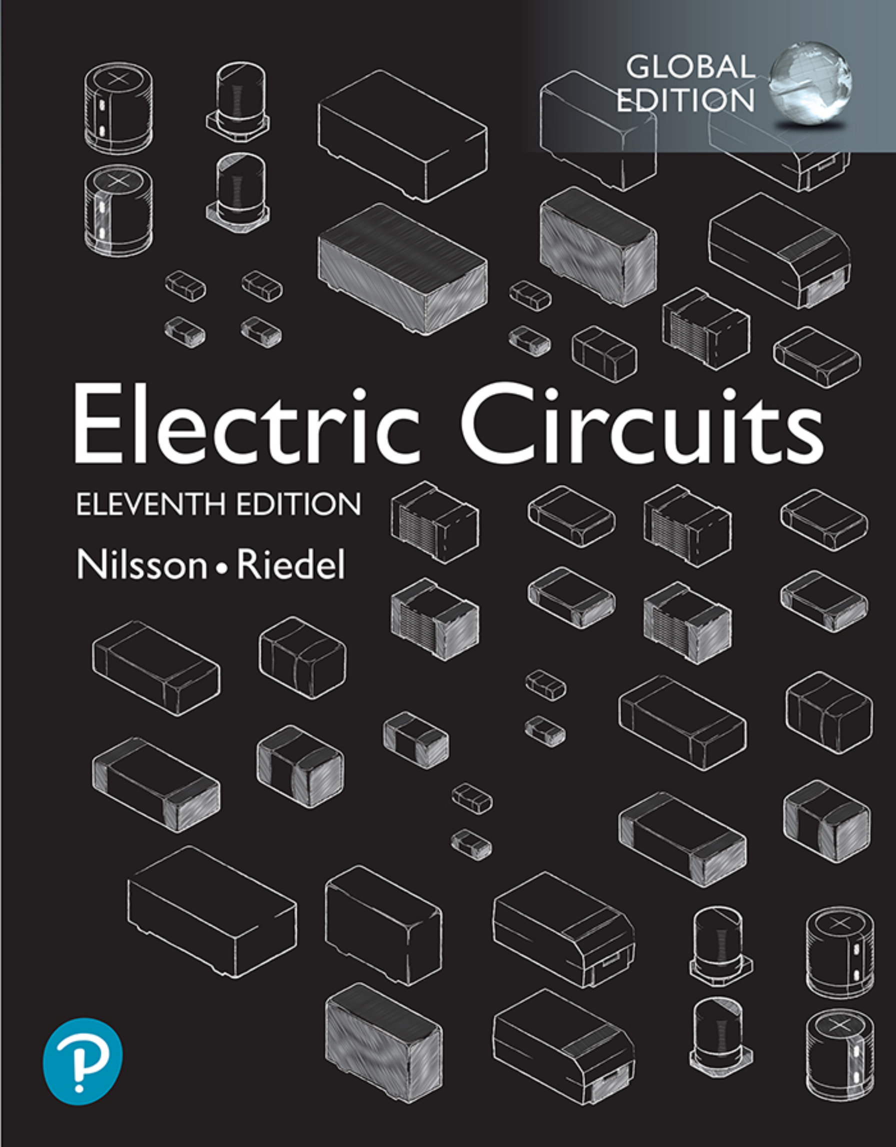


GLOBAL EDITION

Electric Circuits

ELEVENTH EDITION

Nilsson • Riedel

TABLE 4.3 Steps in the Node-Voltage Method and the Mesh-Current Method

	Node-Voltage Method	Mesh-Current Method
Step 1 Identify nodes/meshes	Identify the essential nodes by circling them on the circuit diagram	Identify the meshes by drawing directed arrows inside each mesh
Step 2 Label node voltages/mesh currents Recognize special cases	Pick and label a reference node; then label the remaining essential node voltages • If a voltage source is the only component in a branch connecting the reference node and another essential node, label the essential node with the value of the voltage source • If a voltage source is the only component in a branch connecting two nonreference essential nodes, create a supernode that includes the voltage source and the two nodes on either side	Label each mesh current • If a current source is in a single mesh, label the mesh current with the value of the current source • If a current source is shared by two adjacent meshes, create a supermesh by combining the two adjacent meshes and temporarily eliminating the branch that contains the current source
Step 3 Write the equations	Write the following equations: • A KCL equation for any supernodes • A KCL equation for any remaining essential nodes where the voltage is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the node voltages • A constraint equation for each supernode that equates the difference between the two node voltages in the supernode to the voltage source in the supernode	Write the following equations: • A KVL equation for any supermeshes • A KVL equation for any remaining meshes where the current is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the mesh currents • A constraint equation for each supermesh that equates the difference between the two mesh currents in the supermesh to the current source eliminated to form the supermesh
Step 4 Solve the equations	Solve the equations to find the node voltages	Solve the equations to find the mesh currents
Step 5 Solve for other unknowns	Use the node voltage values to find any unknown voltages, currents, or powers	Use the mesh current values to find any unknown voltages, currents, or powers

ANALYZING A CIRCUIT WITH AN IDEAL OP AMP

1. Check for a negative feedback path. If it exists, assume the op amp operates in its linear region.

2. Write a KCL equation at the inverting input terminal.

3. Solve the KCL equation and use the solution to find the op amp's output voltage.

4. Compare the op amp's output voltage to the power supply voltages to determine if the op amp is operating in its linear region or if it is saturated.

GENERAL METHOD FOR NATURAL AND STEP RESPONSE OF *RL* AND *RC* CIRCUITS

1. Identify the variable $x(t)$, which is the inductor current for RL circuits and capacitor voltage for RC circuits.

2. Calculate the initial value X_0, by analyzing the circuit to find $x(t)$ for $t < 0$.

3. Calculate the time constant τ; for RL circuits $\tau = L/R$ and for RC circuits $\tau = RC$, where R is the equivalent resistance connected to the inductor or capacitor for $t \geq 0$.

4. Calculate the final value X_f, by analyzing the circuit to find $x(t)$ as $t \to \infty$; for the natural response, $X_f = 0$.

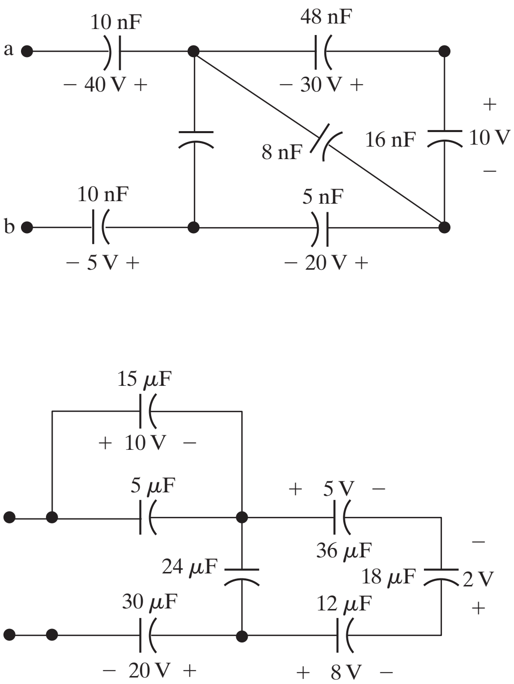
5. Write the equation for $x(t)$,
$x(t) = X_f + (X_0 - X_f)\, e^{-t/\tau}$, for $t \geq 0$.

6. Calculate other quantities of interest using $x(t)$.

6.26 For the circuit shown in Fig. P6.25, how many milliseconds after the switch is opened is the energy delivered to the black box 80% of the total energy delivered?

6.27 Find the equivalent capacitance with respect to the terminals a, b for the circuits shown in Fig. P6.27.

Figure P6.27



6.28 Use realistic capacitor values from Appendix H to construct series and parallel combinations of capacitors to yield the equivalent capacitances specified below. Try to minimize the number of capacitors used. Assume that no initial energy is stored in any of the capacitors.

a) 480 pF;

b) 600 nF;

c) 120 μF.

6.29 Derive the equivalent circuit for a series connection of ideal capacitors. Assume that each capacitor has its own initial voltage. Denote these initial voltages as $v_1(t_0)$, $v_2(t_0)$, and so on. (*Hint*: Sum the voltages across the string of capacitors, recognizing that the series connection forces the current in each capacitor to be the same.)

6.30 Derive the equivalent circuit for a parallel connection of ideal capacitors. Assume that the initial voltage across the paralleled capacitors is $v(t_0)$. (*Hint*: Sum the currents into the string of capacitors, recognizing that the parallel connection forces the voltage across each capacitor to be the same.)

6.31 The two series-connected capacitors in Fig. P6.31 are connected to the terminals of a black box at $t = 0$. The resulting current $i(t)$ for $t > 0$ is known to be $800e^{-25t}\,\mu$A.

a) Replace the original capacitors with an equivalent capacitor and find $v_o(t)$ for $t \geq 0$.

b) Find $v_1(t)$ for $t \geq 0$.

c) Find $v_2(t)$ for $t \geq 0$.

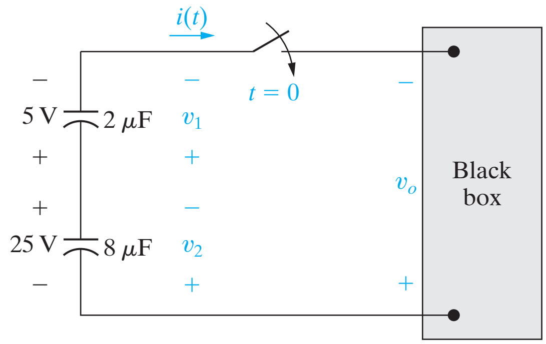
d) How much energy is delivered to the black box in the time interval $0 \leq t < \infty$?

e) How much energy was initially stored in the series capacitors?

f) How much energy is trapped in the ideal capacitors?

g) Show that the solutions for ${}_1$ and ${}_2$ agree with the answer obtained in (f).

Figure P6.31

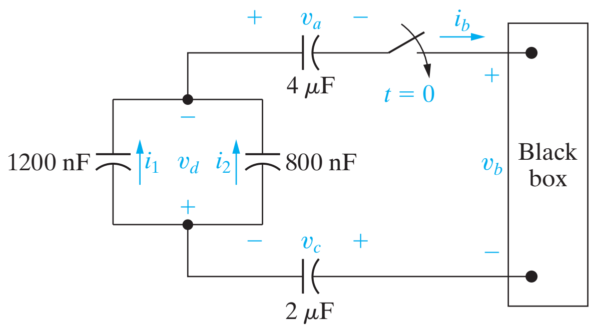


6.32 The four capacitors in the circuit in Fig. P6.32 are connected across the terminals of a black box at $t = 0$. The resulting current i_b for $t > 0$ is known to be

$$i_b = -5e^{-50t}\,\text{mA}.$$

If $v_a(0) = -20$ V, $v_c(0) = -30$ V and $v_d(0) = 250$ V, find the following for $t \geq 0$: (a) $v_b(t)$, (b) $v_a(t)$, (c) $v_c(t)$, (d) $v_d(t)$, (e) $i_1(t)$, and (f) $i_2(t)$.

Figure P6.32



6.33 For the circuit in Fig. P6.32, calculate

a) the initial energy stored in the capacitors;

b) the final energy stored in the capacitors;

c) the total energy delivered to the black box;

d) the percentage of the initial energy stored that is delivered to the black box; and

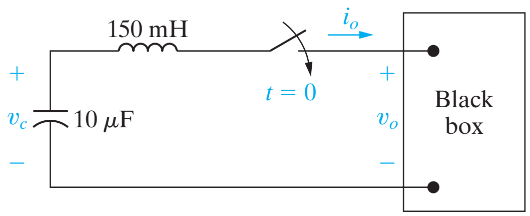
e) the time, in milliseconds, it takes to deliver 7.5 mJ to the black box.

6.34 At $t = 0$, a series-connected capacitor and inductor are placed across the terminals of a black box, as shown in Fig. P6.34. For $t > 0$, it is known that

$$i_o = 200e^{-800t} - 40e^{-200t}\,\text{mA}.$$

If $v_c(0) = 5$ V find $_o$ for $t \geq 0$.

Figure P6.34

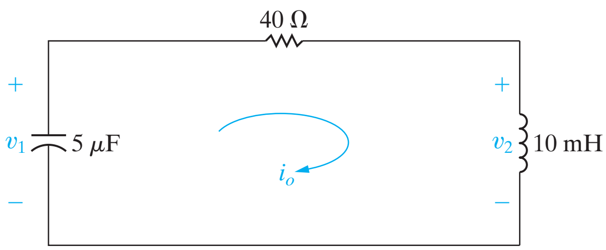


6.35 The current in the circuit in Fig. P6.35 is known to be

$$i_o = 5e^{-2000t}(2\cos 4000t + \sin 4000t)\text{ A}$$

for $t \geq 0^+$. Find $v_1(0^+)$ and $v_2(0^+)$.

Figure P6.35



Section 6.4

6.36 a) Show that the differential equations derived in (a) of Example 6.8 can be rearranged as follows:

$$4\frac{di_1}{dt} + 25i - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt};$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}.$$

b) Show that the solutions for i_1, and i_2 given in (b) of Example 6.8 satisfy the differential equations given in part (a) of this problem.

6.37 Let v_o represent the voltage across the 16 H inductor in the circuit in Fig. 6.29. Assume v_o is positive at the dot. As in Example 6.8, $i_g = 16 - 16e^{-5t}$ A.

a) Can you find v_o without having to differentiate the expressions for the currents? Explain.

b) Derive the expression for v_o.

c) Check your answer in (b) using the appropriate current derivatives and inductances.

6.38 Let v_g represent the voltage across the current source in the circuit in Fig. 6.29. The reference for v_g is positive at the upper terminal of the current source.

a) Find v_g as a function of time when $i_g = 16 - 16e^{-5t}$ A.

b) What is the initial value of v_g?

c) Find the expression for the power developed by the current source.

d) How much power is the current source developing when t is infinite?

e) Calculate the power dissipated in each resistor when t is infinite.

6.39 There is no energy stored in the circuit in Fig. P6.39 at the time the switch is opened.

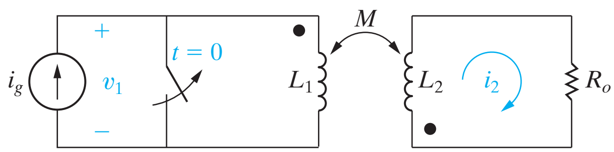
a) Derive the differential equation that governs the behavior of i_2 if $L_1 = 5$ H, $L_2 = 0.2$ H, $M = 0.5$ H, and $R_o = 10\ \Omega$.

b) Show that when $i_g = e^{-10t} - 10$ A, $t \geq 0$, the differential equation derived in (a) is satisfied when $i_2 = 625e^{-10t} - 250e^{-50t}$ mA, $t \geq 0$.

c) Find the expression for the voltage v_1 across the current source.

d) What is the initial value of v_1? Does this make sense in terms of known circuit behavior?

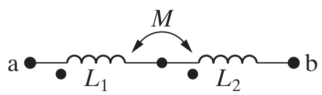
Figure P6.39



6.40 a) Show that the two coupled coils in Fig. P6.40 can be replaced by a single coil having an inductance of $L_{ab} = L_1 + L_2 + 2M$. (*Hint:* Express v_{ab} as a function of i_{ab}.)

b) Show that if the connections to the terminals of the coil labeled L_2 are reversed, $L_{ab} = L_1 + L_2 - 2M$.

Figure P6.40



6.41 a) Show that the two magnetically coupled coils in Fig. P6.41 can be replaced by a single coil having an inductance of

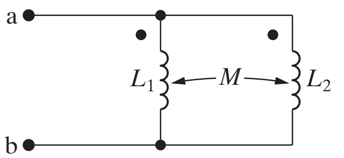
$$L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}.$$

(*Hint:* Let i_1 and i_2 be clockwise mesh currents in the left and right "windows" of Fig. P6.41, respectively. Sum the voltages around the two meshes. In mesh 1 let v_{ab} be the unspecified applied voltage. Solve for di_1/dt as a function of v_{ab}.)

b) Show that if the magnetic polarity of coil 2 is reversed, then

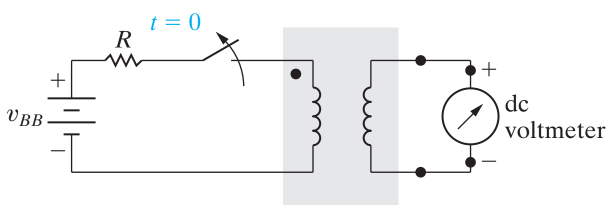
$$L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}.$$

Figure P6.41



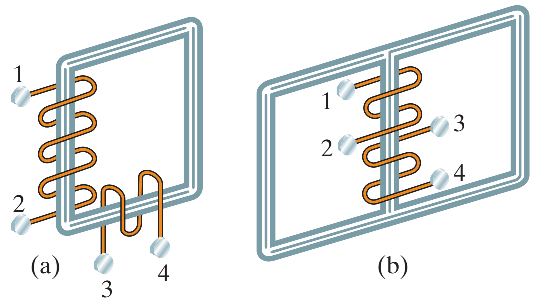
6.42 The polarity markings on two coils are to be determined experimentally. The experimental setup is shown in Fig. P6.42. Assume that the terminal connected to the positive terminal of the battery has been given a polarity mark as shown. When the switch is *opened*, the dc voltmeter kicks downscale. Where should the polarity mark be placed on the coil connected to the voltmeter?

Figure P6.42



6.43 The physical construction of two pairs of magnetically coupled coils is shown in Fig. P6.43. Assume that the magnetic flux is confined to the core material in each structure. Show two possible locations for the dot markings on each pair of coils.

Figure P6.43



Section 6.5

6.44 a) Starting with Eq. 6.26, show that the coefficient of coupling can also be expressed as

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}.$$

b) On the basis of the fractions ϕ_{21}/ϕ_1 and ϕ_{12}/ϕ_2, explain why k is less than 1.0.

6.45 Two magnetically coupled coils have self-inductances of 60 mH and 9.6 mH, respectively. The mutual inductance between the coils is 22.8 mH.

a) What is the coefficient of coupling?

b) For these two coils, what is the largest value that M can have?

c) Assume that the physical structure of these coupled coils is such that $\mathcal{P}_1 = \mathcal{P}_2$. What is the turns ratio N_1/N_2 if N_1 is the number of turns on the 60 mH coil?

6.46 Two magnetically coupled coils are wound on a nonmagnetic core. The self-inductance of coil 1 is 288 mH, the mutual inductance is 90 mH, the coefficient of coupling is 0.75, and the physical structure of the coils is such that $\mathcal{P}_{11} = \mathcal{P}_{22}$.

a) Find L_2 and the turns ratio N_1/N_2.

b) If $N_1 = 1200$, what is the value of $\mathcal{P}_1$ and $\mathcal{P}_2$?

6.47 The self-inductances of the coils in Fig. 6.34 are $L_1 = 18$ mH and $L_2 = 32$ mH. If the coefficient of coupling is 0.85, calculate the energy stored in the system in millijoules when (a) $i_1 = 6$ A, $i_2 = 9$ A; (b) $i_1 = -6$ A, $i_2 = -9$ A; (c) $i_1 = -6$ A, $i_2 = 9$ A; and (d) $i_1 = 6$ A, $i_2 = -9$A.

6.48 The coefficient of coupling in Problem 6.47 is increased to 1.0.

a) If i_1 equals 6 A, what value of i_2 results in zero stored energy?

b) Is there any physically realizable value of i_2 that can make the stored energy negative?

6.49 The self-inductances of two magnetically coupled coils are 72 mH and 40.5 mH, respectively. The 72 mH coil has 250 turns, and the coefficient of coupling between the coils is 2/3. The coupling medium is nonmagnetic. When coil 1 is excited with coil 2 open, the flux linking only coil 1 is 0.2 as large as the flux linking coil 2.

a) How many turns does coil 2 have?

b) What is the value of $\mathcal{P}_2$ in nanowebers per ampere?

c) What is the value of $\mathcal{P}_{11}$ in nanowebers per ampere?

d) What is the ratio (ϕ_{22}/ϕ_{12})?

6.50 The self-inductances of two magnetically coupled coils are $L_1 = 180\,\mu H$ and $L_2 = 500\,\mu H$. The coupling medium is nonmagnetic. If coil 1 has 300 turns and coil 2 has 500 turns, find $\mathcal{P}_{11}$ and $\mathcal{P}_{21}$ (in nanowebers per ampere) when the coefficient of coupling is 0.6.

Sections 6.1–6.5

6.51 PRACTICAL PERSPECTIVE Suppose a capacitive touch screen that uses the mutual-capacitance design, as shown in Fig. 6.37, is touched at the point *x, y*. Determine the mutual capacitance at that point, C'_{mxy}, in terms of the mutual capacitance at the point without a touch, C_{mxy}, and the capacitance introduced by the touch, C_t.

6.52 PRACTICAL PERSPECTIVE a) Assume the parasitic capacitance in the self-capacitance design, $C_p = 30$ pF, and the capacitance introduced by a touch is 15 pF (see Fig. 6.36[b]). What is the capacitance at the touch point with respect to ground for the *x*-grid and *y*-grid electrodes closest to the touch point?

b) Assume the mutual capacitance in the mutual-capacitance design, $C_{mxy} = 30$ pF, and the capacitance introduced by a touch is 15 pF (see Fig. 6.37[b]). What is the mutual capacitance between the *x*- and *y*-grid electrodes closest to the touch point?

c) Compare your results in parts (a) and (b)—does touching the screen increase or decrease the capacitance in these two different capacitive touch screen designs?

6.53 PRACTICAL PERSPECTIVE a) As shown in the Practical Perspective, the self-capacitance design does not permit a true multi-touch screen—if the screen is touched at two difference points, a total of four touch points are identified, the two actual touch points and two ghost points. If a self-capacitance touch screen is touched at the *x, y* coordinates (2.1, 4.3) and (3.2, 2.5), what are the four touch locations that will be identified? (Assume the touch coordinates are measured in inches from the upper left corner of the screen.)

b) A self-capacitance touch screen can still function as a multi-touch screen for several common gestures. For example, suppose at time t_1 the two touch points are those identified in part (a), and at time t_2 four touch points associated with the *x, y* coordinates (1.8, 4.9) and (3.9, 1.8) are identified. Comparing the four points at t_1 with the four points at t_2, software can recognize a pinch gesture—should the screen be zoomed in or zoomed out?

c) Repeat part (b), assuming that at time t_2 four touch points associated with the *x, y* coordinates (2.8, 3.9) and (3.0, 2.8) are identified.

CHAPTER

7

Response of First-Order *RL* and *RC* Circuits

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Be able to determine the natural response of both *RL* and *RC* circuits.
2. Be able to determine the step response of both *RL* and *RC* circuits.
3. Know how to analyze circuits with sequential switching.
4. Be able to analyze op amp circuits containing resistors and a single capacitor.

In this chapter, we focus on circuits that consist only of sources, resistors, and either (but not both) inductors or capacitors. We call these circuits ***RL*** (resistor-inductor) and ***RC*** (resistor-capacitor) **circuits**. In Chapter 6, we saw that inductors and capacitors can store energy. We analyze *RL* and *RC* circuits to determine the currents and voltages that arise when energy is either released or acquired by an inductor or capacitor in response to an abrupt change in a dc voltage or current source.

We divide our analysis of *RL* and *RC* circuits into three phases.

- First Phase: Find the currents and voltages that arise when stored energy in an inductor or capacitor is suddenly released to a resistive network. This happens when the inductor or capacitor is abruptly disconnected from its dc source. Thus, we can reduce the circuit to one of the two equivalent forms shown in Fig. 7.1 on page 250. These currents and voltages characterize the **natural response** of the circuit because the nature of the circuit itself, not external sources of excitation, determines its behavior.
- Second Phase: Find the currents and voltages that arise when energy is being acquired by an inductor or capacitor when a dc voltage or current source is suddenly applied. This response is called the **step response**.
- Third Phase: Develop a general method for finding the response of *RL* and *RC* circuits to any abrupt change in a dc voltage or current source. A general method exists because the process for finding both the natural and step responses is the same.

Figure 7.2 (page 250) shows the four general configurations of *RL* and *RC* circuits. Note that when there are no independent sources in the circuit, the Thévenin voltage or Norton current is zero, and the circuit reduces to one of those shown in Fig. 7.1; that is, we have a natural-response problem.

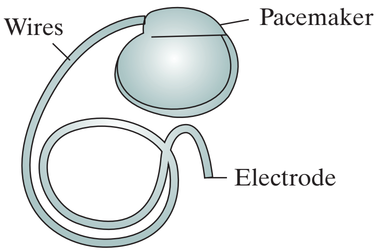
RL and *RC* circuits are also known as **first-order circuits** because their voltages and currents are described by first-order differential equations. No matter how complex a circuit may appear, if it can be reduced to a Thévenin or Norton equivalent connected to the

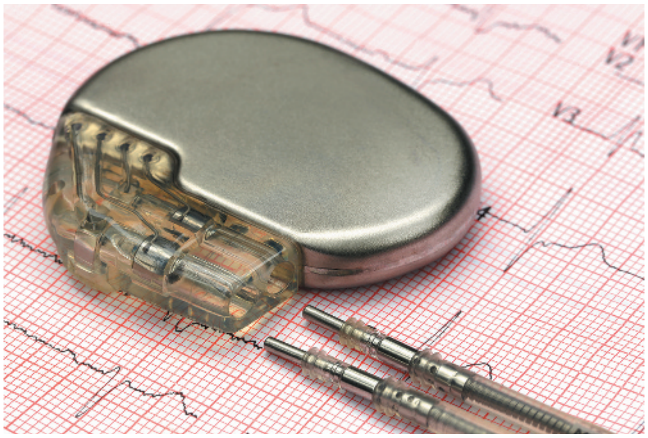
Practical Perspective

Artificial Pacemaker

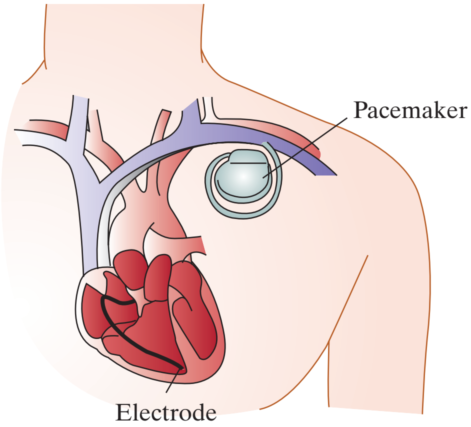
The muscle that makes up the heart contracts due to rhythmical electrical impulses. Pacemaker cells control the impulse frequency. In adults, the pacemaker cells establish a resting heart rate of about 72 beats per minutes. Sometimes, however, damaged pacemaker cells produce a very low resting heart rate (a condition known as bradycardia) or a very high resting heart rate (a condition known as tachycardia). When either happens, a normal heart rhythm can be restored by implanting an artificial pacemaker that mimics the pacemaker cells by delivering electrical impulses to the heart. Examples of internal and external artificial pacemakers are shown in the figures below.

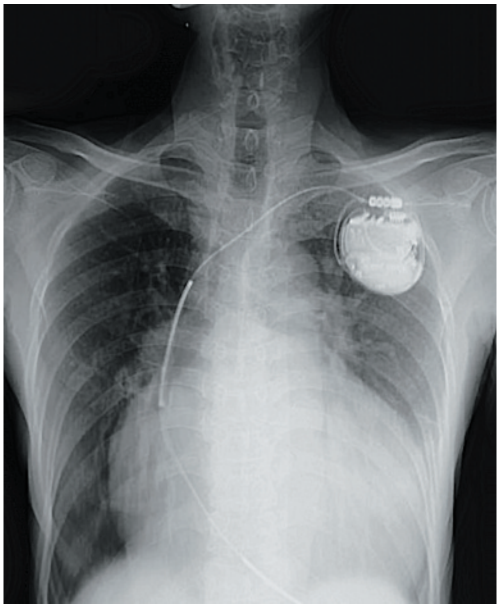
Artificial pacemakers are very small and lightweight. They have a programmable microprocessor that adjusts the heart rate based on several parameters, an efficient battery with a life of up to 15 years, and a circuit that generates the pulse. The simplest circuit consists of a resistor and a capacitor. After we introduce and study the *RC* circuit, we will look at an *RC* circuit design for an artificial pacemaker.





Swapan Photography/Shutterstock





Tewan Banditrukkanka/Shutterstock