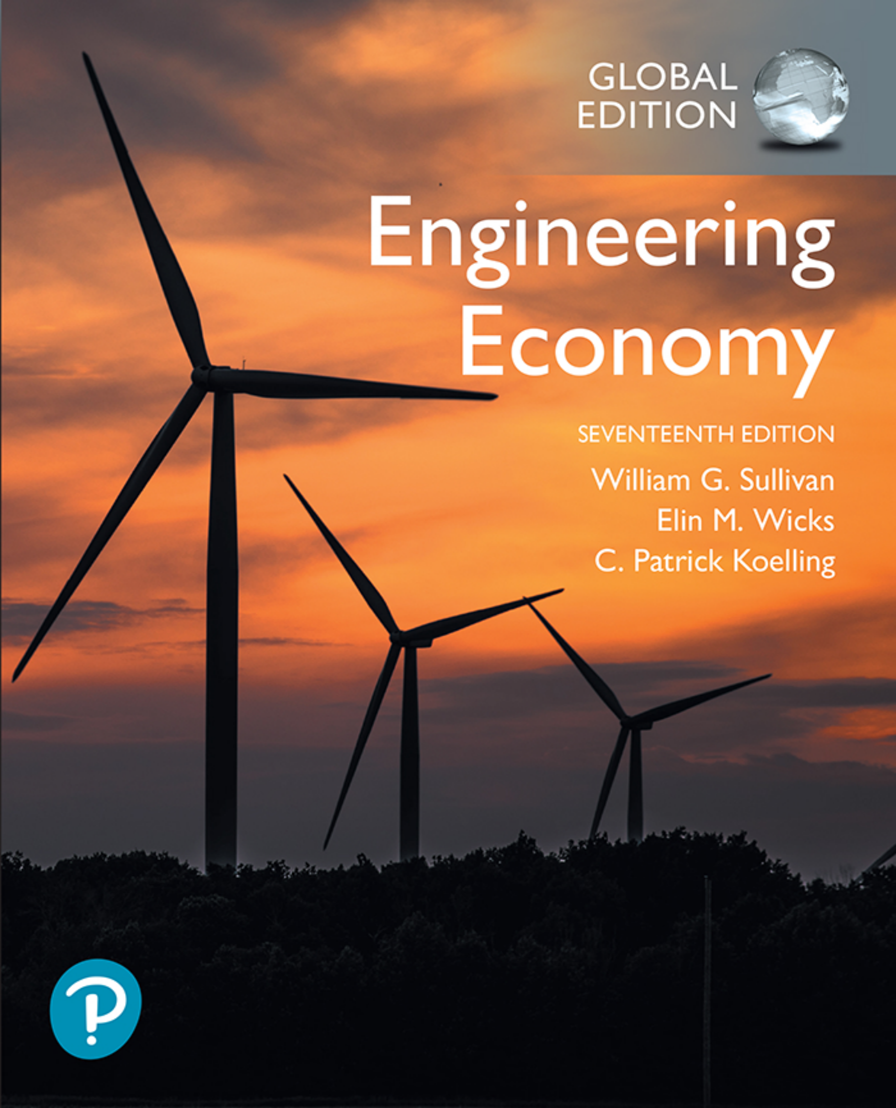

GLOBAL EDITION

# Engineering Economy

SEVENTEENTH EDITION

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# Abbreviations and Notation Summary

## CHAPTER 4

| | |
|---|---|
| APR | annual percentage rate (nominal interest) |
| EOY | end of year |
| $\bar{f}$ | a geometric change from one time period to the next in cash flows or equivalent values |
| $i$ | effective interest rate per interest period |
| $r$ | nominal interest rate per period (usually a year) |

## CHAPTER 5

| | |
|---|---|
| AW($i$%) | equivalent uniform annual worth, computed at $i$% interest, of one or more cash flows |
| CR($i$%) | equivalent annual cost of capital recovery, computed at $i$% interest |
| CW($i$%) | capitalized worth (a present equivalent), computed at $i$% interest |
| EUAC($i$%) | equivalent uniform annual cost, calculated at $i$% interest |
| FW($i$%) | future equivalent worth, calculated at $i$% interest, of one or more cash flows |
| IRR | internal rate of return, also designated $i$% |
| MARR | minimum attractive rate of return |
| $N$ | length of the study period (usually years) |
| PW($i$%) | present equivalent worth, computed at $i$% interest, of one or more cash flows |

## CHAPTER 6

| | |
|---|---|
| $\Delta(B - A)$ | incremental net cash flow (difference) calculated from the cash flow of Alternative $B$ minus the cash flow of Alternative $A$ (read: delta $B$ minus $A$) |

$$
\begin{aligned}
\text{PW} &= -\$10{,}000 + \$130(P/A, 0.5\%, \text{ per month, } 240 \text{ months}) \\
&= -\$10{,}000 + \$130(139.5808) \\
&= \$8{,}145.50.
\end{aligned}
$$

The positive-valued PW signals a favorable investment. Additionally, 13 tons/year $\times$ 20 years = 260 tons of carbon dioxide will be avoided. Can you rework this problem when the MARR is 1% per month? Is the system still a judicious choice?

### 5.3.1 Assumptions of the PW Method

There are several noteworthy assumptions that we make when using PW to model the wealth-creating promise of a capital investment opportunity. First, it is assumed that we know the future with certainty (we don't live in a certain world!). For example, we presume to know with certainty future interest rates and other factors. Second, it is assumed we can borrow and lend money at the same interest rate (i.e., capital markets are perfect). Regrettably, the real world has neither certainty nor perfect (*frictionless*, e.g., no taxes and/or commissions) capital markets.

The PW (and FW and AW, to follow) model is built on these seemingly restrictive assumptions, but it is cost-beneficial in the sense that the cost of using the PW model is less than the benefits of improved decisions resulting from PW analysis. More sophisticated models exist, but they usually do not reverse decisions made with the PW model. Therefore, our goal is to cost-beneficially recommend capital investments that maximize the wealth of a firm to its owners (i.e., stockholders). A positive-valued PW (and FW and AW) means that accepting a project will increase the worth, or value, of the firm.

### 5.3.2 Bond Value

A bond is an IOU where you agree to lend the bond issuer money for a specified length of time (say, 10 years). In return, you receive periodic interest payments (e.g., quarterly) from the issuer plus a promise to return the face value of the bond when it matures. A bond provides an excellent example of commercial value as being the PW of the future net cash flows that are expected to be received through ownership of an interest-bearing certificate. Thus, the value of a bond, at any time, is the PW of future cash receipts. For a bond, let

$Z$ = face, or par, value;
$C$ = redemption or disposal price (usually equal to Z);
$r$ = bond rate (nominal interest) per interest period;
$N$ = number of periods before redemption;
$i$ = bond *yield* rate per period;
$V_N$ = value (price) of the bond $N$ interest periods prior to redemption—this is a PW measure of merit.

The owner of a bond is paid two types of payments by the borrower. The first consists of the series of periodic interest payments he or she will receive until the bond is retired.

There will be $N$ such payments, each amounting to $rZ$. These constitute an annuity of $N$ payments. In addition, when the bond is retired or sold, the bondholder will receive a single payment equal in amount to $C$. The PW of the bond is the sum of PWs of these two types of payments at the bond's yield rate ($i\%$):

$$V_N = C(P/F, i\%, N) + rZ(P/A, i\%, N). \tag{5-2}$$

The most common situations faced by you as a potential investor in bonds are (1) for a desired yield rate, how much should you be willing to pay for the bond and (2) for a stated purchase price, what will your yield be? Examples 5-3 and 5-4 demonstrate how to solve these types of problems.

## EXAMPLE 5-3 Stan Moneymaker Wants to Buy a Bond

Stan Moneymaker has the opportunity to purchase a certain U.S. Treasury bond that matures in eight years and has a face value of \$10,000. This means that Stan will receive \$10,000 cash when the bond's maturity date is reached. The bond stipulates a fixed nominal interest rate of 8% per year, but interest payments are made to the bondholder every three months; therefore, each payment amounts to 2% of the face value.

Stan would like to earn 10% nominal interest (compounded quarterly) per year on his investment, because interest rates in the economy have risen since the bond was issued. How much should Stan be willing to pay for the bond?

### Solution

To establish the value of this bond, in view of the stated conditions, the PW of future cash flows during the next eight years (the study period) must be evaluated. Interest payments are quarterly. Because Stan Moneymaker desires to obtain 10% *nominal interest per year* on the investment, the PW is computed at $i = 10\%/4 = 2.5\%$ per quarter for the remaining $8(4) = 32$ quarters of the bond's life:

$$\begin{aligned} V_N &= \$10{,}000(P/F, 2.5\%, 32) + \$10{,}000(0.02)(P/A, 2.5\%, 32) \\ &= \$4{,}537.71 + \$4{,}369.84 = \$8{,}907.55. \end{aligned}$$

Thus, Stan should pay no more than \$8,907.55 when 10% nominal interest per year is desired.

## EXAMPLE 5-4 Current Price and Annual Yield of Bond Calculations

A bond with a face value of \$5,000 pays interest of 8% per year. This bond will be redeemed at par value at the end of its 20-year life, and the first interest payment is due one year from now.

(a) How much should be paid now for this bond in order to receive a yield of 10% per year on the investment?

(b) If this bond is purchased now for \$4,600, what annual yield would the buyer receive?

### Solution

(a) By using Equation (5-2), the value of $V_N$ can be determined:

$$V_N = \$5{,}000(P/F, 10\%, 20) + \$5{,}000(0.08)(P/A, 10\%, 20)$$
$$= \$743.00 + \$3{,}405.44 = \$4{,}148.44.$$

(b) Here, we are given $V_N = \$4{,}600$, and we must find the value of $i\%$ in Equation (5-2):

$$\$4{,}600 = \$5{,}000(P/F, i'\%, 20) + \$5{,}000(0.08)(P/A, i'\%, 20).$$

To solve for $i'\%$, we can resort to an iterative trial-and-error procedure (e.g., try 8.5%, 9.0%), to determine that $i'\% = 8.9\%$ per year.

## 5.3.3 The Capitalized-Worth Method

One special variation of the PW method discussed in Section 5.3 involves determining the PW of all revenues or expenses over an infinite length of time. This is known as the *Capitalized-Worth* (CW) method. If only expenses are considered, results obtained by this method are sometimes referred to as *capitalized cost*. As will be demonstrated in Chapter 6, the CW method is a convenient basis for comparing mutually exclusive alternatives when the period of needed service is indefinitely long.

The CW of a perpetual series of end-of-period uniform payments $A$, with interest at $i\%$ per period, is $A(P/A, i\%, \infty)$. From the interest formulas, it can be seen that $(P/A, i\%, N) \rightarrow 1/i$ as $N$ becomes very large. Thus, CW $= A/i$ for such a series, as can also be seen from the relation

$$\text{CW}(i\%) = \text{PW}_{N\rightarrow\infty} = A(P/A, i\%, \infty) = A\left[\lim_{N\rightarrow\infty} \frac{(1+i)^N - 1}{i(1+i)^N}\right] = A\left(\frac{1}{i}\right).$$

Hence, the CW of a project with interest at $i\%$ per year is the annual equivalent of the project over its useful life divided by $i$ (as a decimal).

The AW of a series of payments of amount $\$X$ at the end of each $k$th period with interest at $i\%$ per period is $\$X(A/F, i\%, k)$. The CW of such a series can thus be calculated as $\$X(A/F, i\%, k)/i$.

### EXAMPLE 5-5 Determining the Capitalized Worth of a Bridge

A new bridge across the Cumberland River is being planned near a busy highway intersection in the commercial part of a midwestern town. The construction (first) cost of the bridge is \$1,900,000 and annual upkeep is estimated to be \$25,000. In addition to annual upkeep, major maintenance work is anticipated every eight years at a cost of \$350,000 per occurrence. The town government's MARR is 8% per year.

(a) For this problem, what analysis period ($N$) is, practically speaking, defined as forever?

(b) If the bridge has an expected life of 50 years, what is the capitalized worth (CW) of the bridge over a 100-year study period?

### Solution

(a) A practical approximation of "forever" (infinity) is dependent on the interest rate. By examining the ($A/P$, $i$%, $N$) factor as $N$ increases in the Appendix C tables, we observe that this factor approaches a value of $i$ as $N$ becomes large. For $i$ = 8% (Table C-11), the ($A/P$, 8%, 100) factor is 0.08. So $N$ = 100 years is, for practical purposes, "forever" in this example.

(b) The CW is determined as follows:

$$\begin{aligned} \text{CW}(8\%) &= -\$1{,}900{,}000 - \$1{,}900{,}000\,(P/F, 8\%, 50) \\ &\quad - [\$350{,}000\,(A/F, 8\%, 8)]/0.08 - \$25{,}000/0.08. \end{aligned}$$

The CW turns out to be −\$2,664,220 over a 100-year study period, assuming the bridge is replaced at the end of year 50 for \$1,900,000.

## 5.4 The Future Worth Method

Because a primary objective of all time value of money methods is to maximize the future wealth of the owners of a firm, the economic information provided by the FW method is very useful in capital investment decision situations. The FW is based on the equivalent worth of all cash inflows and outflows at the end of the planning horizon (study period) at an interest rate that is generally the MARR. Also, the FW of a project is equivalent to its PW; that is, $\text{FW} = \text{PW}(F/P, i\%, N)$. If $\text{FW} \geq 0$ for a project, it would be economically justified.

FW Decision Rule: If FW ($i$ = MARR) $\geq 0$, the project is economically justified.

Equation (5-3) summarizes the general calculations necessary to determine a project's FW:

$$\begin{aligned} \text{FW}(i\%) &= F_0(1+i)^N + F_1(1+i)^{N-1} + \cdots + F_N(1+i)^0 \\ &= \sum_{k=0}^{N} F_k(1+i)^{N-k}. \end{aligned} \tag{5-3}$$

**EXAMPLE 5-6** **The Relationship between FW and PW**

Evaluate the FW of the potential improvement project described in Example 5-1. Show the relationship between FW and PW for this example.

### Solution

$$\begin{aligned} \text{FW}(20\%) &= -\$25{,}000(F/P, 20\%, 5) \\ &\quad + \$8{,}000(F/A, 20\%, 5) + \$5{,}000 \\ &= \$2{,}324.80. \end{aligned}$$

Again, the project is shown to be a good investment (FW $\geq$ 0). The PW is a multiple of the equivalent FW value:

$$\text{PW}(20\%) = \$2{,}324.80(P/F, 20\%, 5) = \$934.29.$$

To this point, the PW and FW methods have used a known and constant MARR over the study period. Each method produces a measure of merit expressed in dollars and is equivalent to the other. The difference in economic information provided is relative to the point in time used (i.e., the present for the PW versus the future, or end of the study period, for the FW).

**EXAMPLE 5-7** **Sensitivity Analysis Using FW (Example 5-2 Revisited)**

In Example 5-2, the \$110,000 retrofitted space-heating system was projected to save \$30,000 per year in electrical power and be worth \$8,000 at the end of the six-year study period. Use the FW method to determine whether the project is still economically justified if the system has zero market value after six years. The MARR is 15% per year.

### Solution

In this example, we need to find the future equivalent of the \$110,000 investment and the \$30,000 annual savings at an interest rate of 15% per year.

$$\begin{aligned} \text{FW}(15\%) &= -\$110{,}000\,(F/P, 15\%, 6) + \$30{,}000\,(F/A, 15\%, 6) \\ &= -\$110{,}000\,(2.3131) + \$30{,}000\,(8.7537) \\ &= \$8{,}170. \end{aligned}$$

The heating system is still a profitable project (FW $\geq$ 0) even if it has no market value at the end of the study period.

## 5.5 The Annual Worth Method

The AW of a project is an equal annual series of dollar amounts, for a stated study period, that is *equivalent* to the cash inflows and outflows at an interest rate that is generally the MARR. Hence, the AW of a project is annual equivalent revenues or savings ($\underline{R}$) minus annual equivalent expenses ($\underline{E}$), less its annual equivalent capital recovery (CR) amount, which is defined in Equation (5-5). An annual equivalent value of $\underline{R}$, $\underline{E}$, and CR is computed for the study period, $N$, which is usually in years. In equation form, the AW, which is a function of $i$%, is

$$\text{AW}(i\%) = \underline{R} - \underline{E} - \text{CR}(i\%). \tag{5-4}$$

Also, we need to notice that the AW of a project is equivalent to its PW and FW. That is, AW = PW($A/P$, $i$%, $N$), and AW = FW($A/F$, $i$%, $N$). Hence, it can be easily computed for a project from these other equivalent values.

As long as the AW evaluated at the MARR is greater than or equal to zero, the project is economically attractive; otherwise, it is not. An AW of zero means that an annual return exactly equal to the MARR has been earned. Many decision makers prefer the AW method because it is relatively easy to interpret when they are accustomed to working with annual income statements and cash-flow summaries.

AW Decision Rule: If AW ($i$ = MARR) $\geq$ 0, the project is economically justified.

The CR amount for a project is the equivalent uniform annual *cost* of the capital invested. It is an annual amount that covers the following two items:

1. Loss in value of the asset
2. Interest on invested capital (i.e., at the MARR)

As an example, consider a device that will cost \$10,000, last five years, and have a salvage (market) value of \$2,000. Thus, the loss in value of this asset over five years is \$8,000. Additionally, the MARR is 10% per year.

It can be shown that, no matter which method of calculating an asset's loss in value over time is used, the equivalent annual CR amount is the same. For example, if a uniform loss in value is assumed (\$8,000/5 years = \$1,600 per year), the equivalent annual CR amount is calculated to be \$2,310, as shown in Table 5-1.

There are several convenient formulas by which the CR amount (cost) may be calculated to obtain the result in Table 5-1. Probably the easiest formula to understand involves finding the annual equivalent of the initial capital investment and then subtracting the annual equivalent of the salvage value.* Thus,

$$\text{CR}(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N), \tag{5-5}$$

* The following two equations are alternative ways of calculating the CR amount:

$$\text{CR}(i\%) = (I - S)(A/F, i\%, N) + I(i\%);$$

$$\text{CR}(i\%) = (I - S)(A/P, i\%, N) + S(i\%).$$

It is left as a student exercise to show that the above equations are equivalent to Equation (5-5).