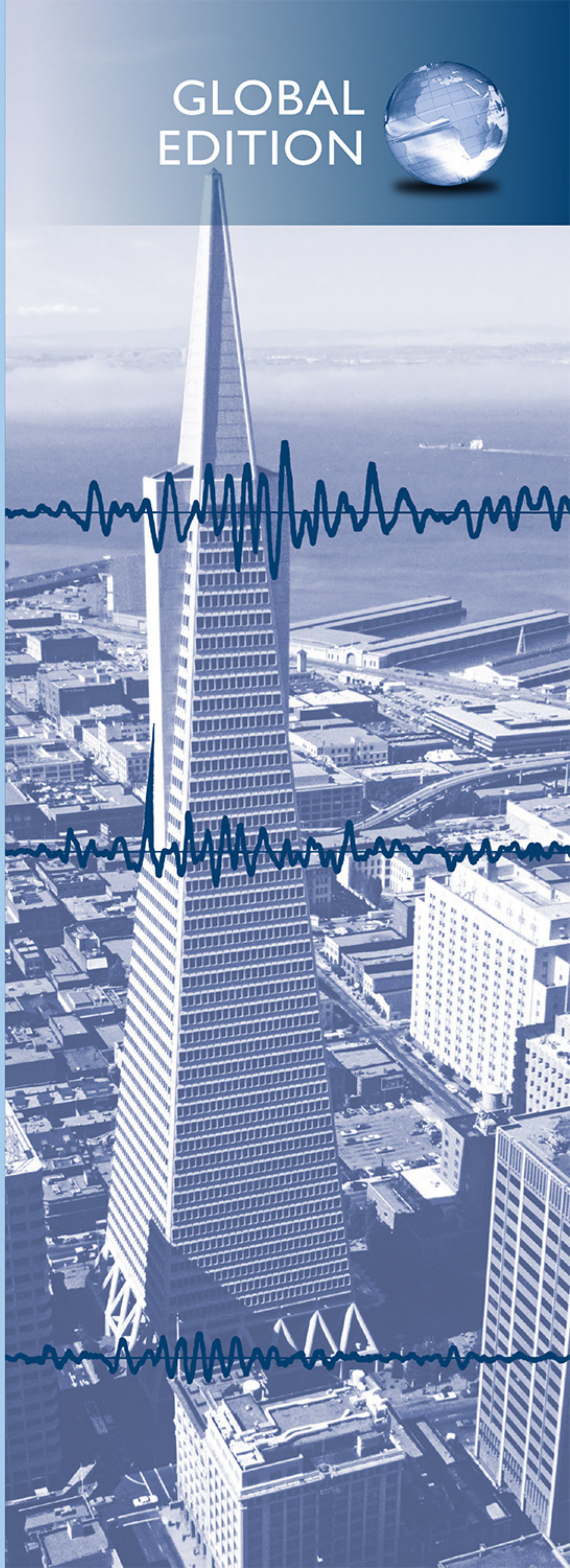
GLOBAL EDITION

Dynamics of Structures

Theory and Applications to Earthquake Engineering

Fifth Edition in SI Units

Anil K. Chopra



DYNAMICS OF STRUCTURES

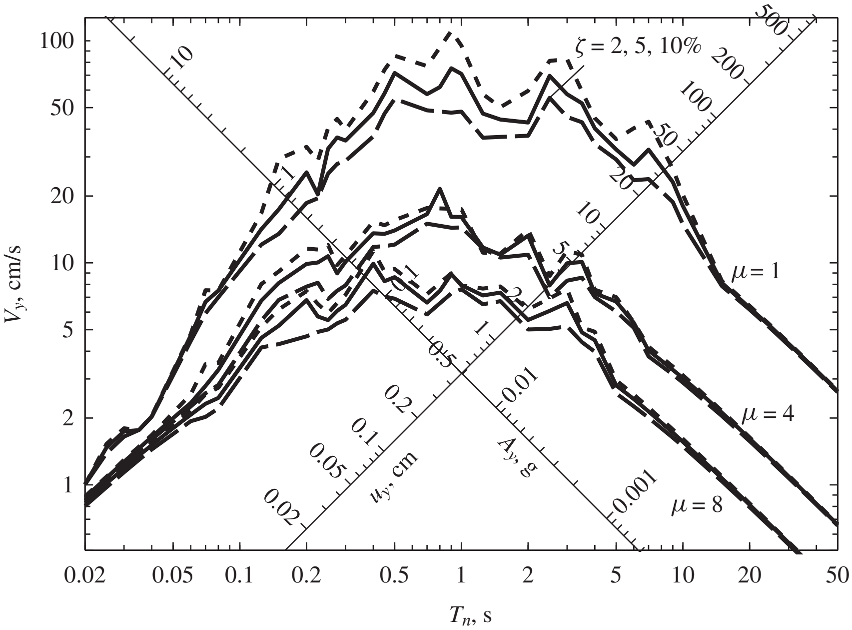


Figure 7.8.1 Response spectra for elastoplastic systems and El Centro ground motion; $\zeta = 2$, 5, and 10% and $\mu = 1$, 4, and 8.

and hence the peak value of the lateral force for which the system should be designed. The relative effectiveness of yielding and damping is quite different, however, in the various spectral regions:

1. Damping has negligible influence on the response of systems with $T_n > T_f$ in the displacement-sensitive region of the spectrum, whereas for such systems the effects of yielding on the design force are very important, but on the peak deformation u_m they are negligible (Fig. 7.4.4).

2. Damping has negligible influence in the response of systems with $T_n < T_a$ in the acceleration-sensitive region of the spectrum, whereas for such systems the effects of yielding on the peak deformation and ductility demand are very important (Figs. 7.4.4 and 7.4.5), but on the design force they are small. In the limit as T_n tends to zero, the pseudo-acceleration A or A_y will approach the peak ground acceleration, implying that this response parameter is unaffected by damping or yielding.

3. Damping is most effective in reducing the response of systems with T_n in the velocity-sensitive region of the spectrum, where yielding is even more effective.

Thus, in general, the effects of yielding cannot be considered in terms of a fixed amount of equivalent viscous damping. If this were possible, the peak response of inelastic systems could be determined directly from the response spectrum for linearly elastic systems, which would have been convenient.

The effectiveness of damping in reducing the response is smaller for inelastic systems and decreases as inelastic deformations increase (Fig. 7.8.1). For example, averaged over the velocity-sensitive spectral region, the percentage reduction in response resulting from increasing the damping ratio from 2% to 10% for systems with $\mu = 4$ is about one-half of the reduction for linearly elastic systems. Thus the added viscoelastic dampers mentioned in Section 6.8 may be less beneficial in reducing the response of inelastic systems compared to elastic systems.

7.9 DISSIPATED ENERGY

The input energy imparted to an inelastic system by an earthquake is dissipated by both viscous damping and yielding. These energy quantities are defined and discussed in this section. The various energy terms can be defined by integrating the equation of motion of an inelastic system, Eq. (7.3.1), as follows:

$$\int_0^u m\ddot{u}(t)\,du + \int_0^u c\dot{u}(t)\,du + \int_0^u f_S(u)\,du = -\int_0^u m\ddot{u}_g(t)\,du \tag{7.9.1}$$

The right side of this equation is the energy input to the structure since the earthquake excitation began:

$$E_I(t) = -\int_0^u m\ddot{u}_g(t)\,du \tag{7.9.2}$$

This is clear by noting that as the structure moves through an increment of displacement du, the energy supplied to the structure by the effective force $p_{\text{eff}}(t) = -m\ddot{u}_g(t)$ is

$$dE_I = -m\ddot{u}_g(t)\,du$$

The first term on the left side of Eq. (7.9.1) is the kinetic energy of the mass associated with its motion relative to the ground:

$$E_K(t) = \int_0^u m\ddot{u}(t)\,du = \int_0^{\dot{u}} m\dot{u}(t)\,d\dot{u} = \frac{m\dot{u}^2}{2} \tag{7.9.3}$$

The second term on the left side of Eq. (7.9.1) is the energy dissipated by viscous damping, defined earlier in Section 3.8:

$$E_D(t) = \int_0^u f_D(t)\,du = \int_0^u c\dot{u}(t)\,du \tag{7.9.4}$$

The third term on the left side of Eq. (7.9.1) is the sum of the energy dissipated by yielding and the recoverable strain energy of the system:

$$E_S(t) = \frac{[f_S(t)]^2}{2k} \tag{7.9.5}$$

where k is the initial stiffness of the inelastic system. Thus the energy dissipated by yielding is

$$E_Y(t) = \int_0^u f_S(u)\,du - E_S(t) \tag{7.9.6}$$

Based on these energy quantities, Eq. (7.9.1) is a statement of energy balance for the system:

$$E_I(t) = E_K(t) + E_D(t) + E_S(t) + E_Y(t) \tag{7.9.7}$$

Concurrent with the earthquake response analysis of a system these energy quantities can be computed conveniently by rewriting the integrals with respect to time. Thus

$$\begin{aligned} E_D(t) &= \int_0^t c[\dot{u}(t)]^2\,dt \\ E_Y(t) &= \left[\int_0^t \dot{u} f_S(u)\,dt\right] - E_S(t) \end{aligned} \tag{7.9.8}$$

The kinetic energy E_K and strain energy E_S at any time t can be computed conveniently from Eqs. (7.9.3) and (7.9.5), respectively.

The foregoing energy analysis is for a structure whose mass is acted upon by a force $p_{\text{eff}}(t) = -m\ddot{u}_g(t)$, not for a structure whose base is excited by acceleration $\ddot{u}_g(t)$. Therefore, the input energy term in Eq. (7.9.1) represents the energy supplied by $p_{\text{eff}}(t)$, not by $\ddot{u}_g(t)$, and the kinetic energy term in Eq. (7.9.1) represents the energy of motion relative to the base rather than that due to the total motion. As it is the relative displacement and velocity that cause forces in a structure, an energy equation expressed in terms of the relative motion is more meaningful than one expressed in terms of absolute velocity and displacement. Furthermore, the energy dissipated in viscous damping or yielding depends only on the relative motion.

Shown in Fig. 7.9.1 is the variation of these energy quantities with time for two SDF systems subjected to the El Centro ground motion. The results presented are for a linearly elastic system with natural period $T_n = 0.5$ sec and damping ratio $\zeta = 0.05$, and for an elastoplastic system with the same properties in the elastic range and normalized strength $\bar{f}_y = 0.25$. Recall that the deformation response of these two systems was presented in Fig. 7.4.3.

The results of Fig. 7.9.1 show that eventually the structure dissipates by viscous damping and yielding all the energy supplied to it. This is indicated by the fact that the kinetic energy and recoverable strain energy diminish near the end of the ground shaking. Viscous damping dissipates less energy from the inelastic system, implying smaller velocities relative to the elastic system. Figure 7.9.1 also indicates that the energy input to a linear system and to an inelastic system, both with the same T_n and ζ, is not the same. Furthermore, the input energy varies with T_n for both systems.

The yielding energy shown in Fig. 7.9.1b indicates a demand imposed on the structure. If this much energy can be dissipated through yielding of the structure, it needs to be designed only for $\bar{f}_y = 0.25$ (i.e., one-fourth the force developed in the corresponding linear system). The

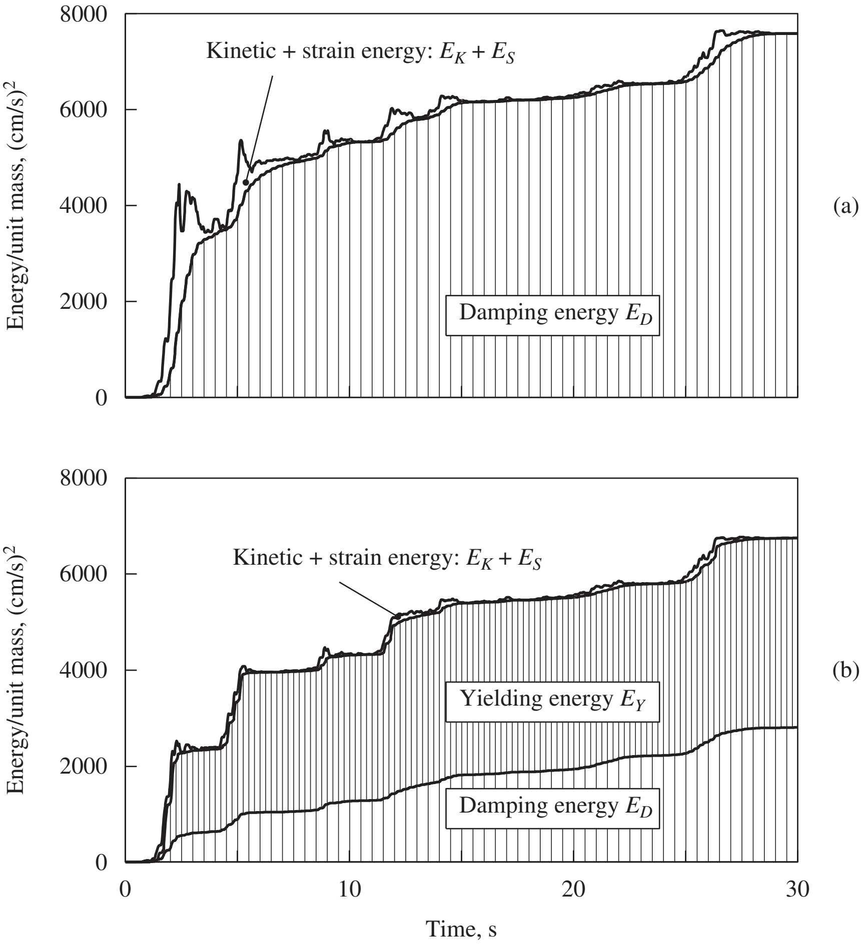


Figure 7.9.1 Time variation of energy dissipated by viscous damping and yielding, and of kinetic plus strain energy; (a) linear system, $T_n = 0.5$ sec, $\zeta = 5\%$; (b) elastoplastic system, $T_n = 0.5$ sec, $\zeta = 5\%, \overline{f}_y = 0.25$.

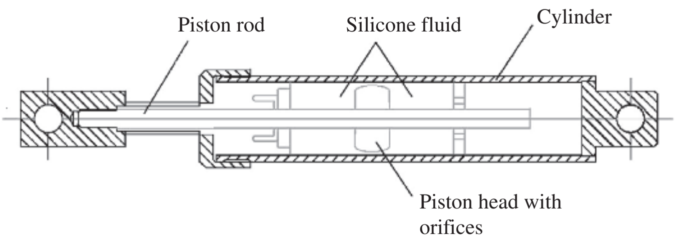
repeated yielding that dissipates energy causes damage to the structure, however, and leaves it in a permanently deformed condition at the end of the earthquake.

7.10 SUPPLEMENTAL ENERGY DISSIPATION DEVICES

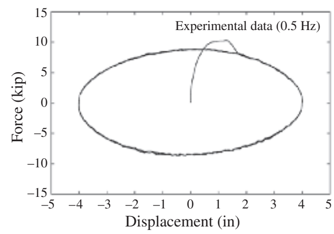
If part of this energy could be dissipated through supplemental devices that can easily be replaced, as necessary, after an earthquake, the structural damage could be reduced. Such devices may be cost-effective in the design of new structures and for seismic protection of existing structures. Available devices can be classified into three main categories: fluid viscous and viscoelastic dampers, metallic yielding dampers, and friction dampers. Only one of the several devices available in each category is described briefly here.

7.10.1 Fluid Viscous and Viscoelastic Dampers

In the most commonly used viscous damper for seismic protection of structures, a viscous fluid, typically silicone-based fluid, is forced to flow through small orifices within a closed container (Fig. 7.10.1a). Energy is dissipated due to friction between the fluid and orifice walls. The damper force–velocity relation, which is a function of the rate of loading, may be linear or nonlinear. Figure 7.10.1b shows an experimentally determined force–displacement relation for a damper subjected to sinusoidal force. An elliptical loop indicates a linear force–velocity relation, as demonstrated analytically in Section 3.10. Fluid viscous dampers are installed within the skeleton of a building frame, typically in line with diagonal bracing (Fig. 7.10.1c) or between the towers (or piers) and the deck of a bridge.



(a)



(b)

(c)

Figure 7.10.1 (a) Fluid viscous damper: schematic drawing; (b) force–displacement relation; and (c) diagonal bracing with fluid viscous damper. [Credits: (a) and (b) Cameron Black; and (c) Taylor Devices, Inc.]

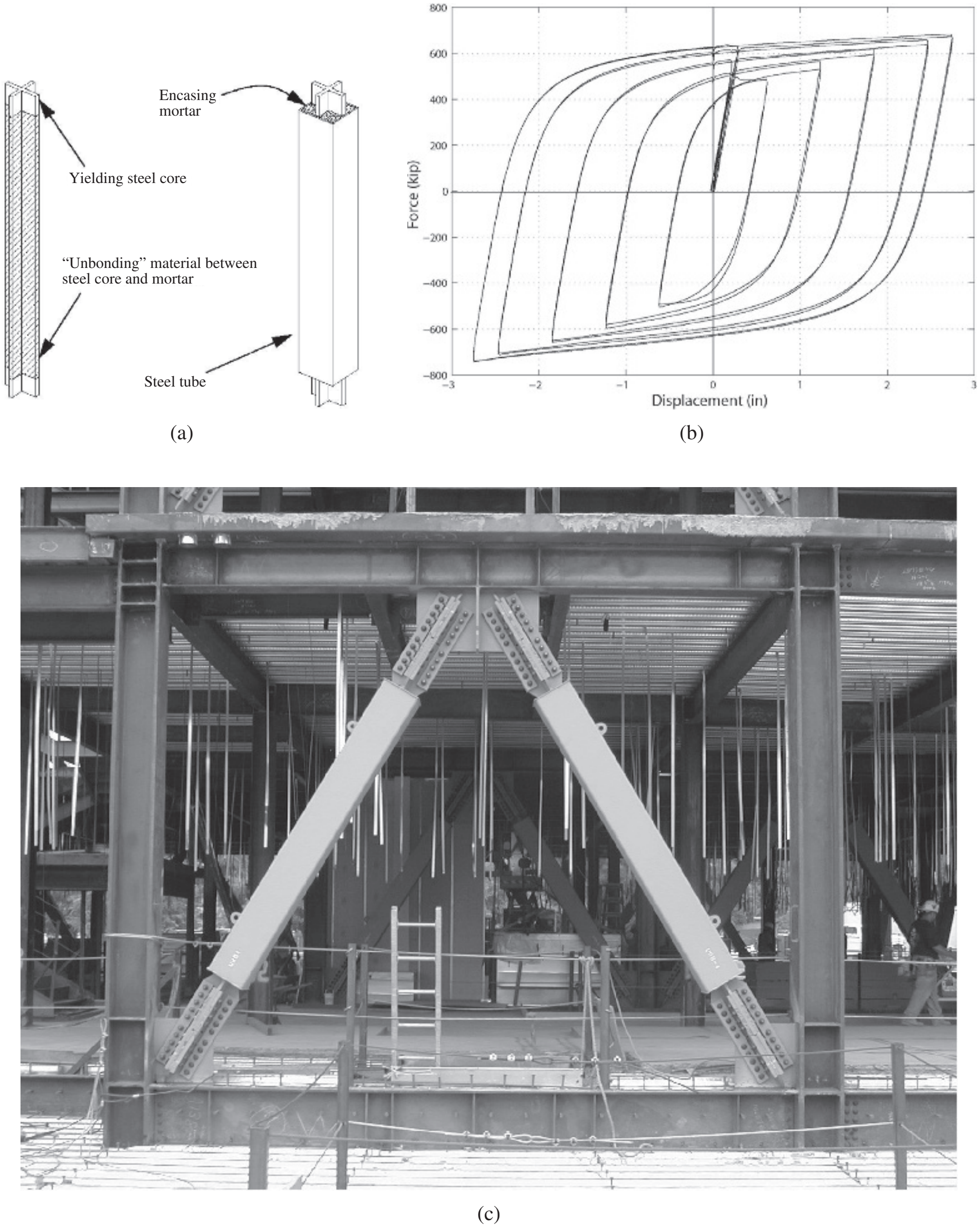


Figure 7.10.2 (a) Buckling restrained brace (BRB): schematic drawings; (b) force–displacement relation; and (c) diagonal bracing with BRB. [Credits: (a) Ian Aiken; (b) Cameron Black; and (c) Ian Aiken.]