

A root locus of the system with this compensator is found from the ensuing Matlab statements

```
s = tf('s');
sysG1=1/((s^2 )*(s + 2));
sysD=(s+0.5)/(s+15);
rlocus(sysG1*sysD)
axis([-3 1 -2 2])
grid on
```

which produces the locus shown in Fig. 5.35.

Comparing the locus with the lines of constant damping shows that it comes very close to the $\zeta = 0.5$ line, and thus most likely will satisfy the requirement that $\zeta \geq 0.44$. Also note that the point on the locus that is closest to the $\zeta = 0.5$ line is approximately at $\omega_n = 1$ rad/sec. Thus, **step 5** consists of verifying this result. This can be carried out by placing your cursor on the Matlab generated root locus at the point of best damping. Doing so shows that

$$K = 30,$$

$$\omega_n = 1.03, \text{ and}$$

$$\zeta = 0.446,$$

which satisfies **step 5** in the design procedure and yields the value of K in the lead compensation. Therefore, we now have the complete set of parameters, and the final design is

$$D_c(s) = 30\frac{s + 0.5}{s + 15}.$$

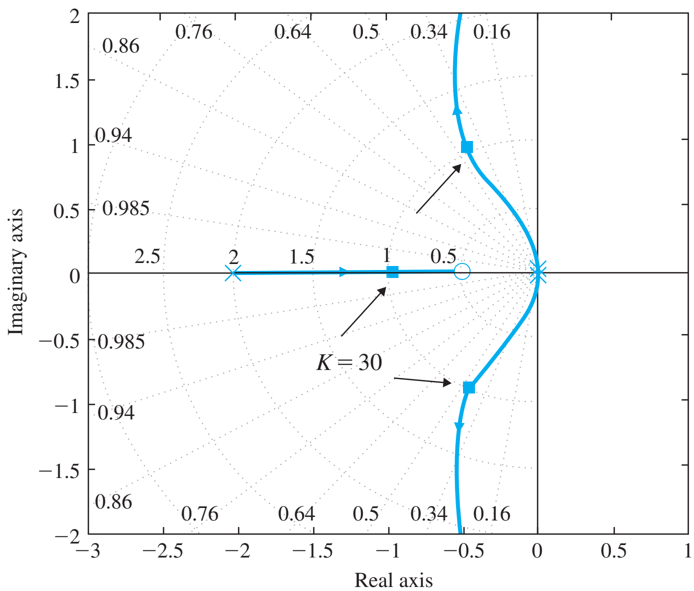

Figure 5.35
Root locus of the quadrotor with $D_c(s) = K\frac{s+0.5}{s+15}$

Thus, all the specifications are now met. Since no time domain or steady-state requirements have been made, **steps 6 and 7** do not apply in this case. Had there been a time response specification that was not met, it would be necessary to return to **step 2** and revise the desired ω_n and ζ so as to improve the situation. A higher value of ω_n would speed up the response time and a higher value of ζ would decrease the overshoot. If the steady-state error requirements had not been met, it is sometimes possible to increase K and still meet the other specifications; however, in this case any increase in K from the selected value of 30 would decrease the damping, ζ, so it would be necessary to add a lag compensator or integral control had a higher value of K been necessary.

EXAMPLE 5.13 *Control of a Small Airplane*

For the Piper Dakota shown in Fig. 5.36, the transfer function between the elevator input and the pitch attitude is

$$G(s) = \frac{\theta(s)}{\delta_e(s)} = \frac{160(s+2.5)(s+0.7)}{(s^2+5s+40)(s^2+0.03s+0.06)}, \tag{5.78}$$

where

$$\theta = \text{pitch attitude, degrees (see Fig. 10.30),}$$

$$\delta_e = \text{elevator angle, degrees.}$$

(For a more detailed discussion of longitudinal aircraft motion, refer to Section 10.3.)

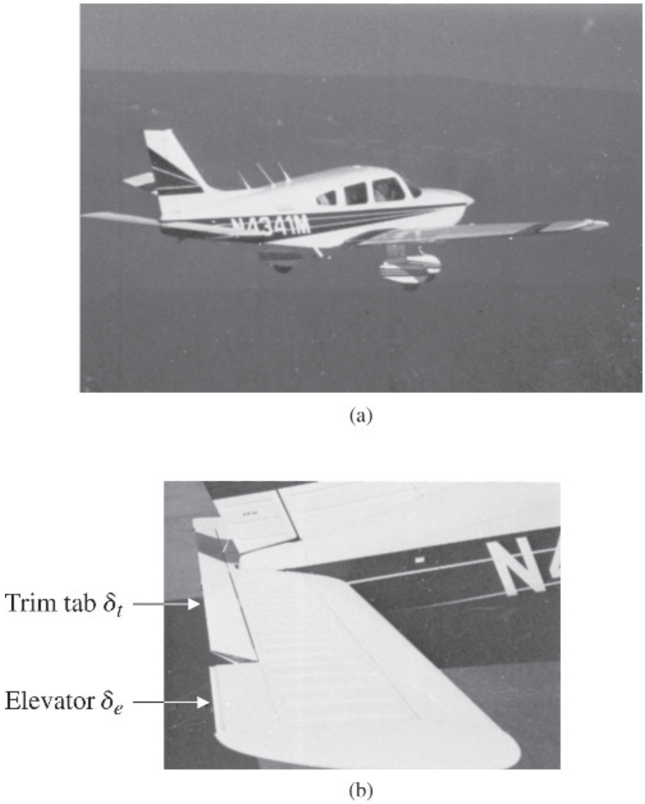

Figure 5.36
Autopilot design in the Piper Dakota, showing elevator and trim tab
Source: Photos courtesy of Denise Freeman

1. Design an autopilot so the response to a step elevator input has a rise time of 1 sec or less and an overshoot less than 10%.
2. When there is a constant disturbing moment acting on the aircraft so the pilot must supply a constant force on the controls for steady flight, it is said to be out of trim. The transfer function between the disturbing moment and the attitude is the same as that due to the elevator; that is,

$$\frac{\theta(s)}{M_d(s)} = \frac{160(s+2.5)(s+0.7)}{(s^2+5s+40)(s^2+0.03s+0.06)}, \tag{5.79}$$

where M_d is the moment acting on the aircraft. There is a separate aerodynamic surface for trimming, δ_t, that can be actuated and will change the moment on the aircraft. It is shown in the close-up of the tail in Fig. 5.36(b), and its influence is depicted in the block diagram shown in Fig. 5.37(a). For both manual and autopilot flight, it is desirable to adjust the trim so there is no steady-state control effort required from the elevator (that is, so $\delta_e = 0$). In manual flight, this means no force is required by the pilot to keep the aircraft at a constant altitude, whereas in autopilot control it means reducing the amount of electrical power required and saving