GLOBAL EDITION

Business STATISTICS

A Decision-Making Approach

TENTH EDITION

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Pearson

BUSINESS Statistics

A Decision-Making Approach

a. the probability that a customer arrives within the next 2 minutes
b. the probability that the time between two arriving customers will be between 1 and 4 minutes
c. the probability that 5 or more minutes will pass between customer arrivals

6-58. Assume that the time required to receive a confirmation that an electronic transfer has occurred is uniformly distributed between 30 and 90 seconds.
a. What is the probability that a randomly selected transfer will take between 30 and 45 seconds?
b. What is the probability that a randomly selected transfer will take between 50 and 90 seconds?
c. What proportion of transfers will take between 40 and 75 seconds?

Business Applications

6-59. Drive-through services are very popular and convenient to consumers today. It provides a business that allows customers to purchase products without leaving their cars. McDonald's has customers arriving at the drive-through window at an average rate of three customers per minute and that their arrivals follow the Poisson model. Use the appropriate exponential distribution to find the probability that the next customer will arrive within 1.5 minutes.

6-60. The Four Brothers Lumber Company is considering buying a machine that planes lumber to the correct thickness. The machine is advertised to produce "6-inch lumber" having a thickness that is normally distributed, with a mean of 6 inches and a standard deviation of 0.1 inch.
a. If building standards in the industry require a 99% chance of a board being between 5.85 and 6.15 inches, should Four Brothers purchase this machine? Why or why not?
b. To what level would the company that manufactures the machine have to reduce the standard deviation for the machine to conform to industry standards?

6-61. Two automatic dispensing machines are being considered for use in a fast-food chain. The first dispenses an amount of liquid that has a normal distribution with a mean of 11.9 ounces and a standard deviation of 0.07 ounce. The second dispenses an amount of liquid that has a normal distribution with a mean of 12.0 ounces and a standard deviation of 0.05 ounce. Acceptable amounts of dispensed liquid are between 11.9 and 12.0 ounces. Calculate the relevant probabilities and determine which machine should be selected.

6-62. Domino's Pizza receives majority of its orders by telephone. The call center supervisor conducted a study on the duration an order had been taken. He found that the amount of time a customer was waiting to place an order was about 10 minutes. Assume that the distribution of time spent in taking orders is approximately exponentially distributed with a mean of 5 minutes. The supervisor is considering adding extra telephone order services to decrease the customers' waiting time. Help the supervisor determine the probability that a second telephone order is coming before the first order has ended. Justify whether the extra telephone order service is necessary.

6-63. Assume that after the first 12 hours, the average remaining useful life of a particular battery (before recharging is required) is 9 hours and that the remaining time is exponentially distributed. What is the probability that a randomly sampled battery of this type will last between 15 and 17 hours?

6-64. Suppose eye shadow and eyeliner have a shelf life of up to three years, and the shelf lives of these two products are exponentially distributed with an average of one year.
a. Calculate the probability that the shelf life of eye shadow will be longer than three years.
b. Determine the probability that at least one of these products will have a shelf life of more than three years.
c. Determine the probability that a purchased eyeliner that is useful after one year will be useful after three years.

6-65. The Shadow Mountain Golf Course is preparing for a major LPGA golf tournament. Since parking near the course is extremely limited (room for only 500 cars), the course officials have contracted with the local community to provide parking and a bus shuttle service. Sunday, the final day of the tournament, will have the largest crowd, and the officials estimate there will be between 8,000 and 12,000 cars needing parking spaces but think no value is more likely than another. The tournament committee is discussing how many parking spots to contract from the city. If they want to limit the chance of not having enough provided parking to 10%, how many spaces do they need from the city on Sunday?

6-66. A product made by a window manufacturer is constructed of two pieces of glass 0.125 inch thick, with a thin layer of vinyl sandwiched between them. The average thickness of the laminated safety glass is 0.25 inch. The thickness of the glass does not vary from the mean by more than 0.10 inch. Assume the thickness of the glass has a uniform distribution.
a. Provide the density for this uniform distribution.
b. If the glass has a thickness that is more than 0.05 inch below the mean, it must be discarded for safety considerations. Determine the probability that a randomly selected glass is discarded due to safety considerations.
c. If the glass thickness is more than 0.075 inch above the mean, it will create installation problems and must be discarded. Calculate the probability that a randomly selected glass will be rejected due to installation concerns.

6-67. A traffic control camera at a busy intersection records, on average, 5 traffic violations per hour. Assume that the

random variable number of recorded traffic violations follows a Poisson distribution.

a. What is the probability that the next recorded violation will occur within 5 minutes?
b. How likely is it that no traffic violations will be recorded within the next 7 minutes?

6-68. A manager of a plywood plant organized a study of the diameters of trees that are shipped to the mill. After collecting a large amount of data on diameters, he concluded that the diameter is approximately normally distributed with a mean of 14.25 inches and a standard deviation of 2.92 inches. Because of the way plywood is made, there is a certain amount of waste on each log because the peeling process leaves a core that is approximately 3 inches in diameter. For this reason, the manager feels that any log less than 10 inches in diameter is not profitable for making plywood. Based on the data collected, what is the probability that a log will be unprofitable?

6-69. The personnel manager for a large company is interested in the distribution of sick-leave hours for employees of her company. A recent study revealed the distribution to be approximately normal, with a mean of 58 hours per year and a standard deviation of 14 hours.

An office manager in one division has reason to believe that during the past year, two of his employees have taken excessive sick leave relative to everyone else. The first employee used 74 hours of sick leave, and the second used 90 hours. What would you conclude about the office manager's claim and why?

6-70. If the number of hours between servicing required for a particular snowmobile engine is exponentially distributed with an average of 118 hours, determine the probability that a randomly selected engine

a. will run at least 145 hours before servicing is needed
b. will run at most 161 hours before servicing is needed

6-71. Assume that the amount of time eighth-graders take to complete an assessment examination is 78 minutes with a standard deviation of 12 minutes.

a. What proportion of eighth-graders complete the assessment examination in 72 minutes or less?
b. What proportion of eighth-graders complete the assessment examination in 82 minutes or more?
c. For what number of minutes would 90% of all eighth-graders complete the assessment examination?

6-72. If the time required to assemble a product is normally distributed with a mean of 180 minutes and a standard deviation of 15 minutes, find

a. the probability that the product's assembly time would be less than 168 minutes
b. the probability that the product's assembly time would be greater than 190 minutes
c. the probability that the product's assembly time would be between 174 and 186 minutes
d. the number of minutes within which 95% of all such products would be assembled

6-73. If the average amount of time customers spent online at an Internet retailer's site is assumed to be normally distributed with mean of 36 minutes and a standard deviation of 6 minutes, determine

a. the probability a customer spends less than 30 minutes on the site
b. the probability a customer spends more than 44 minutes on the site
c. the probability a customer spends between 33 and 42 minutes on the site
d. the amount of time in minutes spent on the site by no more than 5% of all customers

6-74. A manufacturing process applies a coating to a product in order to protect it from scratches. If the thickness of the coating follows a uniform distribution between 15 and 25 microns, determine

a. the mean thickness of the protective coating
b. the standard deviation of the thickness of the protective coating
c. the probability that the protective coating is between 18 and 22 microns thick
d. the probability that the protective coating is less than 23 microns thick

Computer Software Exercises

6-75. The Cozine Corporation runs the landfill operation outside Little Rock, Arkansas. Each day, each of the company's trucks makes several trips from the city to the landfill. On each entry, the truck is weighed. The data file **Cozine** contains a sample of 200 truck weights. Determine the mean and standard deviation for the garbage truck weights. Assume that these sample values are representative of the population of all Cozine garbage trucks, and that the weights are normally distributed.

a. Determine the probability that a truck will arrive at the landfill weighing in excess of 46,000 pounds.
b. Compare the probability in part a to the proportion of trucks in the sample that weighed more than 46,000 pounds. What does this imply to you?
c. Suppose the managers are concerned that trucks are returning to the landfill before they are fully loaded. If they have set a minimum weight of 38,000 pounds before the truck returns to the landfill, what is the probability that a truck will fail to meet the minimum standard?

6-76. The Hydronics Company's research and development department came up with two weight-loss products. To determine whether these products are effective, the company has conducted a test. A total of 300 people who were 30 pounds or more overweight were recruited to participate in the study. Of these, 100 people were given a placebo supplement, 100 people were given Product 1, and 100 people were given Product 2. As might be expected, some people dropped out of the study before the four-week study period was completed.

The weight loss (or gain) for each individual is listed in the data file called **Hydronics**.

a. Develop a frequency histogram for the weight loss (or gain) for those people on Product 1. Does it appear from this graph that weight loss is approximately normally distributed?
b. Referring to part a and assuming that a normal distribution does apply, compute the mean and standard deviation weight loss for the Product 1 users.
c. Referring to parts a and b, assume that the weight-change distribution for Product 1 users is normally distributed and that the sample mean and standard deviation are used to directly represent the population mean and standard deviation. What is the probability that a Product 1 user will lose over 12 pounds in a four-week period?
d. Referring to your answer in part c, would it be appropriate for the company to claim that Product 1 users can expect to lose as much as 12 pounds in four weeks? Discuss.

6-77. Midwest Fan Manufacturing Inc. is a distributor of quality ventilation equipment. Midwest Fan's products include the AXC range hood exhaust fans. The file titled **Fan Life** contains the length of life of 125 randomly chosen AXC fans that were used in an accelerated life-testing experiment.

a. Produce a relative frequency histogram for the data. Does it seem plausible the data came from a population that has an exponential distribution?
b. Calculate the mean and standard deviation of the fans' length of life.
c. Calculate the median length of life of the fans.
d. Determine the probability that a randomly chosen fan will have a life longer than 25,000 hours.

6-78. Team Marketing Report (TMR) produces the Fan Cost Index™ (FCI) survey, which tracks the cost of attendance for a family of four at National Football League (NFL) games. The FCI includes four average-price tickets, four small soft drinks, two small beers, four hot dogs, two game programs, parking, and two adult-size caps. The league's average FCI in 2015 was $480.89 (*source:* www.teammarketing.com). The file **NFL Price** shows the actual costs for 175 randomly chosen families of four.

a. Produce a relative frequency histogram for these data. Does it seem plausible the data were sampled from a population that was normally distributed?
b. Calculate the mean and standard deviation of the league's FCI.
c. Calculate the 90th percentile of the league's FCI.
d. The San Francisco 49ers had an FCI of $640.00. Determine the percentile of the FCI of a randomly chosen family whose FCI is the same as that of the 49ers' average FCI.

6-79. The Future-Vision Digital and Satellite TV Company recently surveyed its customers. A total of 548 responses were received. Among other things, the respondents were asked to indicate their household income. The data from the survey are found in a file named **Future-Vision**.

a. Develop a frequency histogram for the income variable. Does it appear from the graph that income is approximately normally distributed? Discuss.
b. Compute the mean and standard deviation for the income variable.
c. Referring to parts a and b and assuming that income is normally distributed and the sample mean and standard deviation are good substitutes for the population values, what is the probability that a Future-Vision customer will have an income exceeding $40,000?
d. Suppose that Future-Vision managers are thinking about offering a monthly discount to customers who have a household income below a certain level. If the management wants to grant discounts to no more than 7% of the customers, what income level should be used for the cutoff?

6-80. Championship Billiards, owned by D & R Industries, in Lincolnwood, Illinois, provides some of the finest billiard fabrics, cushion rubber, and component parts in the industry. It sells billiard cloth in bolts and half-bolts. A half-bolt of billiard cloth has an average length of 35 yards with widths of either 62 or 66 inches. The file titled **Half Bolts** contains the lengths of 120 randomly selected half-bolts.

a. Produce a relative frequency histogram for these data. Does it seem plausible the data came from a population that has a uniform distribution?
b. Provide the density, $f(x)$, for this uniform distribution.
c. A billiard retailer, Sticks & Stones Billiard Supply, is going to recover the pool tables in the local college pool hall, which has eight tables. It takes approximately 3.8 yards per table. If Championship ships a randomly chosen half-bolt, determine the probability that it will contain enough cloth to recover the eight tables.

Case 6.1 State Entitlement Programs

Franklin Joiner, director of health, education, and welfare, had just left a meeting with the state's newly elected governor and several of the other recently appointed department heads. One of the governor's campaign promises was to try to halt the rising cost of a certain state entitlement program. In several speeches, the governor indicated the state of Idaho should allocate funds only to those individuals ranked in the bottom 10% of the state's income distribution. Now the governor wants to know how much one could earn before being disqualified from the program, and he also wants to know the range of incomes for the middle 95% of the state's income distribution.

Frank had mentioned in the meeting that he thought incomes in the state could be approximated by a normal distribution and that mean per capita income was about $33,000 with a standard deviation of nearly $9,000. The governor was expecting a memo in his office by 3:00 P.M. that afternoon with answers to his questions.

Required Tasks:

1. Assuming that incomes can be approximated using a normal distribution with the specified mean and standard deviation, calculate the income that cut off the bottom 10% of incomes.
2. Assuming that incomes can be approximated using a normal distribution with the specified mean and standard deviation, calculate the middle 95% of incomes. *Hint:* This requires calculating two values.
3. Write a short memo describing your results and how they were obtained. Your memo should clearly state the income that would disqualify people from the program, as well as the range of incomes in the middle 95% of the state's income distribution.

Case 6.2 Credit Data, Inc.

Credit Data, Inc., has been monitoring the amount of time its bill collectors spend on calls that produce contacts with consumers. Management is interested in the distribution of time a collector spends on each call in which he or she initiates contact, informs a consumer about an outstanding debt, discusses a payment plan, and receives payments by phone. Credit Data is mostly interested in how quickly a collector can initiate and end a conversation to move on to the next call. For employees of Credit Data, time is money in the sense that one account may require one call and 2 minutes to collect, whereas another account may take five calls and 10 minutes per call to collect. The company has discovered that the time collectors spend talking to consumers about accounts is approximated by a normal distribution with a mean of 8 minutes and a standard deviation of 2.5 minutes. The managers believe that the mean is too high and should be reduced by more efficient phone call methods. Specifically, they wish to have no more than 10% of all calls require more than 10.5 minutes.

Required Tasks:

1. Assuming that training can affect the average time but not the standard deviation, the managers are interested in knowing to what level the mean call time needs to be reduced in order to meet the 10% requirement.
2. Assuming that the standard deviation can be affected by training but the mean time will remain at 8 minutes, to what level must the standard deviation be reduced in order to meet the 10% requirement?
3. If nothing is done, what percent of all calls can be expected to require more than 10.5 minutes?

Case 6.3 National Oil Company—Part 1

Chad Williams, field geologist for the National Oil Company, settled into his first-class seat on the Sun-Air flight between Los Angeles and Oakland, California. Earlier that afternoon, he had attended a meeting with the design engineering group at the Los Angeles New Product Division. He was now on his way to the home office in Oakland. He was looking forward to the one-hour flight because it would give him a chance to reflect on a problem that surfaced during the meeting. It would also give him a chance to think about the exciting opportunities that lay ahead in Australia.

Chad works with a small group of highly trained people at National Oil who literally walk the Earth looking for new sources of oil. They make use of the latest in electronic equipment to take a wide range of measurements from many thousands of feet below the Earth's surface. It is one of these electronic machines that is the source of Chad's current problem. Engineers in Los Angeles have designed a sophisticated enhancement that will greatly improve the equipment's ability to detect oil. The enhancement requires 800 capacitors, which must operate within ±0.50 micron from the specified standard of 12 microns.

The problem is that the supplier can provide capacitors that operate according to a normal distribution, with a mean of 12 microns and a standard deviation of 1 micron. Thus, Chad knows that not all capacitors will meet the specifications required by the new piece of exploration equipment. This will mean that to have at least 800 usable capacitors, National Oil will have to order more than 800 from the supplier. However, these items are very expensive, so he wants to order as few as possible to meet their needs. At the meeting, the group agreed that they wanted a 98% chance that any order of capacitors would contain the sufficient number of usable items. If the project is to remain on schedule, Chad must place the order by tomorrow. He wants the new equipment ready to go by the time he leaves for an exploration trip in Australia. As he reclined in his seat, sipping a cool lemonade, he wondered whether a basic statistical technique could be used to help determine how many capacitors to order.

7 Introduction to Sampling Distributions

WHY YOU NEED TO KNOW

The director of tourism in a Caribbean country conducted a study that showed that the mean daily expenditure for adult visitors to the country is $318.69. The mean value is based on a statistical sample of 780 adult visitors. The $318.69 is a *statistic*, not a *parameter*, because it is based on a sample rather than an entire population. If you were this official, you might have some questions:

Quick Prep

Review the discussion of random sampling in Chapter 1.

Review the steps for computing means and standard deviations in Chapter 3.

Make sure you are familiar with the normal distribution and how to compute standardized *z*-values as introduced in Chapter 6.

Review the concepts associated with finding probabilities with a standard normal distribution as discussed in Chapter 6.

- Is the actual population mean equal to $318.69?
- If the population mean is not $318.69, how close is $318.69 to the true population mean?
- Is a sample of 780 taken from a population of almost 2 million annual visitors to the country sufficient to provide a "good" estimate of the population mean?

A furniture manufacturer that makes made-to-assemble furniture kits selects a random sample of kits boxed and ready for shipment to customers. These kits are unboxed and inspected to see whether what is in the box matches exactly what is supposed to be in the box. This past week, 150 kits were sampled and 15 had one or more discrepancies. This is a 10% defect rate. Should the quality engineer conclude that exactly 10% of the 6,900 furniture kits made since the first of the year reached the customer with one or more order discrepancies? Is the actual percentage higher or lower than 10% and, if so, by how much? Should the quality engineer request that more furniture kits be sampled?

The questions facing the tourism director and the furniture quality engineer are common to people in business everywhere. You will almost assuredly find yourself in a similar situation many times in the future. To help answer these questions, you need to have an understanding of *sampling distributions*. Whenever decisions are based on samples rather than an entire population, questions about the sample results exist. Anytime we sample from a population, many, many samples could be selected. Each sample will contain different items. Because of this, the sample means for each possible sample can be different, or the sample percentages can be different. The sampling distribution describes the distribution of possible sample outcomes. Knowing what this distribution looks like will help you understand the specific result you obtained from the one sample you selected.

This chapter introduces you to the important concepts of sampling error and sampling distributions and discusses how you can use this knowledge to help answer the questions facing the tourism director and the quality engineer. The information presented here provides an essential building block to understanding statistical estimation and hypothesis testing, which will be covered in upcoming chapters.

7.1 Sampling Error: What It Is and Why It Happens

As discussed in previous chapters, you will encounter many situations in business in which you take a sample from a population and then must analyze the sample data. Chapter 1 introduced several different statistical sampling techniques, including simple random sampling. The objective of random sampling is to gather data that accurately represent a population.

OUTCOME 1

Calculating Sampling Error

Regardless of how careful we are in using random sampling methods, the sample may not be a perfect representation of the population. For example, we might compute a *statistic* such as $\bar{x}$ for sample data. Unless the sample is a perfect replication of the population, the statistic will likely not equal the *parameter*, μ. In this case, the difference between the sample mean and the population mean is called **sampling error**. The sampling error is computed using Equation 7.1.

Sampling Error
The difference between a measure computed from a sample (a statistic) and corresponding measure computed from the population (a parameter).

Sampling Error of the Sample Mean

$$\text{Sampling error} = \bar{x} - \mu \tag{7.1}$$

where:

$\bar{x}$ = Sample mean
μ = Population mean