GLOBAL EDITION

Finite Mathematics

for Business, Economics, Life Sciences, and Social Sciences

FOURTEENTH EDITION

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MyLab Math for *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences,* 14e

(access code required)

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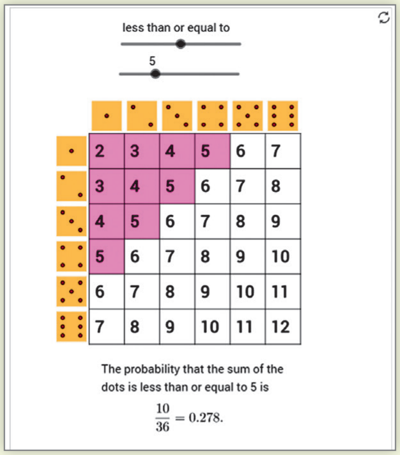

Interactive Figures

A full suite of Interactive Figures has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They have been designed to be used in lecture as well as by students independently.

Questions that Deepen Understanding

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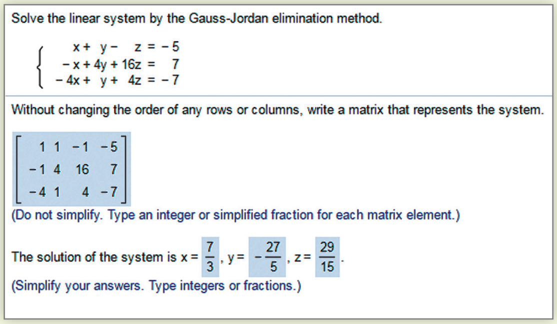

pearson.com/mylab/math

In Problems 19–28, examine the product of the two matrices to determine if each is the inverse of the other.

19. $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}; \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$

20. $\begin{bmatrix} -2 & -1 \\ -4 & 2 \end{bmatrix}; \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$

21. $\begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}; \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

22. $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}; \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

23. $\begin{bmatrix} -5 & 2 \\ -8 & 3 \end{bmatrix}; \begin{bmatrix} 3 & -2 \\ 8 & -5 \end{bmatrix}$

24. $\begin{bmatrix} 7 & 4 \\ -5 & -3 \end{bmatrix}; \begin{bmatrix} 3 & 4 \\ -5 & -7 \end{bmatrix}$

25. $\begin{bmatrix} 1 & -3 & -5 \\ -2 & 3 & 5 \\ -1 & 1 & 2 \end{bmatrix}; \begin{bmatrix} -1 & -1 & 0 \\ 1 & 3 & -5 \\ -1 & -2 & 3 \end{bmatrix}$

26. $\begin{bmatrix} 1 & 0 & 1 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

27. $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 5 & -4 \\ 0 & 2 & 1 \end{bmatrix}; \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & 0 \\ -2 & 1 & 0 \end{bmatrix}$

28. $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}; \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

Without performing any row operations, explain why each of the matrices in Problems 29–38 does not have an inverse.

29. $\begin{bmatrix} 1 & 2 & 0 \\ -3 & 2 & -1 \end{bmatrix}$

30. $\begin{bmatrix} -2 & 3 & -1 \\ 4 & 0 & 1 \end{bmatrix}$

31. $\begin{bmatrix} 1 & -2 \\ 3 & 0 \\ 2 & -1 \end{bmatrix}$

32. $\begin{bmatrix} 0 & -1 \\ 2 & -2 \\ 1 & -3 \end{bmatrix}$

33. $\begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$

34. $\begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$

35. $\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

36. $\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$

37. $\begin{bmatrix} 1 & -3 \\ -3 & 9 \end{bmatrix}$

38. $\begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$

B *Given M in Problems 39–48, find M^{-1} and show that $M^{-1}M = I$.*

39. $\begin{bmatrix} -1 & 0 \\ -3 & 1 \end{bmatrix}$

40. $\begin{bmatrix} 1 & -5 \\ 0 & -1 \end{bmatrix}$

41. $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

42. $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

43. $\begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$

44. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

45. $\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ 2 & -1 & 4 \end{bmatrix}$

46. $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 0 & -1 & -5 \end{bmatrix}$

47. $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$

48. $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$

Find the inverse of each matrix in Problems 49–56, if it exists.

49. $\begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}$

50. $\begin{bmatrix} -4 & 3 \\ -5 & 4 \end{bmatrix}$

51. $\begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$

52. $\begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix}$

53. $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

54. $\begin{bmatrix} -5 & 3 \\ 2 & -2 \end{bmatrix}$

55. $\begin{bmatrix} 3 & -1 \\ -5 & 35 \end{bmatrix}$

56. $\begin{bmatrix} 5 & -10 \\ -2 & 24 \end{bmatrix}$

In Problems 57–60, find the inverse. Note that each inverse can be found mentally, without the use of a calculator or pencil-and-paper calculations.

57. $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

58. $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

59. $\begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

60. $\begin{bmatrix} 3 & 0 \\ 0 & -\frac{1}{4} \end{bmatrix}$

C *Find the inverse of each matrix in Problems 61–68, if it exists.*

61. $\begin{bmatrix} -5 & -2 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

62. $\begin{bmatrix} 2 & -2 & 4 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

63. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$

64. $\begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

65. $\begin{bmatrix} -1 & -2 & 2 \\ 4 & 3 & 0 \\ 4 & 0 & 4 \end{bmatrix}$

66. $\begin{bmatrix} 4 & 2 & 2 \\ 4 & 2 & 0 \\ 5 & 0 & 5 \end{bmatrix}$

67. $\begin{bmatrix} 2 & -1 & -2 \\ -4 & 2 & 8 \\ 6 & -2 & -1 \end{bmatrix}$

68. $\begin{bmatrix} -1 & -1 & 4 \\ 3 & 3 & -22 \\ -2 & -1 & 19 \end{bmatrix}$

69. Show that $(A^{-1})^{-1} = A$ for: $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$

70. Show that $(AB)^{-1} = B^{-1}A^{-1}$ for

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$$

71. Discuss the existence of M^{-1} for 2×2 diagonal matrices of the form

$$M = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

Generalize your conclusions to $n \times n$ diagonal matrices.

72. Discuss the existence of M^{-1} for 2×2 upper triangular matrices of the form

$$M = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

Generalize your conclusions to $n \times n$ upper triangular matrices.

In Problems 73–75, find A^{-1} *and* A^2.

73. $\begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix}$ **74.** $\begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix}$ **75.** $\begin{bmatrix} 5 & -3 \\ 8 & -5 \end{bmatrix}$

76. Based on your observations in Problems 73–75, if $A = A^{-1}$ for a square matrix A, what is A^2? Give a mathematical argument to support your conclusion.

Applications

Problems 77–80 refer to the encoding matrix

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

77. Cryptography. Encode the message "WINGARDIUM LEVIOSA" using matrix A.

78. Cryptography. Encode the message "FINITE INCANTATEM" using matrix A.

79. Cryptography. The following message was encoded with matrix A. Decode this message:

52 70 17 21 5 5 29 43 4 4 52 70 25
35 29 33 15 18 5 5

80. Cryptography. The following message was encoded with matrix A. Decode this message:

36 44 5 5 38 56 55 75 18 23 56 75
22 33 37 55 27 40 53 79 59 81

Problems 81–84 require the use of a graphing calculator or computer. Use the 4×4 *encoding matrix B given below. Form a matrix with 4 rows and as many columns as necessary to accommodate the message.*

$$B = \begin{bmatrix} 2 & 2 & 1 & 3 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 3 & 2 & 3 \end{bmatrix}$$

81. Cryptography. Encode the message "DEPART ISTANBUL ORIENT EXPRESS" using matrix B.

82. Cryptography. Encode the message "SAIL FROM LISBON IN MORNING" using matrix B.

83. Cryptography. The following message was encoded with matrix B. Decode this message:

85 74 27 109 31 27 13 40 139 73 58 154
61 70 18 93 69 59 23 87 18 13 9 22

84. Cryptography. The following message was encoded with matrix B. Decode this message:

75 61 28 94 35 22 13 40 49 21 16 52
42 45 19 64 38 55 10 65 69 75 24 102
67 49 19 82 10 5 5 10

Problems 85–88 require the use of a graphing calculator or a computer. Use the 5×5 *encoding matrix C given below. Form a matrix with 5 rows and as many columns as necessary to accommodate the message.*

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 3 \\ 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$$

85. Cryptography. Encode the message "THE EAGLE HAS LANDED" using matrix C.

86. Cryptography. Encode the message "ONE IF BY LAND AND TWO IF BY SEA" using matrix C.

87. Cryptography. The following message was encoded with matrix C. Decode this message:

37 72 58 45 56 30 67 50 46 60 27 77
41 45 39 28 24 52 14 37 32 58 70 36
76 22 38 70 12 67

88. Cryptography. The following message was encoded with matrix C. Decode this message:

25 75 55 35 50 43 83 54 60 53 25 13
59 9 53 15 35 40 15 45 33 60 60 36
51 15 7 37 0 22

Answers to Matched Problems

1. (A) $\begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 2 \\ 3 & -5 \\ 6 & 8 \end{bmatrix}$

2. (A) $\left[\begin{array}{rrr|rrr} 3 & -1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

(B) $\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -1 & -1 & 2 \end{array}\right]$

(C) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$

4. Does not exist

5. WHO IS WILHELM JORDAN

3.6 Matrix Equations and Systems of Linear Equations

- Matrix Equations
- Matrix Equations and Systems of Linear Equations
- Application

The identity matrix and inverse matrix discussed in the preceding section can be put to immediate use in the solution of certain simple matrix equations. Being able to solve a matrix equation gives us another important method of solving systems of equations, provided that the system is independent and has the same number of variables as equations. If the system is dependent or if it has either fewer or more variables than equations, we must return to the Gauss–Jordan method of elimination.

Matrix Equations

Solving simple matrix equations is similar to solving real number equations but with two important differences:

1. there is *no* operation of division for matrices, and
2. matrix multiplication is *not* commutative.

Compare the real number equation $4x = 9$ and the matrix equation $AX = B$. The real number equation can be solved by dividing both sides of the equation by 4. However, that approach cannot be used for $AX = B$, because there is no operation of division for matrices. Instead, we note that $4x = 9$ can be solved by multiplying both sides of the equation by $\frac{1}{4}$, the multiplicative inverse of 4. So we solve $AX = B$ by multiplying both sides of the equation, *on the left*, by A^{-1}, the inverse of A. Because matrix multiplication is not commutative, multiplying both sides of an equation on the left by A^{-1} is different from multiplying both sides of an equation on the right by A^{-1}. In the case of $AX = B$, it is multiplication on the left that is required. The details are presented in Example 1.

In solving matrix equations, we will be guided by the properties of matrices summarized in Theorem 1.

THEOREM 1 Basic Properties of Matrices

Assuming that all products and sums are defined for the indicated matrices A, B, C, I, and 0, then

Addition Properties

Associative:	$(A + B) + C = A + (B + C)$
Commutative:	$A + B = B + A$
Additive identity:	$A + 0 = 0 + A = A$
Additive inverse:	$A + (-A) = (-A) + A = 0$

Multiplication Properties

Associative property:	$A(BC) = (AB)C$
Multiplicative identity:	$AI = IA = A$
Multiplicative inverse:	If A is a square matrix and A^{-1} exists, then $AA^{-1} = A^{-1}A = I$.

Combined Properties

Left distributive:	$A(B + C) = AB + AC$
Right distributive:	$(B + C)A = BA + CA$

Equality

Addition:	If $A = B$, then $A + C = B + C$.
Left multiplication:	If $A = B$, then $CA = CB$.
Right multiplication:	If $A = B$, then $AC = BC$.

EXAMPLE 1 **Solving a Matrix Equation** Given an $n \times n$ matrix A and $n \times 1$ column matrices B and X, solve $AX = B$ for X. Assume that all necessary inverses exist.

SOLUTION We are interested in finding a column matrix X that satisfies the matrix equation $AX = B$. To solve this equation, we multiply both sides on the left by A^{-1} to isolate X on the left side.

$$\begin{aligned} AX &= B && \text{Use the left multiplication property.} \\ A^{-1}(AX) &= A^{-1}B && \text{Use the associative property.} \\ (A^{-1}A)X &= A^{-1}B && A^{-1}A = I \\ IX &= A^{-1}B && IX = X \\ X &= A^{-1}B \end{aligned}$$

Matched Problem 1 Given an $n \times n$ matrix A and $n \times 1$ column matrices B, C, and X, solve $AX + C = B$ for X. Assume that all necessary inverses exist.

CAUTION Do not mix the left multiplication property and the right multiplication property. If $AX = B$, then

$$A^{-1}(AX) \neq BA^{-1}$$

▲

Matrix Equations and Systems of Linear Equations

Now we show how independent systems of linear equations with the same number of variables as equations can be solved. First, convert the system into a matrix equation of the form $AX = B$, and then use $X = A^{-1}B$ as obtained in Example 1.

EXAMPLE 2 **Using Inverses to Solve Systems of Equations** Use matrix inverse methods to solve the system:

$$\begin{aligned} x_1 - x_2 + x_3 &= 1 \\ 2x_2 - x_3 &= 1 \\ 2x_1 + 3x_2 &= 1 \end{aligned} \tag{1}$$

SOLUTION The inverse of the coefficient matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

provides an efficient method for solving this system. To see how, we convert system (1) into a matrix equation:

$$\overset{A}{\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}} \overset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \overset{B}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} \tag{2}$$

Check that matrix equation (2) is equivalent to system (1) by finding the product of the left side and then equating corresponding elements on the left with those on the right.

We are interested in finding a column matrix X that satisfies the matrix equation $AX = B$. In Example 1 we found that if A^{-1} exists, then

$$X = A^{-1}B$$

The inverse of A was found in Example 2, Section 3.5, to be

$$A^{-1} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

Therefore,

$$\overset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \overset{A^{-1}}{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}} \overset{B}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} = \begin{bmatrix} 5 \\ -3 \\ -7 \end{bmatrix}$$

and we can conclude that $x_1 = 5$, $x_2 = -3$, and $x_3 = -7$. Check this result in system (1).

Matched Problem 2 Use matrix inverse methods to solve the system:

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1 \\ -x_1 + x_2 \qquad &= 3 \\ x_1 \qquad + x_3 &= 2 \end{aligned}$$

[*Note:* The inverse of the coefficient matrix was found in Matched Problem 2, Section 3.5.]

At first glance, using matrix inverse methods seems to require the same amount of effort as using Gauss–Jordan elimination. In either case, row operations must be applied to an augmented matrix involving the coefficients of the system. The advantage of the inverse matrix method becomes readily apparent when solving a number of systems with a common coefficient matrix and different constant terms.

EXAMPLE 3 **Using Inverses to Solve Systems of Equations** Use matrix inverse methods to solve each of the following systems:

(A) $$\begin{aligned} x_1 - x_2 + x_3 &= 3 \\ 2x_2 - x_3 &= 1 \\ 2x_1 + 3x_2 \qquad &= 4 \end{aligned}$$

(B) $$\begin{aligned} x_1 - x_2 + x_3 &= -5 \\ 2x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 \qquad &= -3 \end{aligned}$$

SOLUTION Notice that both systems have the same coefficient matrix A as system (1) in Example 2. Only the constant terms have changed. We can use A^{-1} to solve these systems just as we did in Example 2.

(A)

$$\overset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \overset{A^{-1}}{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}} \overset{B}{\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}} = \begin{bmatrix} 8 \\ -4 \\ -9 \end{bmatrix}$$

$x_1 = 8$, $x_2 = -4$, and $x_3 = -9$.

(B)

$$\overset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \overset{A^{-1}}{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}} \overset{B}{\begin{bmatrix} -5 \\ 2 \\ -3 \end{bmatrix}} = \begin{bmatrix} -6 \\ 3 \\ 4 \end{bmatrix}$$

$x_1 = -6$, $x_2 = 3$, and $x_3 = 4$.

Matched Problem 3 Use matrix inverse methods to solve each of the following systems (see Matched Problem 2):

(A)
$$\begin{aligned} 3x_1 - x_2 + x_3 &= 3 \\ -x_1 + x_2 &= -3 \\ x_1 + x_3 &= 2 \end{aligned}$$

(B)
$$\begin{aligned} 3x_1 - x_2 + x_3 &= -5 \\ -x_1 + x_2 &= 1 \\ x_1 + x_3 &= -4 \end{aligned}$$

As Examples 2 and 3 illustrate, inverse methods are very convenient for hand calculations because once the inverse is found, it can be used to solve any new system formed by changing only the constant terms. Since most graphing calculators and computers can compute the inverse of a matrix, this method also adapts readily to graphing calculator and spreadsheet solutions (Fig. 1). However, if your graphing calculator (or spreadsheet) also has a built-in procedure for finding the reduced form of an augmented matrix, it is just as convenient to use Gauss–Jordan elimination. Furthermore, Gauss–Jordan elimination can be used in all cases and, as noted below, matrix inverse methods cannot always be used.

	A	B	C	D	E	F	G	H	I	J	K
1			A			B	X		B	X	
2		1	-1	1		3	8		-5	-6	
3		0	2	-1		1	-4		2	3	
4		2	3	0		4	-9		-3	4	
5											

Figure 1 Using inverse methods on a spreadsheet: The values in G2:G4 are produced by the command MMULT (MINVERSE(B2:D4),F2:F4)

SUMMARY Using Inverse Methods to Solve Systems of Equations

If the number of equations in a system equals the number of variables and the coefficient matrix has an inverse, then the system will always have a unique solution that can be found by using the inverse of the coefficient matrix to solve the corresponding matrix equation.

Matrix equation	Solution
$AX = B$	$X = A^{-1}B$

CONCEPTUAL INSIGHT

There are two cases where inverse methods will not work:

Case 1. The coefficient matrix is singular.

Case 2. The number of variables is not the same as the number of equations.

In either case, use Gauss–Jordan elimination.

Application

The following application illustrates the usefulness of the inverse matrix method for solving systems of equations.

EXAMPLE 4 **Investment Analysis** An investment advisor currently has two types of investments available for clients: a conservative investment A that pays 5% per year and a higher risk investment B that pays 10% per year. Clients may divide their investments between the two to achieve any total return desired between 5% and 10%. However, the higher the desired return, the higher the risk. How should each client invest to achieve the indicated return?