GLOBAL EDITION

Electrical Engineering

Principles and Applications

SEVENTH EDITION

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where $V_{R\text{rms}}$ is the rms value of the voltage *across the resistance.* (Notice in Figure 5.22 that the source voltage does not appear across the resistance, because the reactance is in series with the resistance.)

Similarly, we have

In Equation 5.72, $V_{X\text{rms}}$ is the rms voltage across the reactance.

$$Q = \frac{V^2_{X\text{rms}}}{X} \tag{5.72}$$

where $V_{X\text{rms}}$ is the rms value of the voltage *across the reactance*. Here again, X is positive for an inductance and negative for a capacitance.

Complex Power

Consider the portion of a circuit shown in Figure 5.27. The **complex power**, denoted as **S**, delivered to this circuit is defined as one half the product of the phasor voltage **V** and the complex conjugate of the phasor current **I***.

$$\mathbf{S} = \frac{1}{2}\mathbf{VI}^* \tag{5.73}$$

The phasor voltage is $\mathbf{V} = V_m \angle \theta_v$ in which V_m is the peak value of the voltage and θ_v is the phase angle of the voltage. Furthermore, the phasor current is $\mathbf{I} = I_m \angle \theta_i$ where I_m is the peak value and θ_i is the phase angle of the current. Substituting into Equation 5.73, we have

$$\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = \frac{1}{2}(V_m \angle \theta_v) \times (I_m \angle -\theta_i) = \frac{V_m I_m}{2} \angle \theta_v - \theta_i = \frac{V_m I_m}{2} \angle \theta \tag{5.74}$$

where, as before, $\theta = \theta_v - \theta_i$ is the power angle. Expanding the right-hand term of Equation 5.74 into real and imaginary parts, we have

$$\mathbf{S} = \frac{V_m I_m}{2}\cos(\theta) + j\frac{V_m I_m}{2}\sin(\theta)$$

However, the first term on the right-hand side is the average power P delivered to the circuit and the second term is j times the reactive power. Thus, we can write:

$$\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = P + jQ \tag{5.75}$$

If we know the complex power **S**, then we can find the power, reactive power, and apparent power:

$$P = \text{Re}(\mathbf{S}) = \text{Re}\left(\frac{1}{2}\mathbf{VI}^*\right) \tag{5.76}$$

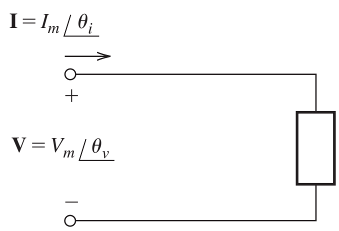

Figure 5.27 The complex power delivered to this circuit element is $\mathbf{S} = \frac{1}{2}\mathbf{VI}^*$.

$$Q = \mathrm{Im}(\mathbf{S}) = \mathrm{Im}\left(\frac{1}{2}\mathbf{V}\mathbf{I}^*\right) \tag{5.77}$$

$$\text{apparent power} = |\mathbf{S}| = \left|\frac{1}{2}\mathbf{V}\mathbf{I}^*\right| \tag{5.78}$$

where Re(**S**) denotes the real part of **S** and Im(**S**) denotes the imaginary part of **S**.

Example 5.9 AC Power Calculations

Compute the power and reactive power taken from the source for the circuit of Example 5.6. Also, compute the power and reactive power delivered to each element in the circuit. For convenience, the circuit and the currents that were computed in Example 5.6 are shown in Figure 5.28.

Solution To find the power and reactive power for the source, we must first find the power angle which is given by Equation 5.62:

$$\theta = \theta_v - \theta_i$$

The angle of the source voltage is $\theta_v = -90°$, and the angle of the current delivered by the source is $\theta_i = -135°$. Therefore, we have

$$\theta = -90° - (-135°) = 45°$$

The rms source voltage and current are

$$V_{s\text{rms}} = \frac{|\mathbf{V}_s|}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071 \text{ V}$$

$$I_{\text{rms}} = \frac{|\mathbf{I}|}{\sqrt{2}} = \frac{0.1414}{\sqrt{2}} = 0.1 \text{ A}$$

Now, we use Equations 5.60 and 5.63 to compute the power and reactive power delivered by the source:

$$P = V_{s\text{rms}} I_{\text{rms}} \cos(\theta)$$
$$= 7.071 \times 0.1 \cos(45°) = 0.5 \text{ W}$$
$$Q = V_{s\text{rms}} I_{\text{rms}} \sin(\theta)$$
$$= 7.071 \times 0.1 \sin(45°) = 0.5 \text{ VAR}$$

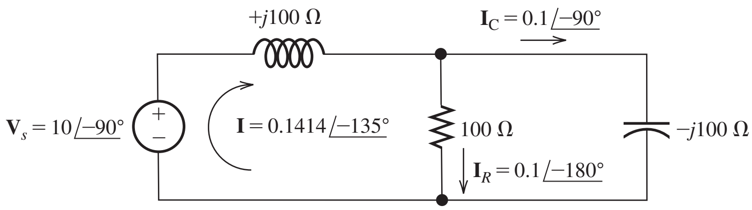

Figure 5.28 Circuit and currents for Example 5.9.

An alternative and more compact method for computing P and Q is to first find the complex power and then take the real and imaginary parts:

$$\mathbf{S} = \frac{1}{2}\mathbf{V}_s\mathbf{I}^* = \frac{1}{2}(10\,\underline{/-90^\circ})(0.1414\,\underline{/135^\circ}) = 0.707\,\underline{/45^\circ} = 0.5 + j0.5$$

$$P = \text{Re}(\mathbf{S}) = 0.5 \text{ W}$$

$$Q = \text{Im}(\mathbf{S}) = 0.5 \text{ VAR}$$

We can use Equation 5.70 to compute the reactive power delivered to the inductor, yielding

$$Q_L = I_{\text{rms}}^2 X_L = (0.1)^2(100) = 1.0 \text{ VAR}$$

For the capacitor, we have

$$Q_C = I_{C\text{rms}}^2 X_C = \left(\frac{0.1}{\sqrt{2}}\right)^2 (-100) = -0.5 \text{ VAR}$$

Notice that we have used the rms value of the current through the capacitor in this calculation. Furthermore, notice that the reactance X_C of the capacitance is negative. As expected, the reactive power is negative for a capacitance. The reactive power for the resistance is zero. As a check, we can verify that the reactive power delivered by the source is equal to the sum of the reactive powers absorbed by the inductance and capacitance. This is demonstrated by

$$Q = Q_L + Q_C$$

The power delivered to the resistance is

$$P_R = I_{R\,\text{rms}}^2 R = \left(\frac{|\mathbf{I}_R|}{\sqrt{2}}\right)^2 R = \left(\frac{0.1}{\sqrt{2}}\right)^2 100$$

$$= 0.5 \text{ W}$$

The power absorbed by the capacitance and inductance is given by

$$P_L = 0$$

$$P_C = 0$$

Thus, all of the power delivered by the source is absorbed by the resistance. ■

In power distribution systems, we typically encounter much larger values of power, reactive power, and apparent power than the small values of the preceding example. For example, a large power plant may generate 1000 MW. A 100-hp motor used in an industrial application absorbs approximately 85 kW of electrical power under full load.

A typical residence absorbs a *peak* power in the range of 10 to 40 kW. The *average* power for my home (which is of average size, has two residents, and does not use electrical heating) is approximately 600 W. It is interesting to keep your average power consumption and the power used by various appliances in mind because it gives you a clear picture of the economic and environmental impact of turning off lights, computers, and so on, that are not being used.

Example 5.10 Using Power Triangles

Consider the situation shown in Figure 5.29. Here, a voltage source delivers power to two loads connected in parallel. Find the power, reactive power, and power factor for the source. Also, find the phasor current **I**.

Solution By the units given in the figure, we see that load A has an *apparent power* of 10 kVA. On the other hand, the *power* for load B is specified as 5 kW.

Furthermore, load A has a power factor of 0.5 leading, which means that the current leads the voltage in load A. Another way to say this is that load A is capacitive. Similarly, load B has a power factor of 0.7 lagging (or inductive).

Our approach is to find the power and reactive power for each load. Then, we add these values to find the power and reactive power for the source. Finally, we compute the power factor for the source and then find the current.

Calculations for load A

Because load A has a leading (capacitive) power factor, we know that the reactive power Q_A and power angle θ_A are negative. The power triangle for load A is shown in Figure 5.30(a). The power factor is

$$\cos(\theta_A) = 0.5$$

The power is

$$P_A = V_{\text{rms}} I_{A\text{rms}} \cos(\theta_A) = 10^4(0.5) = 5\text{ kW}$$

Solving Equation 5.64 for reactive power, we have

$$\begin{aligned} Q_A &= \sqrt{(V_{\text{rms}} I_{A\text{rms}})^2 - P_A^2} \\ &= \sqrt{(10^4)^2 - (5000)^2} \\ &= -8.660\text{ kVAR} \end{aligned}$$

Notice that we have selected the negative value for Q_A, because we know that reactive power is negative for a capacitive (leading) load.

Calculations for load B

The power triangle for load B is shown in Figure 5.30(b). Since load B has a lagging (inductive) power factor, we know that the reactive power Q_B and power angle θ_B are positive. Thus,

$$\theta_B = \arccos(0.7) = 45.57°$$

Applying trigonometry, we can write

$$Q_B = P_B \tan(\theta_B) = 5000\tan(45.57°)$$

$$Q_B = 5.101\text{ kVAR}$$

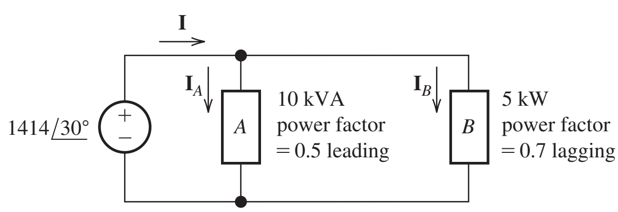

Figure 5.29 Circuit for Example 5.10.

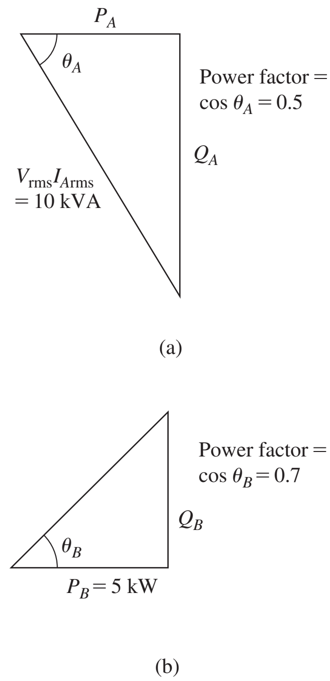

Figure 5.30 Power triangles for loads *A* and *B* of Example 5.10.

Total power is obtained by adding the powers for the various loads. Similarly, the reactive powers are added.

At this point, as shown here we can find the power and reactive power delivered by the source:

$$P = P_A + P_B = 5 + 5 = 10\text{ kW}$$

$$Q = Q_A + Q_B = -8.660 + 5.101 = -3.559\text{ kVAR}$$

Power calculations for the source.

Because Q is negative, we know that the power angle is negative. Thus, we have

$$\theta = \arctan\left(\frac{Q}{P}\right) = \arctan\left(\frac{-3.559}{10}\right) = -19.59°$$

The power factor is

$$\cos(\theta) = 0.9421$$

Power-system engineers frequently express power factors as percentages and would state this power factor as 94.21 percent leading.

The complex power delivered by the source is

$$\mathbf{S} = P + jQ = 10 - j3.559 = 10.61 \angle -19.59°\text{ kVA}$$

Thus, we have

$$\mathbf{S} = \frac{1}{2}\mathbf{V}_s\mathbf{I}^* = \frac{1}{2}(1414 \angle 30°)\mathbf{I}^* = 10.61 \times 10^3 \angle -19.59°\text{ kVA}$$

Solving for the phasor current, we obtain:

$$\mathbf{I} = 15.0 \angle 49.59°\text{ A}$$

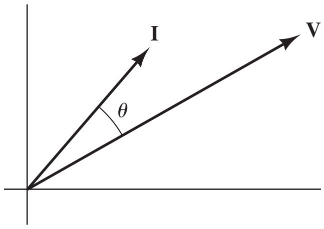

Figure 5.31 Phasor diagram for Example 5.10.

The phasor diagram for the current and voltage is shown in Figure 5.31. Notice that the current is leading the voltage. ■

Power-Factor Correction

We have seen that large currents can flow in energy-storage devices (inductance and capacitance) without average power being delivered. In heavy industry, many loads are partly inductive, and large amounts of reactive power flow. This reactive power causes higher currents in the power distribution system. Consequently, the lines and transformers must have higher ratings than would be necessary to deliver the same average power to a resistive (100 percent power factor) load.

Power-factor correction can provide a significant economic advantage for consumers of large amounts of electrical energy.

Energy rates charged to industry depend on the power factor, with higher charges for energy delivered at lower power factors. (Power factor is not taken into account for residential customers.) Therefore, it is advantageous to choose loads that operate at near unity power factor. A common approach is to place capacitors in parallel with an inductive load to increase the power factor.

Example 5.11 Power-Factor Correction

A 50-kW load operates from a 60-Hz 10-kV-rms line with a power factor of 60 percent lagging. Compute the capacitance that must be placed in parallel with the load to achieve a 90 percent lagging power factor.

Solution First, we find the load power angle:

$$\theta_L = \arccos(0.6) = 53.13°$$

Then, we use the power-triangle concept to find the reactive power of the load. Hence,

$$Q_L = P_L \tan(\theta_L) = 66.67 \text{ kVAR}$$

After adding the capacitor, the power will still be 50 kW and the power angle will become

$$\theta_{\text{new}} = \arccos(0.9) = 25.84°$$

The new value of the reactive power will be

$$Q_{\text{new}} = P_L \tan(\theta_{\text{new}}) = 24.22 \text{ kVAR}$$

Thus, the reactive power of the capacitance must be

$$Q_C = Q_{\text{new}} - Q_L = -42.45 \text{ kVAR}$$