GLOBAL EDITION

Calculus

for Business, Economics, Life Sciences, and Social Sciences

FOURTEENTH EDITION

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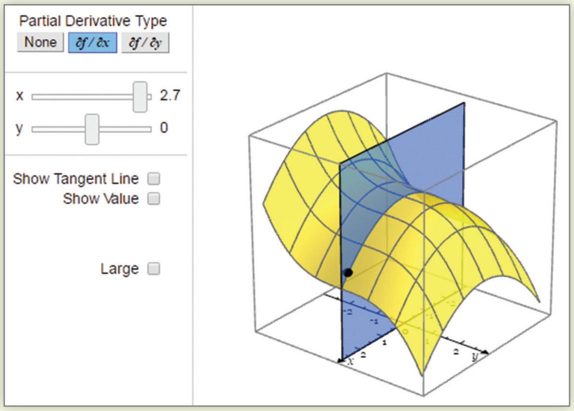
Pearson MyLab

MyLab Math for *Calculus for Business, Economics, Life Sciences, and Social Sciences,* 14e

(access code required)

Used by over 3 million students a year, MyLab™ Math is the world's leading online program for teaching and learning mathematics. MyLab Math delivers assessment, tutorials, and multimedia resources that provide engaging and personalized experiences for each student, so learning can happen in any environment.

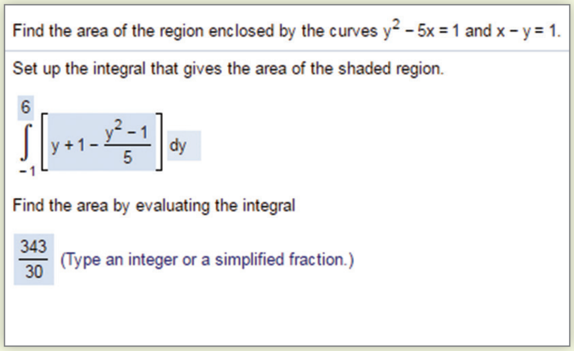


Interactive Figures

A full suite of Interactive Figures has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They have been designed to be used in lecture as well as by students independently.

Questions that Deepen Understanding

MyLab Math includes a variety of question types designed to help students succeed in the course. In Setup & Solve questions, students show how they set up a problem as well as the solution, better mirroring what is required on tests. Additional Conceptual Questions provide support for assessing concepts and vocabulary. Many of these questions are application oriented.


Find the area of the region enclosed by the curves $y^2 - 5x = 1$ and $x - y = 1$.

Set up the integral that gives the area of the shaded region.

$$\int_{-1}^{6} \left[y + 1 - \frac{y^2 - 1}{5} \right] dy$$

Find the area by evaluating the integral

$\frac{343}{30}$ (Type an integer or a simplified fraction.)

pearson.com/mylab/math

In Problems 67–70, find $f'(x)$ and find the value(s) of x where $f'(x) = 0$.

67. $f(x) = (2x - 15)(x^2 + 18)$

68. $f(x) = (2x + 9)(x^2 - 54)$

69. $f(x) = \dfrac{x}{x^2 + 1}$

70. $f(x) = \dfrac{x}{x^2 + 9}$

In Problems 71–74, find $f'(x)$ in two ways: (1) using the product or quotient rule and (2) simplifying first.

71. $f(x) = x^3(x^4 - 1)$

72. $f(x) = x^4(x^3 - 1)$

73. $f(x) = \dfrac{x^3 + 9}{x^3}$

74. $f(x) = \dfrac{x^4 + 4}{x^4}$

C *In Problems 75–92, find each indicated derivative and simplify.*

75. $f'(w)$ for $f(w) = (w - 4)3$

76. $g'(w)$ for $g(w) = (w - 5)\log_3 w$

77. $\dfrac{dy}{dx}$ for $y = 9x^{1/3}(x^3 + 5)$

78. $\dfrac{d}{dx}[(4x^{1/2} - 1)(3x^{1/3} + 2)]$

79. y' for $y = \dfrac{\log_2 x}{1 + x^2}$

80. $\dfrac{dy}{dx}$ for $y = \dfrac{10^x}{1 + x^4}$

81. $f'(x)$ for $f(x) = \dfrac{6\sqrt[3]{x}}{x^2 - 3}$

82. y' for $y = \dfrac{2\sqrt{x}}{x^2 - 3x + 1}$

83. $g'(t)$ if $g(t) = \dfrac{0.2t}{3t^2 - 1}$

84. $h'(t)$ if $h(t) = \dfrac{-0.05t^2}{2t + 1}$

85. $\dfrac{d}{dx}[4x \log x^5]$

86. $\dfrac{d}{dt}[10^t \log t]$

87. $\dfrac{dy}{dx}$ for $y = (x + 3)(x^2 - 3x + 5)$

88. $f'(x)$ for $f(x) = (x^4 + x^2 + 1)(x^2 - 1)$

89. y' for $y = (x^2 + x + 1)(x^2 - x + 1)$

90. $g'(t)$ for $g(t) = (t + 1)(t^4 - t^3 + t^2 - t + 1)$

91. $\dfrac{dy}{dt}$ for $y = \dfrac{t \ln t}{e^t}$

92. $\dfrac{dy}{du}$ for $y = \dfrac{u^2 e^u}{1 + \ln u}$

Applications

93. Sales analysis. The total sales S (in thousands) of a video game are given by

$$S(t) = \frac{90t^2}{t^2 + 50}$$

where t is the number of months since the release of the game.

(A) Find $S'(t)$.

(B) Find $S(10)$ and $S'(10)$. Write a brief interpretation of these results.

(C) Use the results from part (B) to estimate the total sales after 11 months.

94. Sales analysis. A communications company has installed a new cable television system in a city. The total number N (in thousands) of subscribers t months after the installation of the system is given by

$$N(t) = \frac{180t}{t + 4}$$

(A) Find $N'(t)$.

(B) Find $N(16)$ and $N'(16)$. Write a brief interpretation of these results.

(C) Use the results from part (B) to estimate the total number of subscribers after 17 months.

95. Price–demand equation. According to economic theory, the demand x for a quantity in a free market decreases as the price p increases (see figure). Suppose that the number x of DVD players people are willing to buy per week from a retail chain at a price of \$$p$ is given by

$$x = \frac{4{,}000}{0.1p + 1} \qquad 10 \le p \le 70$$

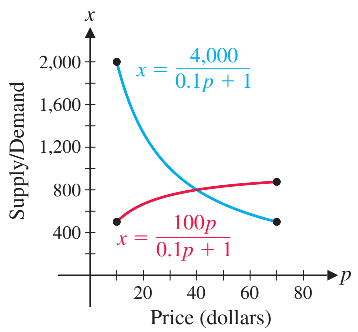


Figure for 95 and 96

(A) Find dx/dp.

(B) Find the demand and the instantaneous rate of change of demand with respect to price when the price is \$40. Write a brief interpretation of these results.

(C) Use the results from part (B) to estimate the demand if the price is increased to \$41.

96. Price–supply equation. According to economic theory, the supply x of a quantity in a free market increases as the price p increases (see figure). Suppose that the number x of DVD players a retail chain is willing to sell per week at a price of \$$p$ is given by

$$x = \frac{100p}{0.1p + 1} \qquad 10 \le p \le 70$$

(A) Find dx/dp.

(B) Find the supply and the instantaneous rate of change of supply with respect to price when the price is \$40. Write a brief verbal interpretation of these results.

(C) Use the results from part (B) to estimate the supply if the price is increased to \$41.

97. Medicine. A drug is injected into a patient's bloodstream through her right arm. The drug concentration (in milligrams per cubic centimeter) in the bloodstream of the left arm t hours after the injection is given by

$$C(t) = \frac{0.14t}{t^2 + 1}$$

(A) Find $C'(t)$.

(B) Find $C'(0.5)$ and $C'(3)$, and interpret the results.

98. Drug sensitivity. One hour after a dose of x milligrams of a particular drug is administered to a person, the change in body temperature $T(x)$, in degrees Fahrenheit, is given approximately by

$$T(x) = x^2\left(1 - \frac{x}{9}\right) \qquad 0 \le x \le 7$$

The rate $T'(x)$ at which T changes with respect to the size of the dosage x is called the *sensitivity* of the body to the dosage.

(A) Use the product rule to find $T'(x)$.

(B) Find $T'(1)$, $T'(3)$, and $T'(6)$.

Answers to Matched Problems

1. $30x^4 - 36x^3 + 9x^2$

2. (A) $y = 84x - 297$

(B) $x = -4, x = 1$

3. (A) $5x^8e^x + e^x(40x^7) = 5x^7(x + 8)e^x$

(B) $x^7 \cdot \frac{1}{x} + \ln x\,(7x^6) = x^6\,(1 + 7\ln x)$

4. (A) $\frac{(x^2 + 3)2 - (2x)(2x)}{(x^2 + 3)^2} = \frac{6 - 2x^2}{(x^2 + 3)^2}$

(B) $\frac{(t^2 - 4)(3t^2 - 3) - (t^3 - 3t)(2t)}{(t^2 - 4)^2} = \frac{t^4 - 9t^2 + 12}{(t^2 - 4)^2}$

(C) $-\frac{6}{x^4}$

5. (A) $\frac{(e^x + 2)\,3x^2 - x^3e^x}{(e^x + 2)^2}$

(B) $\frac{(1 + \ln x)\,4 - 4x\frac{1}{x}}{(1 + \ln x)^2} = \frac{4\ln x}{(1 + \ln x)^2}$

6. (A) $S'(t) = \frac{450}{(t + 3)^2}$

(B) $S(12) = 120$; $S'(12) = 2$. After 12 months, the total sales are 120,000 games, and sales are increasing at the rate of 2,000 games per month.

(C) 122,000 games

3.5 The Chain Rule

- Composite Functions
- General Power Rule
- The Chain Rule

The word *chain* in the name "chain rule" comes from the fact that a function formed by composition involves a chain of functions—that is, a function of a function. The *chain rule* enables us to compute the derivative of a composite function in terms of the derivatives of the functions making up the composite. In this section, we review composite functions, introduce the chain rule by means of a special case known as the *general power rule,* and then discuss the chain rule itself.

Composite Functions

The function $m(x) = (x^2 + 4)^3$ is a combination of a quadratic function and a cubic function. To see this more clearly, let

$$y = f(u) = u^3 \qquad \text{and} \qquad u = g(x) = x^2 + 4$$

We can express y as a function of x:

$$y = f(u) = f[g(x)] = [x^2 + 4]^3 = m(x)$$

The function m is the *composite* of the two functions f and g.

DEFINITION Composite Functions

A function m is a **composite** of functions f and g if

$$m(x) = f[g(x)]$$

The domain of m is the set of all numbers x such that x is in the domain of g, and $g(x)$ is in the domain of f.

The composite m of functions f and g is pictured in Figure 1. The domain of m is the shaded subset of the domain of g (Fig. 1); it consists of all numbers x such that x is in the domain of g and $g(x)$ is in the domain of f. Note that the functions f and g play different roles. The function g, which is on the *inside* or *interior* of the square brackets in $f[g(x)]$, is applied first to x. Then function f, which appears on the *outside* or *exterior* of the square brackets, is applied to $g(x)$, provided $g(x)$ is in the domain of f. Because f and g play different roles, the composite of f and g is usually a different function than the composite of g and f, as illustrated by Example 1.

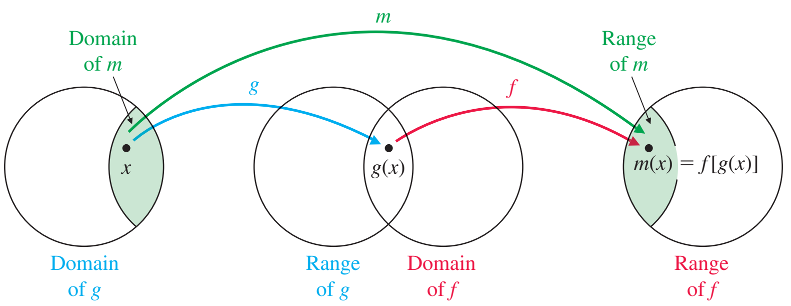


Figure 1 **The composite m of f and g**

EXAMPLE 1 **Composite Functions** Let $f(u) = e^u$ and $g(x) = -3x$. Find $f[g(x)]$ and $g[f(u)]$.

SOLUTION

$$f[g(x)] = f(-3x) = e^{-3x}$$
$$g[f(u)] = g(e^u) = -3e^u$$

Matched Problem 1 Let $f(u) = 2u$ and $g(x) = e^x$. Find $f[g(x)]$ and $g[f(u)]$.

EXAMPLE 2 **Composite Functions** Write each function as a composite of two simpler functions.

(A) $y = 100e^{0.04x}$ (B) $y = \sqrt{4 - x^2}$

SOLUTION

(A) Let

$$y = f(u) = 100e^u$$
$$u = g(x) = 0.04x$$

Check: $y = f[g(x)] = f(0.04x) = 100e^{0.04x}$

(B) Let

$$y = f(u) = \sqrt{u}$$
$$u = g(x) = 4 - x^2$$

Check: $y = f[g(x)] = f(4 - x^2) = \sqrt{4 - x^2}$

Matched Problem 2 Write each function as a composite of two simpler functions.

(A) $y = 50e^{-2x}$ (B) $y = \sqrt[3]{1 + x^3}$

CONCEPTUAL INSIGHT

There can be more than one way to express a function as a composite of simpler functions. Choosing $y = f(u) = 100u$ and $u = g(x) = e^{0.04x}$ in Example 2A produces the same result:

$$y = f[g(x)] = 100g(x) = 100e^{0.04x}$$

Since we will be using composition as a means to an end (finding a derivative), usually it will not matter which functions you choose for the composition.

General Power Rule

We have already made extensive use of the power rule,

$$\frac{d}{dx}x^n = nx^{n-1} \qquad (1)$$

Can we apply rule (1) to find the derivative of the composite function $m(x) = p[u(x)] = [u(x)]^n$, where p is the power function $p(u) = u^n$ and $u(x)$ is a differentiable function? In other words, is rule (1) valid if x is replaced by $u(x)$?

Explore and Discuss 1

Let $u(x) = 2x^2$ and $m(x) = [u(x)]^3 = 8x^6$. Which of the following is $m'(x)$?

(A) $3[u(x)]^2$ (B) $3[u'(x)]^2$ (C) $3[u(x)]^2u'(x)$

The calculations in Explore and Discuss 1 show that we cannot find the derivative of $[u(x)]^n$ simply by replacing x with $u(x)$ in equation (1).

How can we find a formula for the derivative of $[u(x)]^n$, where $u(x)$ is an arbitrary differentiable function? Let's begin by considering the derivatives of $[u(x)]^2$ and $[u(x)]^3$ to see if a general pattern emerges. Since $[u(x)]^2 = u(x)u(x)$, we use the product rule to write

$$\frac{d}{dx}[u(x)]^2 = \frac{d}{dx}[u(x)u(x)]$$
$$= u(x)u'(x) + u(x)u'(x)$$
$$= 2u(x)u'(x) \qquad (2)$$

Because $[u(x)]^3 = [u(x)]^2 u(x)$, we use the product rule and the result in equation (2) to write

$$\begin{aligned}\frac{d}{dx}[u(x)]^3 &= \frac{d}{dx}\{[u(x)]^2 u(x)\} \\ &= [u(x)]^2 \frac{d}{dx}u(x) + u(x)\frac{d}{dx}[u(x)]^2 \\ &= [u(x)]^2 u'(x) + u(x)[2u(x)u'(x)] \\ &= 3[u(x)]^2 u'(x)\end{aligned}$$

Use equation (2) to substitute for $\frac{d}{dx}[u(x)]^2$.

Continuing in this fashion, we can show that

$$\frac{d}{dx}[u(x)]^n = n[u(x)]^{n-1}u'(x) \qquad n \text{ a positive integer} \qquad (3)$$

Using more advanced techniques, we can establish formula (3) for all real numbers n, obtaining the **general power rule**.

THEOREM 1 General Power Rule

If $u(x)$ is a differentiable function, n is any real number, and

$$y = f(x) = [u(x)]^n$$

then

$$f'(x) = n[u(x)]^{n-1}u'(x)$$

Using simplified notation,

$$y' = nu^{n-1}u' \qquad \text{or} \qquad \frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx} \qquad \text{where } u = u(x)$$

EXAMPLE 3 **Using the General Power Rule** Find the indicated derivatives:

(A) $f'(x)$ if $f(x) = (3x + 1)^4$

(B) y' if $y = (x^3 + 4)^7$

(C) $\frac{d}{dt}\frac{1}{(t^2 + t + 4)^3}$

(D) $\frac{dh}{dw}$ if $h(w) = \sqrt{3 - w}$

SOLUTION

(A) $f(x) = (3x + 1)^4$ — Apply general power rule.

$f'(x) = 4(3x + 1)^3(3x + 1)'$ — Substitute $(3x + 1)' = 3$.

$= 4(3x + 1)^3 3$ — Simplify.

$= 12(3x + 1)^3$

(B) $y = (x^3 + 4)^7$ — Apply general power rule.

$y' = 7(x^3 + 4)^6(x^3 + 4)'$ — Substitute $(x^3 + 4)' = 3x^2$.

$= 7(x^3 + 4)^6 3x^2$ — Simplify.

$= 21x^2(x^3 + 4)^6$

(C) $\frac{d}{dt}\frac{1}{(t^2+t+4)^3}$

$= \frac{d}{dt}(t^2+t+4)^{-3}$ Apply general power rule.

$= -3(t^2+t+4)^{-4}(t^2+t+4)'$ Substitute $(t^2+t+4)' = 2t+1$.

$= -3(t^2+t+4)^{-4}(2t+1)$ Simplify.

$= \frac{-3(2t+1)}{(t^2+t+4)^4}$

(D) $h(w) = \sqrt{3-w} = (3-w)^{1/2}$ Apply general power rule.

$\frac{dh}{dw} = \frac{1}{2}(3-w)^{-1/2}(3-w)'$ Substitute $(3-w)' = -1$.

$= \frac{1}{2}(3-w)^{-1/2}(-1)$ Simplify.

$= -\frac{1}{2(3-w)^{1/2}}$ or $-\frac{1}{2\sqrt{3-w}}$

Matched Problem 3 Find the indicated derivatives:

(A) $h'(x)$ if $h(x) = (5x+2)^3$

(B) y' if $y = (x^4-5)^5$

(C) $\frac{d}{dt}\frac{1}{(t^2+4)^2}$

(D) $\frac{dg}{dw}$ if $g(w) = \sqrt{4-w}$

Notice that we used two steps to differentiate each function in Example 3. First, we applied the general power rule, and then we found du/dx. As you gain experience with the general power rule, you may want to combine these two steps. If you do this, be certain to multiply by du/dx. For example,

$$\frac{d}{dx}(x^5+1)^4 = 4(x^5+1)^3 5x^4 \quad \text{Correct}$$

$$\frac{d}{dx}(x^5+1)^4 \neq 4(x^5+1)^3 \quad du/dx = 5x^4 \text{ is missing}$$

CONCEPTUAL INSIGHT

If we let $u(x) = x$, then $du/dx = 1$, and the general power rule reduces to the (ordinary) power rule discussed in Section 2.5. Compare the following:

$$\frac{d}{dx}x^n = nx^{n-1} \quad \text{Yes—power rule}$$

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx} \quad \text{Yes—general power rule}$$

$$\frac{d}{dx}u^n \neq nu^{n-1} \quad \text{Unless } u(x) = x + k\text{, so that } du/dx = 1$$