GLOBAL EDITION

College Mathematics

for Business, Economics, Life Sciences, and Social Sciences

FOURTEENTH EDITION

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Pearson MyLab

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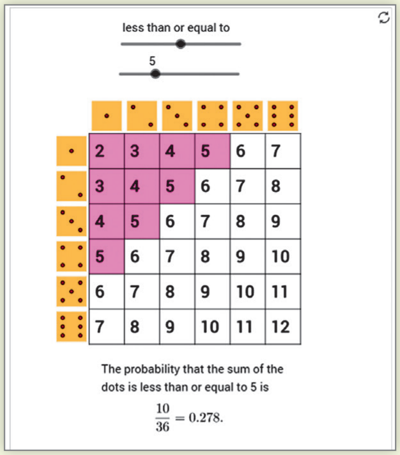

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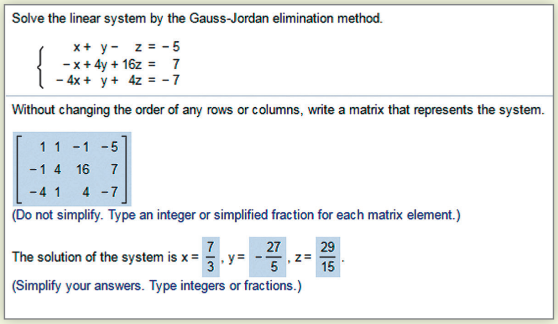

pearson.com/mylab/math

7. Maximize $P = 5x_1 + 10x_2$
subject to
$$\begin{aligned} x_1 + x_2 &\le 3 \\ 2x_1 + 3x_2 &\ge 12 \\ x_1, x_2 &\ge 0 \end{aligned}$$

8. Maximize $P = 4x_1 + 6x_2$
subject to
$$\begin{aligned} x_1 + x_2 &\le 2 \\ 3x_1 + 5x_2 &\ge 15 \\ x_1, x_2 &\ge 0 \end{aligned}$$

B *Use the big M method to solve Problems 9–22.*

9. Minimize and maximize $P = 2x_1 - x_2$
subject to
$$\begin{aligned} x_1 + x_2 &\le 8 \\ 5x_1 + 3x_2 &\ge 30 \\ x_1, x_2 &\ge 0 \end{aligned}$$

10. Minimize and maximize $P = -4x_1 + 16x_2$
subject to
$$\begin{aligned} 3x_1 + x_2 &\le 28 \\ x_1 + 2x_2 &\ge 16 \\ x_1, x_2 &\ge 0 \end{aligned}$$

11. Maximize $P = 2x_1 + 5x_2$
subject to
$$\begin{aligned} x_1 + 2x_2 &\le 18 \\ 2x_1 + x_2 &\le 21 \\ x_1 + x_2 &\ge 10 \\ x_1, x_2 &\ge 0 \end{aligned}$$

12. Maximize $P = 6x_1 + 2x_2$
subject to
$$\begin{aligned} x_1 + 2x_2 &\le 20 \\ 2x_1 + x_2 &\le 16 \\ x_1 + x_2 &\ge 9 \\ x_1, x_2 &\ge 0 \end{aligned}$$

13. Maximize $P = 10x_1 + 12x_2 + 20x_3$
subject to
$$\begin{aligned} 3x_1 + x_2 + 2x_3 &\ge 12 \\ x_1 - x_2 + 2x_3 &= 6 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

14. Maximize $P = 5x_1 + 7x_2 + 9x_3$
subject to
$$\begin{aligned} x_1 - x_2 + x_3 &\ge 20 \\ 2x_1 + x_2 + 5x_3 &= 35 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

15. Minimize $C = -5x_1 - 12x_2 + 16x_3$
subject to
$$\begin{aligned} x_1 + 2x_2 + x_3 &\le 10 \\ 2x_1 + 3x_2 + x_3 &\ge 6 \\ 2x_1 + x_2 - x_3 &= 1 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

16. Minimize $C = -3x_1 + 15x_2 - 4x_3$
subject to
$$\begin{aligned} 2x_1 + x_2 + 3x_3 &\le 24 \\ x_1 + 2x_2 + x_3 &\ge 6 \\ x_1 - 3x_2 + x_3 &= 2 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

17. Maximize $P = 3x_1 + 5x_2 + 6x_3$
subject to
$$\begin{aligned} 2x_1 + x_2 + 2x_3 &\le 8 \\ 2x_1 + x_2 - 2x_3 &= 0 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

18. Maximize $P = 3x_1 + 6x_2 + 2x_3$
subject to
$$\begin{aligned} 2x_1 + 2x_2 + 3x_3 &\le 12 \\ 2x_1 - 2x_2 + x_3 &= 0 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

19. Maximize $P = 2x_1 + 3x_2 + 4x_3$
subject to
$$\begin{aligned} x_1 + 2x_2 + x_3 &\le 25 \\ 2x_1 + x_2 + 2x_3 &\le 60 \\ x_1 + 2x_2 - x_3 &\ge 10 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

20. Maximize $P = 5x_1 + 2x_2 + 9x_3$
subject to
$$\begin{aligned} 2x_1 + 4x_2 + x_3 &\le 150 \\ 3x_1 + 3x_2 + x_3 &\le 90 \\ -x_1 + 5x_2 + x_3 &\ge 120 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

21. Maximize $P = x_1 + 2x_2 + 5x_3$
subject to
$$\begin{aligned} x_1 + 3x_2 + 2x_3 &\le 60 \\ 2x_1 + 5x_2 + 2x_3 &\ge 50 \\ x_1 - 2x_2 + x_3 &\ge 40 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

22. Maximize $P = 2x_1 + 4x_2 + x_3$
subject to
$$\begin{aligned} 2x_1 + 3x_2 + 5x_3 &\le 280 \\ 2x_1 + 2x_2 + x_3 &\ge 140 \\ 2x_1 + x_2 &\ge 150 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

23. Solve Problems 5 and 7 by graphing (the geometric method).

24. Solve Problems 6 and 8 by graphing (the geometric method).

C *Problems 25–32 are mixed. Some can be solved by the methods presented in Sections 5.2 and 5.3 while others must be solved by the big M method.*

25. Minimize $C = 10x_1 - 40x_2 - 5x_3$
subject to
$$\begin{aligned} x_1 + 3x_2 &\le 6 \\ 4x_2 + x_3 &\le 3 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

26. Maximize $P = 7x_1 - 5x_2 + 2x_3$
subject to
$$\begin{aligned} x_1 - 2x_2 + x_3 &\ge -8 \\ x_1 - x_2 + x_3 &\le 10 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

27. Maximize $P = -5x_1 + 10x_2 + 15x_3$
subject to
$$\begin{aligned} 2x_1 + 3x_2 + x_3 &\le 24 \\ x_1 - 2x_2 - 2x_3 &\ge 1 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

28. Minimize $C = -5x_1 + 10x_2 + 15x_3$

subject to
$$\begin{aligned} 2x_1 + 3x_2 + x_3 &\le 24 \\ x_1 - 2x_2 - 2x_3 &\ge 1 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

29. Minimize $C = 10x_1 + 40x_2 + 5x_3$

subject to
$$\begin{aligned} x_1 + 3x_2 &\ge 6 \\ 4x_2 + x_3 &\ge 3 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

30. Maximize $P = 8x_1 + 2x_2 - 10x_3$

subject to
$$\begin{aligned} x_1 + x_2 - 3x_3 &\le 6 \\ 4x_1 - x_2 + 2x_3 &\le -7 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

31. Maximize $P = 12x_1 + 9x_2 + 5x_3$

subject to
$$\begin{aligned} x_1 + 3x_2 + x_3 &\le 40 \\ 2x_1 + x_2 + 3x_3 &\le 60 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

32. Minimize $C = 10x_1 + 12x_2 + 28x_3$

subject to
$$\begin{aligned} 4x_1 + 2x_2 + 3x_3 &\ge 20 \\ 3x_1 - x_2 - 4x_3 &\le 10 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

Applications

In Problems 33–38, construct a mathematical model in the form of a linear programming problem. (The answers in the back of the book for these application problems include the model.) Then solve the problem using the big M method.

33. Advertising. An advertising company wants to attract new customers by placing a total of at most 10 ads in 3 newspapers. Each ad in the *Sentinel* costs $200 and will be read by 2,000 people. Each ad in the *Journal* costs $200 and will be read by 500 people. Each ad in the *Tribune* costs $100 and will be read by 1,500 people. The company wants at least 16,000 people to read its ads. How many ads should it place in each paper in order to minimize the advertising costs? What is the minimum cost?

34. Advertising. Discuss the effect on the solution to Problem 33 if the *Tribune* will not accept more than 4 ads from the company.

35. Human nutrition. A person on a high-protein, low-carbohydrate diet requires at least 100 units of protein and at most 24 units of carbohydrates daily. The diet will consist entirely of three special liquid diet foods: *A*, *B*, and *C*. The contents and costs of the diet foods are given in the table. How many bottles of each brand of diet food should be consumed daily in order to meet the protein and carbohydrate requirements at minimal cost? What is the minimum cost?

	Units per Bottle		
	A	*B*	*C*
Protein	10	10	20
Carbohydrates	2	3	4
Cost per bottle ($)	0.60	0.40	0.90

36. Human nutrition. Discuss the effect on the solution to Problem 35 if the cost of brand *C* liquid diet food increases to $1.50 per bottle.

37. Plant food. A farmer can use three types of plant food: mix *A*, mix *B*, and mix *C*. The amounts (in pounds) of nitrogen, phosphoric acid, and potash in a cubic yard of each mix are given in the table. Tests performed on the soil indicate that the field needs at least 800 pounds of potash. The tests also indicate that no more than 700 pounds of phosphoric acid should be added to the field. The farmer plans to plant a crop that requires a great deal of nitrogen. How many cubic yards of each mix should be added to the field in order to satisfy the potash and phosphoric acid requirements and maximize the amount of nitrogen? What is the maximum amount of nitrogen?

	Pounds per Cubic Yard		
	A	*B*	*C*
Nitrogen	12	16	8
Phosphoric acid	12	8	16
Potash	16	8	16

38. Plant food. Discuss the effect on the solution to Problem 37 if the limit on phosphoric acid is increased to 1,000 pounds.

In Problems 39–47, construct a mathematical model in the form of a linear programming problem. Do not solve.

39. Manufacturing. A company manufactures car and truck frames at plants in Milwaukee and Racine. The Milwaukee plant has a daily operating budget of $50,000 and can produce at most 300 frames daily in any combination. It costs $150 to manufacture a car frame and $200 to manufacture a truck frame at the Milwaukee plant. The Racine plant has a daily operating budget of $35,000, and can produce a maximum combined total of 200 frames daily. It costs $135 to manufacture a car frame and $180 to manufacture a truck frame at the Racine plant. Based on past demand, the company wants to limit production to a maximum of 250 car frames and 350 truck frames per day. If the company realizes a profit of $50 on each car frame and $70 on each truck frame, how many frames of each type should be produced at each plant to maximize the daily profit?

40. Loan distributions. A savings and loan company has $3 million to lend. The types of loans and annual returns offered are given in the table. State laws require that at least 50% of the money loaned for mortgages must be for first mortgages and that at least 30% of the total amount loaned must be for either first or second mortgages. Company policy requires that the amount of signature and automobile loans cannot exceed 25% of the total amount loaned and that signature loans cannot exceed 15% of the total amount loaned. How much money should be allocated to each type of loan in order to maximize the company's return?

Type of Loan	Annual Return (%)
Signature	18
First mortgage	12
Second mortgage	14
Automobile	16

41. Oil refining. A refinery produces two grades of gasoline, regular and premium, by blending together three components: *A*, *B*, and *C*. Component *A* has an octane rating of 90 and costs \$28 a barrel, component *B* has an octane rating of 100 and costs \$30 a barrel, and component *C* has an octane rating of 110 and costs \$34 a barrel. The octane rating for regular must be at least 95 and the octane rating for premium must be at least 105. Regular gasoline sells for \$38 a barrel and premium sells for \$46 a barrel. The company has 40,000 barrels of component *A*, 25,000 barrels of component *B*, and 15,000 barrels of component *C*. It must produce at least 30,000 barrels of regular and 25,000 barrels of premium. How should the components be blended in order to maximize profit?

42. Trail mix. A company makes two brands of trail mix, regular and deluxe, by mixing dried fruits, nuts, and cereal. The recipes for the mixes are given in the table. The company has 1,200 pounds of dried fruits, 750 pounds of nuts, and 1,500 pounds of cereal for the mixes. The company makes a profit of \$0.40 on each pound of regular mix and \$0.60 on each pound of deluxe mix. How many pounds of each ingredient should be used in each mix in order to maximize the company's profit?

Type of Mix	Ingredients
Regular	At least 20% nuts
	At most 40% cereal
Deluxe	At least 30% nuts
	At most 25% cereal

43. Investment strategy. An investor is planning to divide her investments among high-tech mutual funds, global mutual funds, corporate bonds, municipal bonds, and CDs. Each of these investments has an estimated annual return and a risk factor (see the table). The risk level for each choice is the product of its risk factor and the percentage of the total funds invested in that choice. The total risk level is the sum of the risk levels for all the investments. The investor wants at least 20% of her investments to be in CDs and does not want the risk level to exceed 1.8. What percentage of her total investments should be invested in each choice to maximize the return?

Investment	Annual Return (%)	Risk Factor
High-tech funds	11	2.7
Global funds	10	1.8
Corporate bonds	9	1.2
Muncipal bonds	8	0.5
CDs	5	0

44. Investment strategy. Refer to Problem 43. Suppose the investor decides that she would like to minimize the total risk factor, as long as her return does not fall below 9%. What percentage of her total investments should be invested in each choice to minimize the total risk level?

45. Human nutrition. A dietitian arranges a special diet using foods *L*, *M*, and *N*. The table gives the nutritional contents and cost of 1 ounce of each food. The diet's daily requirements are at least 400 units of calcium, at least 200 units of iron, at least 300 units of vitamin A, at most 150 units of cholesterol, and at most 900 calories. How many ounces of each food should be used in order to meet the diet's requirements at a minimal cost?

	Units per Bottle		
	L	*M*	*N*
Calcium	30	10	30
Iron	10	10	10
Vitamin A	10	30	20
Cholesterol	8	4	6
Calories	60	40	50
Cost per ounce (\$)	0.40	0.60	0.80

46. Mixing feed. A farmer grows three crops: corn, oats, and soybeans. He mixes them to feed his cows and pigs. At least 40% of the feed mix for the cows must be corn. The feed mix for the pigs must contain at least twice as much soybeans as corn. He has harvested 1,000 bushels of corn, 500 bushels of oats, and 1,000 bushels of soybeans. He needs 1,000 bushels of each feed mix for his livestock. The unused corn, oats, and soybeans can be sold for \$4, \$3.50, and \$3.25 a bushel, respectively (thus, these amounts also represent the cost of the crops used to feed the livestock). How many bushels of each crop should be used in each feed mix in order to produce sufficient food for the livestock at a minimal cost?

47. Transportation. Three towns are forming a consolidated school district with two high schools. Each high school has a maximum capacity of 2,000 students. Town *A* has 500 high school students, town *B* has 1,200, and town *C* has 1,800. The weekly costs of transporting a student from each town to each school are given in the table. In order to balance the enrollment, the school board decided that each high school must enroll at least 40% of the total student population. Furthermore, no more than 60% of the students in any town should be sent to the same high school. How many students from each town should be enrolled in each school in order to meet these requirements and minimize the cost of transporting the students?

	Weekly Transportation Cost per Student (\$)	
	School I	School II
Town *A*	4	8
Town *B*	6	4
Town *C*	3	9

Answers to Matched Problems

1. Maximize $P = 3x_1 - 2x_2 + x_3 - Ma_1 - Ma_2 - Ma_3$
subject to
$$\begin{aligned} x_1 - 2x_2 + x_3 - s_1 + a_1 &= 5 \\ x_1 + 3x_2 - 4x_3 - s_2 + a_2 &= 10 \\ 2x_1 + 4x_2 + 5x_3 + s_3 &= 20 \\ -3x_1 + x_2 + x_3 + a_3 &= 15 \\ x_1, x_2, x_3, s_1, a_1, s_2, a_2, s_3, a_3 &\ge 0 \end{aligned}$$

2. Max $P = 22$ at $x_1 = 6, x_2 = 4, x_3 = 0$

3. No optimal solution

4. A minimum cost of \$200 is realized when no type J, 6 type K, and 2 type L stones are processed each day.

5. Maximize $P = 9x_1 + 15x_2 - x_3 + 5x_4$
subject to
$$\begin{aligned} x_1 + x_2 &\le 35{,}000 \\ x_3 + x_4 &\le 15{,}000 \\ x_1 + x_3 &\ge 20{,}000 \\ x_2 + x_4 &\ge 10{,}000 \\ 5x_1 - 15x_3 &\le 0 \\ 15x_2 - 5x_4 &\le 0 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

Chapter 5 Summary and Review

Important Terms, Symbols, and Concepts

5.1 The Table Method: An Introduction to the Simplex Method

EXAMPLES

- A linear programming problem is said to be a **standard maximization problem in standard form** if its mathematical model is of the following form: Maximize the objective function

$$P = c_1x_1 + c_2x_2 + \cdots + c_kx_k$$

subject to problem constraints of the form

$$a_1x_1 + a_2x_2 + \cdots + a_kx_k \le b \qquad b \ge 0$$

with nonnegative constraints

$$x_1, x_2, \ldots, x_k \ge 0$$

- The system of inequalities (***i*-system**) of a linear programming problem is converted to a system of equations (***e*-system**) by means of **slack variables**. A solution of the *e*-system is a **feasible solution** if the values of all decision variables and slack variables are nonnegative. The feasible solutions of the *e*-system correspond to the points in the feasible region of the *i*-system. A **basic solution** of the *e*-system is found by setting k of the variables equal to 0, where k is the number of decision variables $x_1, x_2, \ldots, x_k$. A solution of the *e*-system that is both basic and feasible is called a **basic feasible solution**. The **table method** for solving a linear programming problem consists of constructing a table of all basic solutions, determining which of the basic solutions are feasible, and then maximizing the objective function over the basic feasible solutions. A procedure for carrying out the table method in the case of $k = 2$ decision variables is given on page 272. For an arbitrary number of decision variables, see the procedure on page 278. Ex. 1, p. 273 Ex. 2, p. 274 Ex. 3, p. 275

- The **fundamental theorem of linear programming** can be formulated in terms of basic feasible solutions. It states that an optimal solution to the linear programming problem, if one exists, must occur at one or more of the basic feasible solutions.

- The k variables that are assigned the value 0, in order to generate a basic solution, are called **nonbasic variables**. The remaining variables are called **basic variables**. So the classification of variables as basic or nonbasic depends on the basic solution under consideration. Ex. 4, p. 277

- The benefit of the table method is that it gives a procedure for **finding all corner points of the feasible region without drawing a graph**. But the table method has a drawback: If the number of decision variables and problem constraints is large, then the number of rows in the table (that is, the number of basic solutions) becomes too large for practical implementation. The *simplex method,* discussed in Section 5.2, gives a practical method for solving large linear programming problems.

5.2 The Simplex Method: Maximization with Problem Constraints of the Form ≤

- Adding the objective function to the system of constraint equations produces the **initial system**. Negative values of the objective function variable are permitted in a basic feasible solution as long as all other variables are nonnegative. The fundamental theorem of linear programming also applies to initial systems.
- The augmented matrix of the initial system is called the **initial simplex tableau**. The **simplex method** consists of performing **pivot operations**, starting with the initial simplex tableau, until an optimal solution is found (if one exists). The procedure is illustrated in Figure 2 (p. 289). Ex. 1, p. 289 Ex. 2, p. 291 Ex. 3, p. 292

5.3 The Dual Problem: Minimization with Problem Constraints of the Form ≥

- By the **Fundamental Principle of Duality**, a linear programming problem that asks for the minimum of the objective function over a region described by ≥ problem constraints can be solved by first forming the **dual problem** and then using the simplex method. Ex. 1, p. 300 Ex. 2, p. 303 Ex. 3, p. 305

5.4 Maximization and Minimization with Mixed Problem Constraints

- The **big *M* method** can be used to find the maximum of any objective function on any feasible region. The solution process involves the introduction of two new types of variables, **surplus variables** and **artificial variables**, and a modification of the objective function. The result is an initial tableau that can be transformed into the tableau of a **modified problem**. Ex. 1, p. 316
- Applying the simplex method to the modified problem produces a solution to the original problem, if one exists. Ex. 2, p. 318 Ex. 3, p. 319
- The dual method can be used to solve *only* certain minimization problems. But *all* minimization problems can be solved by using the big *M* method to find the maximum of the negative of the objective function. The big *M* method also lends itself to computer implementation. Ex. 4, p. 321 Ex. 5, p. 323

Review Exercises

Work through all the problems in this chapter review and check your answers in the back of the book. Answers to all review problems are there along with section numbers in italics to indicate where each type of problem is discussed. Where weaknesses show up, review appropriate sections in the text.

A *Problems 1–7 refer to the partially completed table of the six basic solutions to the e-system*

$$2x_1 + 5x_2 + s_1 = 32$$

$$x_1 + 2x_2 + s_2 = 14$$

	x_1	x_2	s_1	s_2
(A)	0	0	32	14
(B)	0	6.4	0	1.2
(C)	0	7	−3	0
(D)	16	0	0	−2
(E)		0		0
(F)			0	0

1. In basic solution (B), which variables are basic?

2. In basic solution (D), which variables are nonbasic?

3. Find basic solution (E).

4. Find basic solution (F).

5. Which of the six basic solutions are feasible?

6. Describe geometrically the set of points in the plane such that $s_1 < 0$.

7. Use the basic feasible solutions to maximize $P = 50x_1 + 60x_2$.

8. A linear programming problem has 6 decision variables and 3 problem constraints. How many rows are there in the table of basic solutions of the corresponding *e*-system?

9. Given the linear programming problem

$$\begin{aligned} \text{Maximize} \quad & P = 6x_1 + 2x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \le 8 \\ & x_1 + 2x_2 \le 10 \\ & x_1, x_2 \ge 0 \end{aligned}$$

convert the problem constraints into a system of equations using slack variables.

10. How many basic variables and how many nonbasic variables are associated with the system in Problem 9?

11. Find all basic solutions for the system in Problem 9, and determine which basic solutions are feasible.

12. Write the simplex tableau for Problem 9, and circle the pivot element. Indicate the entering and exiting variables.

13. Solve Problem 9 using the simplex method.

14. For the simplex tableau below, identify the basic and nonbasic variables. Find the pivot element, the entering and exiting variables, and perform one pivot operation.

x_1	x_2	x_3	s_1	s_2	s_3	P	
2	1	3	−1	0	0	0	20
3	0	4	1	1	0	0	30
2	0	5	2	0	1	0	10
−8	0	−5	3	0	0	1	50

15. Find the basic solution for each tableau. Determine whether the optimal solution has been reached, additional pivoting is required, or the problem has no optimal solution.

(A)

x_1	x_2	s_1	s_2	P	
4	1	0	0	0	2
2	0	1	1	0	5
−2	0	3	0	1	12

(B)

x_1	x_2	s_1	s_2	P	
−1	3	0	1	0	7
0	2	1	0	0	0
−2	1	0	0	1	22

(C)

x_1	x_2	s_1	s_2	P	
1	−2	0	4	0	6
0	2	1	6	0	15
0	3	0	2	1	10

16. Form the dual problem of

$$\begin{aligned} \text{Minimize} \quad & C = 5x_1 + 2x_2 \\ \text{subject to} \quad & x_1 + 3x_2 \geq 15 \\ & 2x_1 + x_2 \geq 20 \\ & x_1, x_2 \geq 0 \end{aligned}$$

17. Write the initial system for the dual problem in Problem 16.

18. Write the first simplex tableau for the dual problem in Problem 16 and label the columns.

19. Use the simplex method to find the optimal solution of the dual problem in Problem 16.

20. Use the final simplex tableau from Problem 19 to find the optimal solution of the linear programming problem in Problem 16.

B **21.** Solve the linear programming problem using the simplex method.

$$\begin{aligned} \text{Maximize} \quad & P = 3x_1 + 4x_2 \\ \text{subject to} \quad & 2x_1 + 4x_2 \leq 24 \\ & 3x_1 + 3x_2 \leq 21 \\ & 4x_1 + 2x_2 \leq 20 \\ & x_1, x_2 \geq 0 \end{aligned}$$

22. Form the dual problem of the linear programming problem

$$\begin{aligned} \text{Minimize} \quad & C = 3x_1 + 8x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 10 \\ & x_1 + 2x_2 \geq 15 \\ & x_2 \geq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

23. Solve Problem 22 by applying the simplex method to the dual problem.

Solve the linear programming Problems 24 and 25.

24.
$$\begin{aligned} \text{Maximize} \quad & P = 5x_1 + 3x_2 - 3x_3 \\ \text{subject to} \quad & x_1 - x_2 - 2x_3 \leq 3 \\ & 2x_1 + 2x_2 - 5x_3 \leq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

25.
$$\begin{aligned} \text{Maximize} \quad & P = 5x_1 + 3x_2 - 3x_3 \\ \text{subject to} \quad & x_1 - x_2 - 2x_3 \leq 3 \\ & x_1 + x_2 \leq 5 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

26. Solve the linear programming problem using the table method:

$$\begin{aligned} \text{Maximize} \quad & P = 10x_1 + 7x_2 + 8x_3 \\ \text{subject to} \quad & 2x_1 + x_2 + 3x_3 \leq 12 \\ & x_1, x_2 \geq 0 \end{aligned}$$

27. Refer to Problem 26. How many pivot operations are required to solve the linear programming problem using the simplex method?

In Problems 28 and 29,

(A) Introduce slack, surplus, and artificial variables and form the modified problem.

(B) Write the preliminary simplex tableau for the modified problem and find the initial simplex tableau.

(C) Find the optimal solution of the modified problem by applying the simplex method to the initial simplex tableau.

(D) Find the optimal solution of the original problem, if it exists.

28.
$$\begin{aligned} \text{Maximize} \quad & P = x_1 + 3x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 6 \\ & x_1 + 2x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

29.
$$\begin{aligned} \text{Maximize} \quad & P = x_1 + x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 5 \\ & x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

30. Find the modified problem for the following linear programming problem. (Do not solve.)

$$\begin{aligned} \text{Maximize} \quad & P = 2x_1 + 3x_2 + x_3 \\ \text{subject to} \quad & x_1 - 3x_2 + x_3 \leq 7 \\ & -x_1 - x_2 + 2x_3 \leq -2 \\ & 3x_1 + 2x_2 - x_3 = 4 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$