GLOBAL EDITION

Calculus
& Its Applications

FOURTEENTH EDITION

Goldstein • Lay • Schneider • Asmar

Pearson

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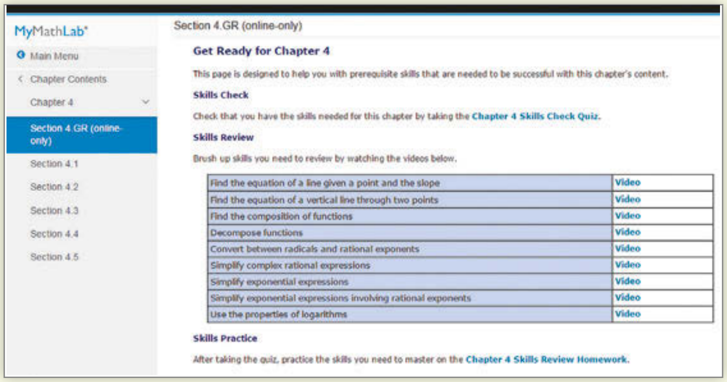

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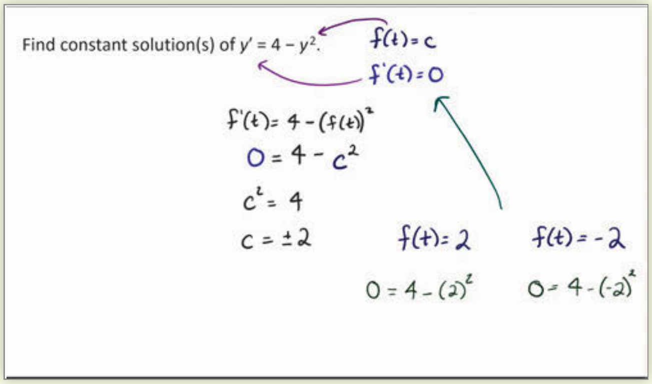

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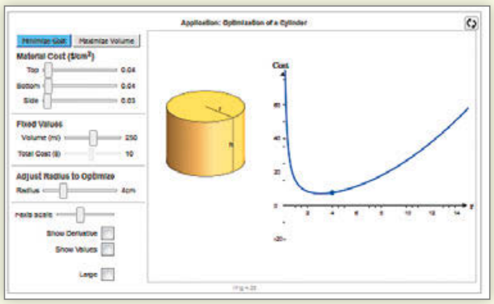

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The concept of a demand curve applies to an entire industry (with many producers) as well as to a single monopolistic firm. In this case, many producers offer the same product for sale. If x denotes the total output of the industry, $f(x)$ is the market price per unit of output and $x \cdot f(x)$ is the total revenue earned from the sale of the x units.

EXAMPLE 2

Maximizing Revenue The demand equation for a certain product is $p = 6 - \frac{1}{2}x$ thousand dollars. Find the level of production that results in maximum revenue.

SOLUTION In this case, the revenue function, $R(x)$, is

$$R(x) = x \cdot p = x\left(6 - \frac{1}{2}x\right) = 6x - \frac{1}{2}x^2$$

thousand dollars. The marginal revenue is given by

$$R'(x) = 6 - x.$$

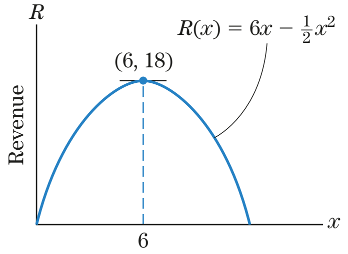

Figure 6 Maximizing revenue.

The graph of $R(x)$ is a parabola that opens downward. (See Fig. 6.) It has a horizontal tangent precisely at those x for which $R'(x) = 0$—that is, for those x at which marginal revenue is 0. The only such x is $x = 6$. The corresponding value of revenue is

$$R(6) = 6 \cdot 6 - \frac{1}{2}(6)^2 = \$18{,}000.$$

Thus, the rate of production resulting in maximum revenue is $x = 6$, which results in total revenue of \$18,000.

» Now Try Exercise 3

EXAMPLE 3

Setting Up a Demand Equation The WMA Bus Lines offers sightseeing tours of Washington, D.C. One tour, priced at \$7 per person, had an average demand of about 1000 customers per week. When the price was lowered to \$6, the weekly demand jumped to about 1200 customers. Assuming that the demand equation is linear, find the tour price that should be charged per person to maximize the total revenue each week.

SOLUTION First, we must find the demand equation. Let x be the number of customers per week and let p be the price of a tour ticket. Then $(x_1, p_1) = (1000, 7)$ and $(x_2, p_2) = (1200, 6)$ are on the demand curve. (See Fig. 7.) Using the point–slope formula for the line through these two points, we have

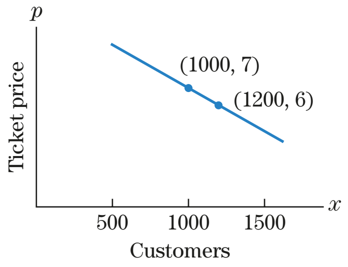

Figure 7 A demand curve.

$$\begin{aligned} p - 7 &= \frac{7-6}{1000-1200} \cdot (x - 1000) \\ &= -\frac{1}{200}(x - 1000) \\ &= -\frac{1}{200}x + 5, \end{aligned}$$

so

$$p = 12 - \frac{1}{200}x. \tag{2}$$

From equation (1), we obtain the revenue function:

$$R(x) = x \cdot p = x\left(12 - \frac{1}{200}x\right) = 12x - \frac{1}{200}x^2.$$

The marginal revenue function is

$$R'(x) = 12 - \frac{1}{100}x = -\frac{1}{100}(x - 1200).$$

Using $R(x)$ and $R'(x)$, we can sketch the graph of $R(x)$. (See Fig. 8.) The maximum revenue occurs when the marginal revenue is zero, that is, when $x = 1200$. The price corresponding to this number of customers is found from demand equation (2):

$$p = 12 - \frac{1}{200}(1200) = 6 \text{ dollars.}$$

Thus, the price of \$6 is most likely to bring the greatest revenue per week.

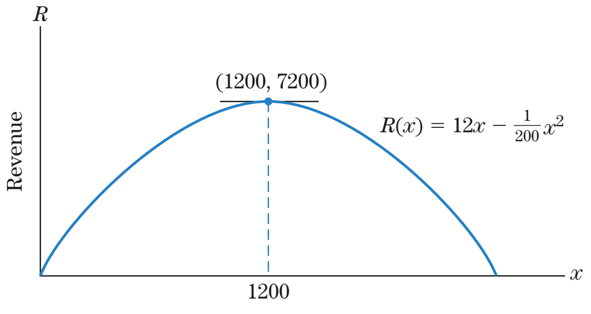

Figure 8 Maximizing revenue.

» *Now Try Exercise 11*

Profit Functions

Once we know the cost function $C(x)$ and the revenue function $R(x)$, we can compute the profit function $P(x)$ from

$$P(x) = R(x) - C(x).$$

EXAMPLE 4 **Maximizing Profits** Suppose that the demand equation for a monopolist is $p = 100 - .01x$ and the cost function is $C(x) = 50x + 10{,}000$. (See Fig. 9.) Find the value of x that maximizes the profit, and determine the corresponding price and total profit for this level of production.

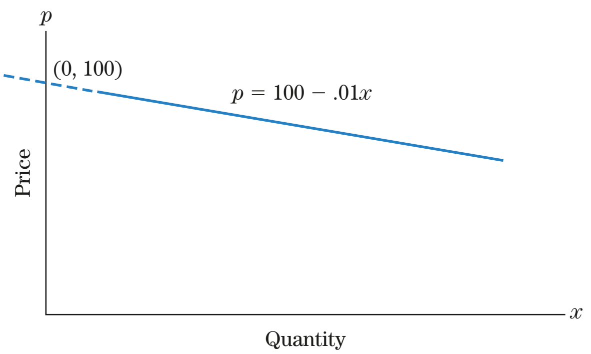

Figure 9 A demand curve.

SOLUTION The total revenue function is

$$R(x) = x \cdot p = x(100 - .01x) = 100x - .01x^2.$$

Hence, the profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 100x - .01x^2 - (50x + 10{,}000) \\ &= -.01x^2 + 50x - 10{,}000. \end{aligned}$$

The graph of this function is a parabola that opens downward. (See Fig. 10.) Its highest point will be where the curve has zero slope, that is, where the marginal profit $P'(x)$ is zero. Now,

$$P'(x) = -.02x + 50 = -.02(x - 2500).$$

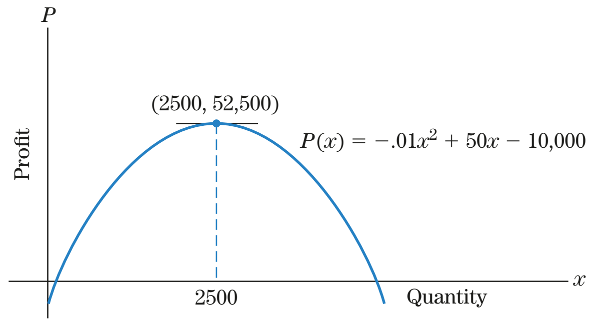

Figure 10 Maximizing profit.

So $P'(x) = 0$ when $x = 2500$. The profit for this level of production is

$$P(2500) = -.01(2500)^2 + 50(2500) - 10{,}000 = \$52{,}500.$$

Finally, we return to the demand equation to find the highest price that can be charged per unit to sell all 2500 units:

$$p = 100 - .01(2500) = 100 - 25 = \$75.$$

Thus, to maximize the profit, produce 2500 units and sell them at \$75 per unit. The profit will be \$52,500. **>> Now Try Exercise 17**

EXAMPLE 5 Rework Example 4 under the condition that the government has imposed an excise tax of \$10 per unit.

SOLUTION For each unit sold, the manufacturer will have to pay \$10 to the government. In other words, $10x$ dollars are added to the cost of producing and selling x units. The cost function is now

$$C(x) = (50x + 10{,}000) + 10x = 60x + 10{,}000.$$

The demand equation is unchanged by this tax, so the revenue is still

$$R(x) = 100x - .01x^2.$$

Proceeding as before, we have

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 100x - .01x^2 - (60x + 10{,}000) \\ &= -.01x^2 + 40x - 10{,}000. \\ P'(x) &= -.02x + 40 = -.02(x - 2000). \end{aligned}$$

The graph of $P(x)$ is still a parabola that opens downward, and the highest point is where $P'(x) = 0$, that is, where $x = 2000$. (See Fig. 11.) The corresponding profit is

$$P(2000) = -.01(2000)^2 + 40(2000) - 10{,}000 = \$30{,}000.$$

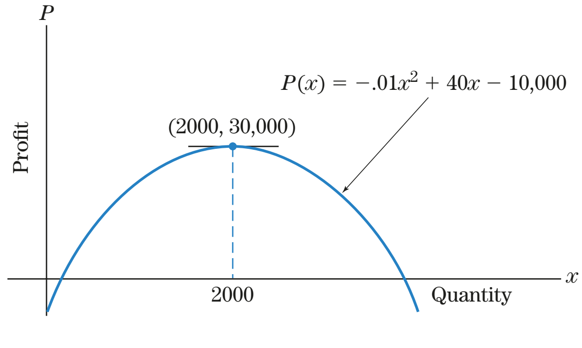

Figure 11 Profit after an excise tax.

From the demand equation, $p = 100 - .01x$, we find the price that corresponds to $x = 2000$:

$$p = 100 - .01(2000) = 80 \text{ dollars.}$$

To maximize profit, produce 2000 units and sell them at \$80 per unit. The profit will be \$30,000. «

Notice in Example 5 that the optimal price is raised from \$75 to \$80. If the monopolist wishes to maximize profits, he or she should pass only half the \$10 tax on to the customer. The monopolist cannot avoid the fact that profits will be substantially lowered by the imposition of the tax. This is one reason why industries lobby against taxation.

Setting Production Levels

Suppose that a firm has cost function $C(x)$ and revenue function $R(x)$. In a free-enterprise economy, the firm will set production x in such a way as to maximize the profit function

$$P(x) = R(x) - C(x).$$

We have seen that if $P(x)$ has a maximum at $x = a$, then $P'(a) = 0$. In other words, since $P'(x) = R'(x) - C'(x)$,

$$R'(a) - C'(a) = 0$$
$$R'(a) = C'(a).$$

Thus, profit is maximized at a production level for which marginal revenue equals marginal cost. (See Fig. 12.)

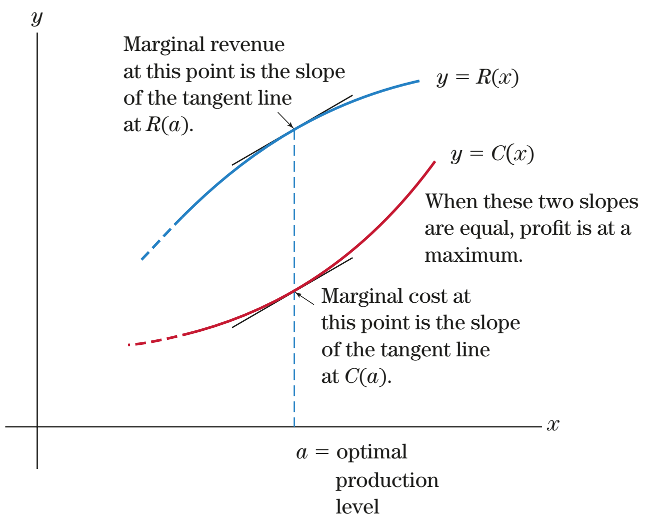

Figure 12

Check Your Understanding 2.7

Solutions can be found following the section exercises.

1. Rework Example 4 by finding the production level at which marginal revenue equals marginal cost.
2. Rework Example 4 under the condition that the fixed cost is increased from \$10,000 to \$15,000.
3. On a certain route, a regional airline carries 8000 passengers per month, each paying \$50. The airline wants to increase the fare. However, the market research department estimates that for each \$1 increase in fare, the airline will lose 100 passengers. Determine the price that maximizes the airline's revenue.

EXERCISES 2.7

1. **Minimizing Marginal Cost** Given the cost function $C(x) = x^3 - 6x^2 + 13x + 15$, find the minimum marginal cost.
2. **Minimizing Marginal Cost** If a total cost function is $C(x) = .0001x^3 - .06x^2 + 12x + 100$, is the marginal cost increasing, decreasing, or not changing at $x = 100$? Find the minimum marginal cost.

3. **Maximizing Revenue Cost** The revenue function for a one-product firm is
$$R(x) = 200 - \frac{1600}{x + 8} - x.$$
Find the value of x that results in maximum revenue.

4. **Maximizing Revenue** The revenue function for a particular product is $R(x) = x(4 - .0001x)$. Find the largest possible revenue.

5. **Cost and Profit** A one-product firm estimates that its daily total cost function (in suitable units) is $C(x) = x^3 - 6x^2 + 13x + 15$ and its total revenue function is $R(x) = 28x$. Find the value of x that maximizes the daily profit.

6. **Maximizing Profit** A small tie shop sells ties for \$3.50 each. The daily cost function is estimated to be $C(x)$ dollars, where x is the number of ties sold on a typical day and $C(x) = .0006x^3 - .03x^2 + 2x + 20$. Find the value of x that will maximize the store's daily profit.

7. **Demand and Revenue** The demand equation for a certain commodity is
$$p = \frac{1}{12}x^2 - 10x + 300,$$
$0 \le x \le 60$. Find the value of x and the corresponding price p that maximize the revenue.

8. **Maximizing Revenue** The demand equation for a product is $p = 2 - .001x$. Find the value of x and the corresponding price, p, that maximize the revenue.

9. **Profit** Some years ago, it was estimated that the demand for steel approximately satisfied the equation $p = 256 - 50x$, and the total cost of producing x units of steel was $C(x) = 182 + 56x$. (The quantity x was measured in millions of tons and the price and total cost were measured in millions of dollars.) Determine the level of production and the corresponding price that maximize the profits.

10. **Maximizing Area** Consider a rectangle in the xy-plane, with corners at $(0, 0)$, $(a, 0)$, $(0, b)$, and (a, b). If (a, b) lies on the graph of the equation $y = 30 - x$, find a and b such that the area of the rectangle is maximized. What economic interpretations can be given to your answer if the equation $y = 30 - x$ represents a demand curve and y is the price corresponding to the demand x?

11. **Demand, Revenue, and Profit** Until recently hamburgers at the city sports arena cost \$4 each. The food concessionaire sold an average of 10,000 hamburgers on a game night. When the price was raised to \$4.40, hamburger sales dropped off to an average of 8000 per night.
    (a) Assuming a linear demand curve, find the price of a hamburger that will maximize the nightly hamburger revenue.
    (b) If the concessionaire has fixed costs of \$1000 per night and the variable cost is \$.60 per hamburger, find the price of a hamburger that will maximize the nightly hamburger profit.

12. **Demand and Revenue** The average ticket price for a concert at the opera house was \$50. The average attendance was 4000. When the ticket price was raised to \$52, attendance declined to an average of 3800 persons per performance. What should the ticket price be to maximize the revenue for the opera house? (Assume a linear demand curve.)

13. **Demand and Revenue** An artist is planning to sell signed prints of her latest work. If 50 prints are offered for sale, she can charge \$400 each. However, if she makes more than 50 prints, she must lower the price of all the prints by \$5 for each print in excess of the 50. How many prints should the artist make to maximize her revenue?

14. **Demand and Revenue** A swimming club offers memberships at the rate of \$200, provided that a minimum of 100 people join. For each member in excess of 100, the membership fee will be reduced \$1 per person (for each member). At most, 160 memberships will be sold. How many memberships should the club try to sell to maximize its revenue?

15. **Profit** In the planning of a sidewalk café, it is estimated that for 12 tables, the daily profit will be \$10 per table. Because of overcrowding, for each additional table the profit per table (for every table in the café) will be reduced by \$.50. How many tables should be provided to maximize the profit from the café?

16. **Demand and Revenue** A certain toll road averages 36,000 cars per day when charging \$1 per car. A survey concludes that increasing the toll will result in 300 fewer cars for each cent of increase. What toll should be charged to maximize the revenue?

17. **Price Setting** The monthly demand equation for an electric utility company is estimated to be
$$p = 60 - (10^{-5})x,$$
where p is measured in dollars and x is measured in thousands of kilowatt-hours. The utility has fixed costs of 7 million dollars per month and variable costs of \$30 per 1000 kilowatt-hours of electricity generated, so the cost function is
$$C(x) = 7 \cdot 10^6 + 30x.$$
    (a) Find the value of x and the corresponding price for 1000 kilowatt-hours that maximize the utility's profit.
    (b) Suppose that rising fuel costs increase the utility's variable costs from \$30 to \$40, so its new cost function is
$$C_1(x) = 7 \cdot 10^6 + 40x.$$
    Should the utility pass all this increase of \$10 per thousand kilowatt-hours on to consumers? Explain your answer.

18. **Taxes, Profit, and Revenue** The demand equation for a company is $p = 200 - 3x$, and the cost function is
$$C(x) = 75 + 80x - x^2, \qquad 0 \le x \le 40.$$
    (a) Determine the value of x and the corresponding price that maximize the profit.
    (b) If the government imposes a tax on the company of \$4 per unit quantity produced, determine the new price that maximizes the profit.
    (c) The government imposes a tax of T dollars per unit quantity produced (where $0 \le T \le 120$), so the new cost function is
$$C(x) = 75 + (80 + T)x - x^2, \quad 0 \le x \le 40.$$
    Determine the new value of x that maximizes the company's profit as a function of T. Assuming that the company cuts back production to this level, express the tax revenues received by the government as a function of T. Finally, determine the value of T that will maximize the tax revenue received by the government.

19. **Interest Rate** A savings and loan association estimates that the amount of money on deposit will be 1 million times the percentage rate of interest. For instance, a 4% interest rate will generate \$4 million in deposits. If the savings and loan association can loan all the money it takes in at 10% interest, what interest rate on deposits generates the greatest profit?

20. **Analyzing Profit** Let $P(x)$ be the annual profit for a certain product, where x is the amount of money spent on advertising. (See Fig. 13.)
    (a) Interpret $P(0)$
    (b) Describe how the marginal profit changes as the amount of money spent on advertising increases.
    (c) Explain the economic significance of the inflection point.

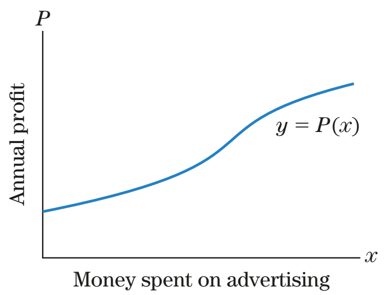

Figure 13 Profit as a function of advertising.

21. **Revenue** The revenue for a manufacturer is $R(x)$ thousand dollars, where x is the number of units of goods produced (and sold) and R and R' are the functions given in Figs. 14(a) and 14(b).

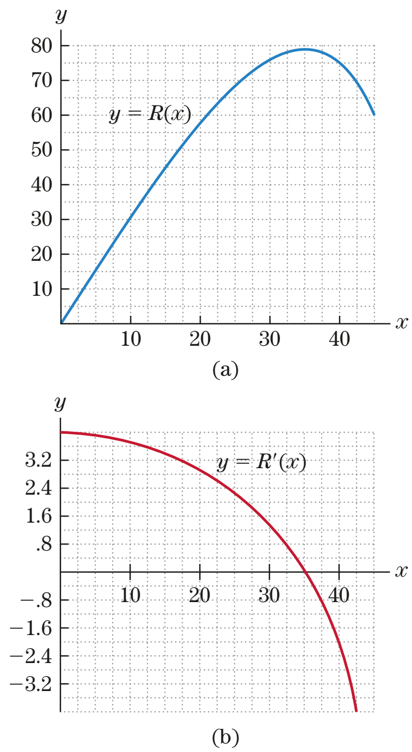

Figure 14 Revenue function and its first derivative.

(a) What is the revenue from producing 40 units of goods?
(b) What is the marginal revenue when 17.5 units of goods are produced?
(c) At what level of production is the revenue \$45,000?
(d) At what level(s) of production is the marginal revenue \$800?
(e) At what level of production is the revenue greatest?

22. **Cost and Marginal Cost** The cost function for a manufacturer is $C(x)$ dollars, where x is the number of units of goods produced and C, C', and C'' are the functions given in Fig. 15.

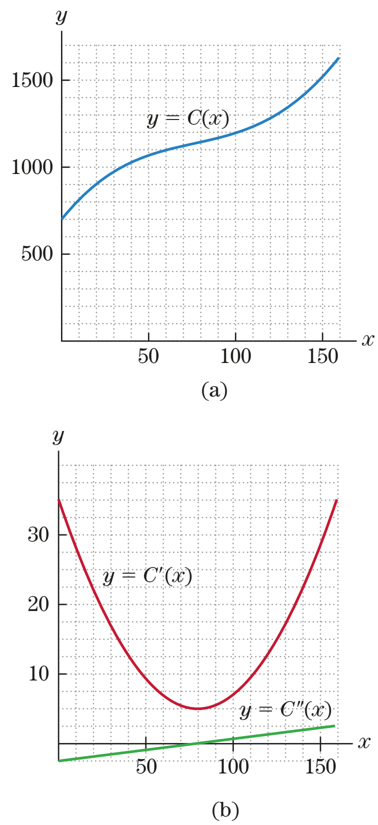

Figure 15 Cost function and its derivatives.

(a) What is the cost of manufacturing 60 units of goods?
(b) What is the marginal cost when 40 units of goods are manufactured?
(c) At what level of production is the cost \$1200?
(d) At what level(s) of production is the marginal cost \$22.50?
(e) At what level of production does the marginal cost have the least value? What is the marginal cost at this level of production?