

Mechanical Vibrations

Sixth Edition in SI Units

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3.38 A turbine rotor is mounted on a stepped shaft that is fixed at both ends as shown in Fig. 3.51. The torsional stiffnesses of the two segments of the shaft are given by $k_{t1} = 3000 \text{ N-m/rad}$ and $k_{t2} = 4000 \text{ N-m/rad}$. The turbine generates a harmonic torque given by $M(t) = M_0 \cos \omega t$ about the shaft axis with $M_0 = 200 \text{ N-m}$ and $\omega = 500 \text{ rad/s}$. The mass moment of inertia of the rotor about the shaft axis is $J_0 = 0.05 \text{ kg-m}^2$. Assuming the equivalent torsional damping constant of the system as $c_t = 2.5 \text{ N-m-s/rad}$, determine the steady-state response of the rotor, $\theta(t)$.

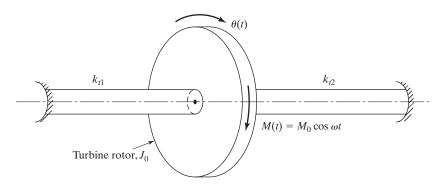


FIGURE 3.51 Turbine rotor subjected to harmonic torque.

- **3.39** It is required to design an electromechanical system to achieve a natural frequency of 1000 Hz and a Q factor of 1200. Determine the damping factor and the bandwidth of the system.
- **3.40** Show that, for small values of damping, the damping ratio ζ can be expressed as

$$\zeta = \frac{\omega_2 - \omega_1}{\omega_2 + \omega_1}$$

where ω_1 and ω_2 are the frequencies corresponding to the half-power points.

- **3.41** A torsional system consists of a disc of mass moment of inertia $J_0 = 10 \text{ kg-m}^2$, a torsional damper of damping constant $c_t = 300 \text{ N-m-s/rad}$, and a steel shaft of diameter 4 cm and length 1 m (fixed at one end and attached to the disc at the other end). A steady angular oscillation of amplitude 2° is observed when a harmonic torque of magnitude 1000 N-m is applied to the disc. (a) Find the frequency of the applied torque, and (b) find the maximum torque transmitted to the support.
- **3.42** For a vibrating system, m = 10 kg, k = 2500 N/m, and c = 45 N-s/m. A harmonic force of amplitude 180 N and frequency 3.5 Hz acts on the mass. If the initial displacement and velocity of the mass are 15 mm and 5 m/s, respectively, find the complete solution representing the motion of the mass.
- **3.43** The peak amplitude of a single-degree-of-freedom system, under a harmonic excitation, is observed to be 5 mm. If the undamped natural frequency of the system is 5 Hz, and the static deflection of the mass under the maximum force is 2.5 mm, (a) estimate the damping ratio of the system, and (b) find the frequencies corresponding to the amplitudes at half power.

3.44 The landing gear of an airplane can be idealized as the spring-mass-damper system shown in Fig. 3.52 [3.16]. If the runway surface is described $y(t) = y_0 \cos \omega t$, determine the values of k and c that limit the amplitude of vibration of the airplane (x) to 0.1 m. Assume m = 2000 kg, $y_0 = 0.2 \text{ m}$, and $\omega = 157.08 \text{ rad/s}$.

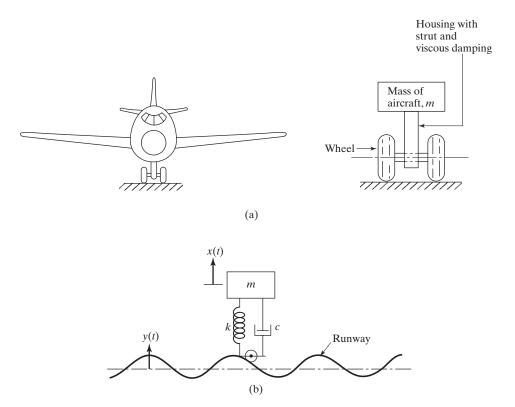


FIGURE 3.52 Modeling of landing gear.

- 3.45 A precision grinding machine (Fig. 3.53) is supported on an isolator that has a stiffness of 1 MN/m and a viscous damping constant of 1 kN-s/m. The floor on which the machine is mounted is subjected to a harmonic disturbance due to the operation of an unbalanced engine in the vicinity of the grinding machine. Find the maximum acceptable displacement amplitude of the floor if the resulting amplitude of vibration of the grinding wheel is to be restricted to 10⁻⁶ m. Assume that the grinding machine and the wheel are a rigid body of weight 5000 N.
- **3.46** Derive the equation of motion and find the steady-state response of the system shown in Fig. 3.54 for rotational motion about the hinge O for the following data: k = 5000 N/m, l = 1 m, c = 1000 N-s/m, m = 10 kg, $M_0 = 100 \text{ N-m}$, $\omega = 1000 \text{ rpm}$.

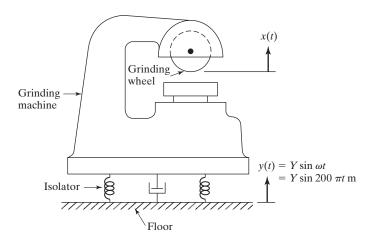


FIGURE 3.53 Grinding machine on isolator.

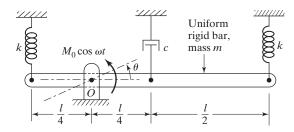


FIGURE 3.54 Harmonic torque applied to spring supported bar.

- 3.47 An air compressor of mass 100 kg is mounted on an elastic foundation. It has been observed that, when a harmonic force of amplitude 100 N is applied to the compressor, the maximum steady-state displacement of 5 mm occurred at a frequency of 300 rpm. Determine the equivalent stiffness and damping constant of the foundation.
- **3.48** Find the steady-state response of the system shown in Fig. 3.55 for the following data: $k_1=1000$ N/m, $k_2=500$ N/m, c=500 N-s/m, m=10 kg, r=5 cm, $J_0=1$ kg-m², $F_0=50$ N, $\omega=20$ rad/s.
- **3.49** A uniform slender bar of mass m may be supported in one of two ways as shown in Fig. 3.56. Determine the arrangement that results in a reduced steady-state response of the bar under a harmonic force, $F_0 \sin \omega t$, applied at the middle of the bar, as shown in the figure.

FIGURE 3.55 Spring-mass-damper connected to pulley.

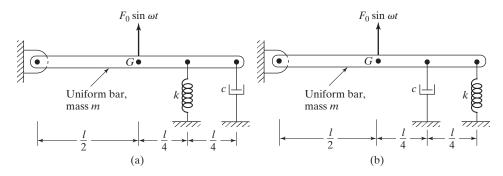


FIGURE 3.56 Slender bar supported in two ways.

- **3.50** Determine the steady state response of the mass of a spring-mass-damper system subjected to a harmonic force, f(t), for the following data: $m = 1 \text{ kg}, c = 50 \text{ N-s/m}, k = 50000 \text{ N/m}, f(t) = 50 \cos 400 t \text{ N}.$
- 3.51 By denoting the amplitudes of velocity and acceleration of the mass of a viscously damped system subjected to a harmonic force (shown in Fig. 3.1), as \dot{X} and \ddot{X} , respectively, find expressions for the ratios $\frac{\dot{X}}{F_0/\sqrt{k\,m}}$ and $\frac{\ddot{X}}{F_0/m}$ in terms of r and ζ . The nondimensional ratios $\frac{\dot{X}}{F_0/\sqrt{k\,m}}$ and $\frac{\ddot{X}}{F_0/m}$ are called the velocity and acceleration frequency responses of the system, respectively.
- **3.52** Find the force transmitted to the base of a viscously damped system subjected to a harmonic force, in the steady state, by using the relation $f_T = F_0 \cos \omega t m\ddot{x}$ instead of $f_T = kx + c\dot{x}$ and the steady state response of the system given by Eq. (3.25).

Section 3.5 Response of a Damped System Under $F(t) = F_0 e$

- 3.53 Derive the expression for the complex frequency response of an undamped torsional system.
- **3.54** A damped single-degree-of-freedom system, with parameters m = 150 kg, k = 25 kN/m, and c = 2000 N-s/m, is subjected to the harmonic force $f(t) = 100 \cos 20t \text{ N}$. Find the amplitude and phase angle of the steady-state response of the system using a graphical method.

Section 3.6 Response of a System Under the Harmonic Motion of the Base

3.55 A single-story building frame is subjected to a harmonic ground acceleration, as shown in Fig. 3.57. Find the steady-state motion of the floor (mass m).

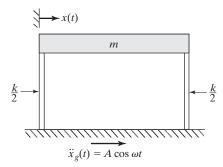


FIGURE 3.57 Single story building subjected to ground acceleration.

- **3.56** Find the horizontal displacement of the floor (mass m) of the building frame shown in Fig. 3.57 when the ground acceleration is given by $\ddot{x}_g = 100 \sin \omega t \text{ mm/s}^2$. Assume m = 2000 kg, k = 0.1 MN/m, $\omega = 25 \text{ rad/s}$, and $x_g(t = 0) = \dot{x}_g(t = 0) = x(t = 0) = \dot{x}(t = 0) = 0$.
- 3.57 If the ground in Fig. 3.57, is subjected to a horizontal harmonic displacement with frequency $\omega = 200$ rad/s and amplitude $X_g = 15$ mm, find the amplitude of vibration of the floor (mass m). Assume the mass of the floor as 2000 kg and the stiffness of the columns as 0.5 MN/m.
- 3.58 An automobile is modeled as a single-degree-of-freedom system vibrating in the vertical direction. It is driven along a road whose elevation varies sinusoidally. The distance from peak to trough is 0.2 m and the distance along the road between the peaks is 35 m. If the natural frequency of the automobile is 2 Hz and the damping ratio of the shock absorbers is 0.15, determine the amplitude of vibration of the automobile at a speed of 60 km/hour. If the speed of the automobile is varied, find the most unfavorable speed for the passengers.
- **3.59** Derive Eq. (3.74).
- **3.60** A single-story building frame is modeled by a rigid floor of mass m and columns of stiffness k, as shown in Fig. 3.58. It is proposed that a damper shown in the figure is attached to absorb vibrations due to a horizontal ground motion $y(t) = Y \cos \omega t$. Derive an expression for the damping constant of the damper that absorbs maximum power.

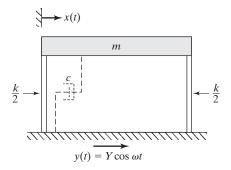


FIGURE 3.58 Damped single story building subjected to ground acceleration.

3.61 A uniform bar of mass m is pivoted at point O and supported at the ends by two springs, as shown in Fig. 3.59. End P of spring PQ is subjected to a sinusoidal displacement, $x(t) = x_0 \sin \omega t$. Find the steady-state angular displacement of the bar when l = 1 m, k = 1000 N/m, m = 10 kg, $x_0 = 1$ cm, and $\omega = 10$ rad/s.

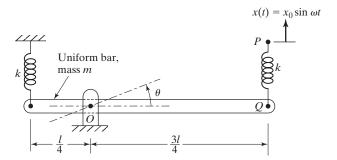


FIGURE 3.59 Spring subjected to sinusoidal motion.

3.62 A uniform bar of mass m is pivoted at point O and supported at the ends by two springs, as shown in Fig. 3.60. End P of spring PQ is subjected to a sinusoidal displacement, $x(t) = x_0 \sin \omega t$. Find the steady-state angular displacement of the bar when l = 1 m, k = 1000 N/m, c = 500 N-s/m, m = 10 kg, $x_0 = 1$ cm, and $\omega = 10$ rad/s.

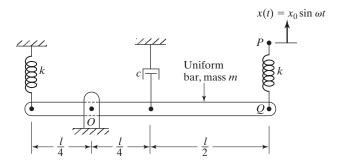


FIGURE 3.60 Spring of damped system subjected to sinusoidal motion.