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Mechanics of Materials

Tenth Edition in SI Units

R. C. Hibbeler

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Mechanics of Materials

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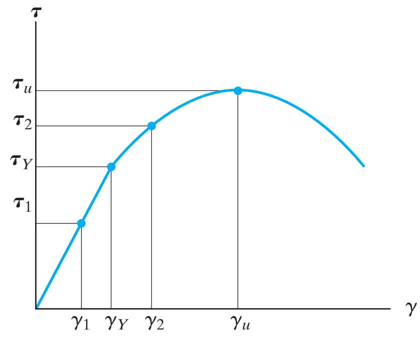
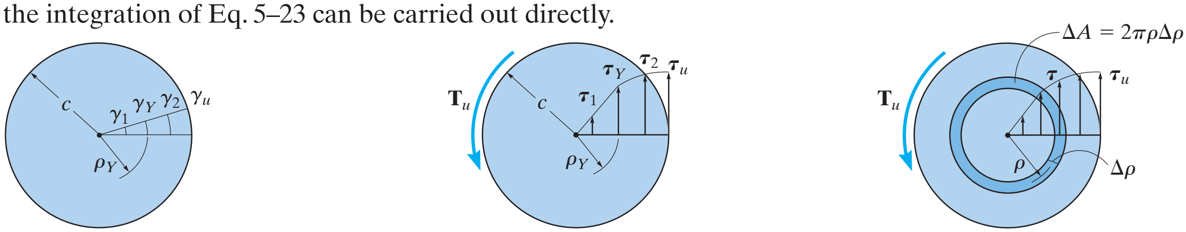
Ultimate Torque. In the general case, most engineering materials will have a shear stress–strain diagram as shown in Fig. 5–36*a*. Consequently, if T is increased so that the maximum shear strain in the shaft becomes $\gamma = \gamma_u$, Fig. 5–36*b*, then, by proportion γ_Y occurs at $\rho_Y = (\gamma_Y/\gamma_u)c$. Likewise, the shear strains at, say, $\rho = \rho_1$ and $\rho = \rho_2$, can be found by proportion, i.e., $\gamma_1 = (\rho_1/c)\gamma_u$ and $\gamma_2 = (\rho_2/c)\gamma_u$. If the corresponding values of τ_1, τ_Y, τ_2, and τ_u are taken from the τ–γ diagram and plotted, we obtain the shear-stress distribution, which acts along a radial line on the cross section, Fig. 5–36*c*. The torque produced by this stress distribution is called the ***ultimate torque***, T_u.

The magnitude of $\mathbf{T}_u$ can be determined by "graphically" integrating Eq. 5–23. To do this, the cross-sectional area of the shaft is segmented into a finite number of small rings, such as the one shown shaded in Fig. 5–36*d*. The area of this ring, $\Delta A = 2\pi\rho\,\Delta\rho$, is multiplied by the shear stress τ that acts on it, so that the force $\Delta F = \tau\,\Delta A$ can be determined. The torque created by this force is then $\Delta T = \rho\,\Delta F = \rho(\tau\,\Delta A)$. The addition of all the torques for the entire cross section, as determined in this manner, gives the ultimate torque T_u; that is, Eq. 5–23 becomes $T_u \approx 2\pi\Sigma\tau\rho^2\,\Delta\rho$. Of course, if the stress distribution can be expressed as an analytical function, $\tau = f(\rho)$, as in the elastic and plastic torque cases, then the integration of Eq. 5–23 can be carried out directly.



(a)



Ultimate shear-strain distribution

(b)

Ultimate shear-stress distribution

(c)

(d)

Fig. 5–36

IMPORTANT POINTS

- The *shear-strain distribution* along a radial line on the cross section of a shaft is based on geometric considerations, and it is found to *always* vary linearly along the radial line. Once it is established, the shear-stress distribution can then be determined using the shear stress–strain diagram.
- If the shear-stress distribution for the shaft is established, then the resultant torque it produces is equivalent to the resultant internal torque acting on the cross section.
- *Perfectly plastic behavior* assumes the shear-stress distribution is *constant* over each radial line. When it occurs, the shaft will continue to twist with no increase in torque. This torque is called the *plastic torque*.

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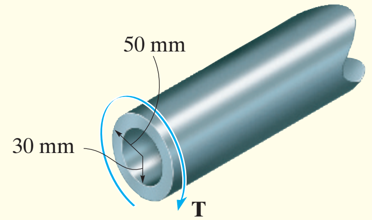
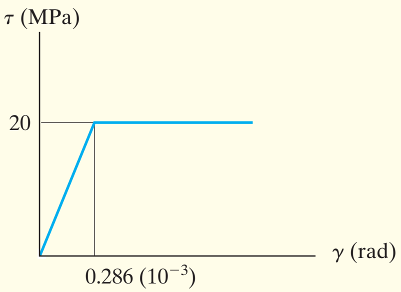
EXAMPLE 5.15

The tubular shaft in Fig. 5–37*a* is made of an aluminum alloy that is assumed to have an elastic perfectly plastic τ–γ diagram as shown. Determine the maximum torque that can be applied to the shaft without causing the material to yield, and the plastic torque that can be applied to the shaft. Also, what should the minimum shear strain at the outer wall be in order to develop a fully plastic torque?





(a)

SOLUTION

Maximum Elastic Torque. We require the shear stress at the outer fiber to be 20 MPa. Using the torsion formula, we have

$$\tau_Y = \frac{T_Y c}{J}; \qquad 20(10^6)\ \mathrm{N/m^2} = \frac{T_Y(0.05\ \mathrm{m})}{(\pi/2)[(0.05\ \mathrm{m})^4 - (0.03\ \mathrm{m})^4]}$$

$$T_Y = 3.42\ \mathrm{kN\cdot m} \qquad \textit{Ans.}$$

The shear-stress and shear-strain distributions for this case are shown in Fig. 5–37*b*. The values at the tube's inner wall have been obtained by proportion.

Plastic Torque. The shear-stress distribution in this case is shown in Fig. 5–37*c*. Application of Eq. 5–23 requires $\tau = \tau_Y$. We have

$$T_p = 2\pi \int_{0.03\ \mathrm{m}}^{0.05\ \mathrm{m}} [20(10^6)\ \mathrm{N/m^2}]\rho^2\, d\rho = 125.66(10^6)\frac{1}{3}\rho^3 \Big|_{0.03\ \mathrm{m}}^{0.05\ \mathrm{m}}$$

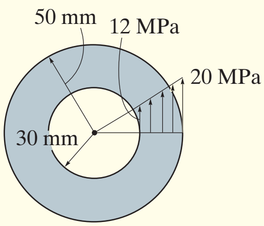
$$= 4.11\ \mathrm{kN\cdot m} \qquad \textit{Ans.}$$

For this tube T_p represents a 20% increase in torque capacity compared with the elastic torque T_Y.

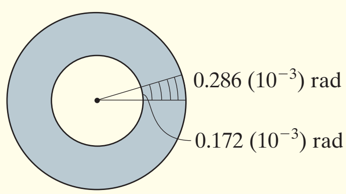
Outer Radius Shear Strain. The tube becomes fully plastic when the shear strain at the *inner wall* becomes $0.286(10^{-3})$ rad, as shown in Fig. 5–37*c*. Since the shear strain *remains linear* over the cross section, the plastic strain at the outer fibers of the tube in Fig. 5–37*c* is determined by proportion.

$$\frac{\gamma_o}{50\ \mathrm{mm}} = \frac{0.286(10^{-3})\ \mathrm{rad}}{30\ \mathrm{mm}}$$

$$\gamma_o = 0.477(10^{-3})\ \mathrm{rad} \qquad \textit{Ans.}$$



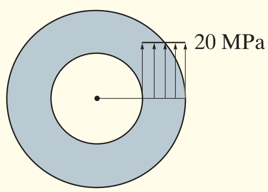
Elastic shear-stress distribution



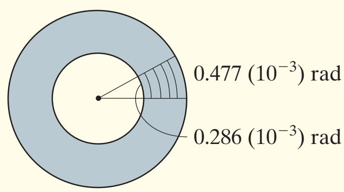
Elastic shear–strain distribution

(b)

Fig. 5–37



Plastic shear-stress distribution



Initial plastic shear–strain distribution

(c)

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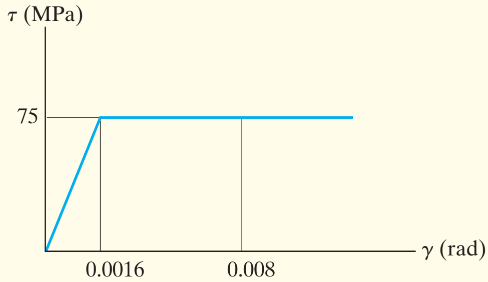
EXAMPLE 5.16

A solid circular shaft has a radius of 20 mm and length of 1.5 m. The material has an elastic perfectly plastic τ–γ diagram as shown in Fig. 5–38*a*. Determine the torque needed to twist the shaft $\phi = 0.6$ rad.



(a)

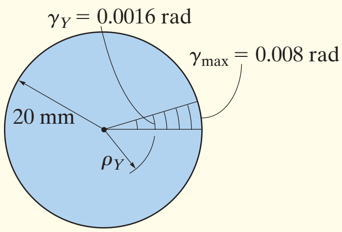
SOLUTION

We will first obtain the shear-strain distribution based on the required twist, then establish the shear-stress distribution. Once this is known, the applied torque can be determined.

The maximum shear strain occurs at the surface of the shaft, $\rho = c$. Since the angle of twist is $\phi = 0.6$ rad for the entire 1.5-m length of the shaft, then using Eq. 5–13, for the entire length we have

$$\phi = \gamma \frac{L}{\rho}; \qquad 0.6 = \frac{\gamma_{\max}(1.5\text{ m})}{0.02\text{ m}}$$

$$\gamma_{\max} = 0.008\text{ rad}$$



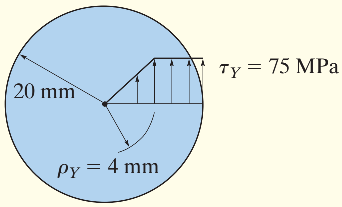
Shear–strain distribution

(b)

The shear-strain distribution is shown in Fig. 5–38*b*. Note that yielding of the material occurs since $\gamma_{\max} > \gamma_Y = 0.0016$ rad in Fig. 5–38*a*. The radius of the elastic core, ρ_Y, can be obtained by proportion. From Fig. 5–38*b*,

$$\frac{\rho_Y}{0.0016} = \frac{0.02\text{ m}}{0.008}$$

$$\rho_Y = 0.004\text{ m} = 4\text{ mm}$$



Shear-stress distribution

(c)

Fig. 5–38

Based on the shear-strain distribution, the shear-stress distribution, plotted over a radial line segment, is shown in Fig. 5–38*c*. The torque can now be obtained using Eq. 5–25. Substituting in the numerical data yields

$$T = \frac{\pi\tau_Y}{6}(4c^3 - \rho_Y^3)$$

$$= \frac{\pi[75(10^6)\text{ N/m}^2]}{6}[4(0.02\text{ m})^3 - (0.004\text{ m})^3]$$

$$= 1.25\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

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EXAMPLE 5.17

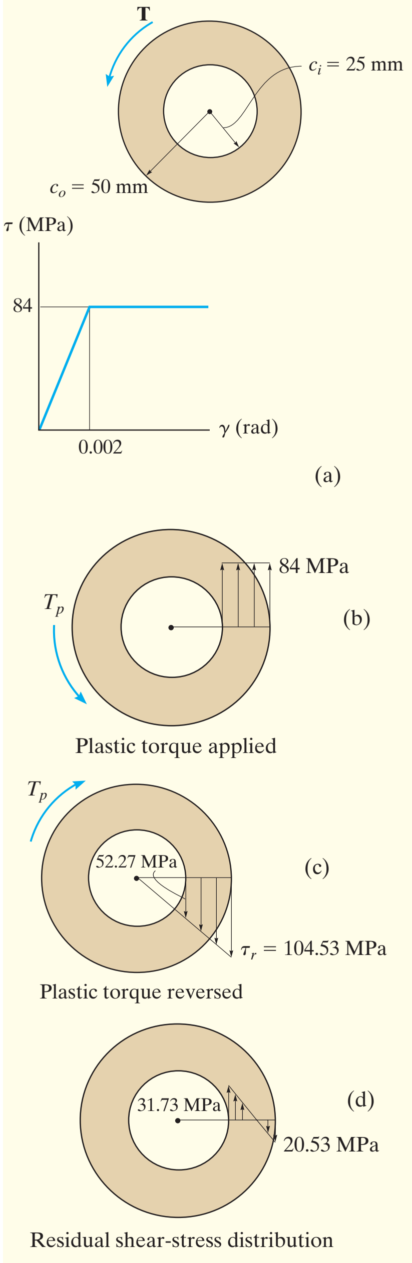

Fig. 5–39

A tube in Fig. 5–39*a* has a length of 1.5 m and its material has an elastic-plastic $\tau-\gamma$ diagram, also shown in Fig. 5–39*a*. Determine the plastic torque T_p. What is the residual shear-stress distribution if $\mathbf{T}_p$ is removed *just after* the tube becomes fully plastic?

SOLUTION

Plastic Torque. The plastic torque $\mathbf{T}_p$ will strain the tube such that all the material yields. Hence the stress distribution will appear as shown in Fig. 5–39*b*. Applying Eq. 5–23, we have

$$T_p = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2\, d\rho = \frac{2\pi}{3}\tau_Y(c_o^3 - c_i^3)$$

$$= \frac{2\pi}{3}\,[84(10^6)\ \text{N/m}^2][(0.050\ \text{m})^3 - (0.025\ \text{m})^3]$$

$$= 19.24(10^3)\ \text{N}\cdot\text{m} = 19.2\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

When the tube just becomes fully plastic, yielding has started at the inner wall, i.e., at $c_i = 0.025$ m, $\gamma_Y = 0.002$ rad, Fig. 5–39*a*. The angle of twist that occurs can be determined from Eq. 5–25, which for the entire tube becomes

$$\phi_p = \gamma_Y \frac{L}{c_i} = \frac{(0.002)(1.5\ \text{m})}{(0.025\ \text{m})} = 0.120\ \text{rad}\circlearrowleft$$

When $\mathbf{T}_p$ is *removed,* or in effect reapplied in the opposite direction, then the "fictitious" linear shear-stress distribution shown in Fig. 5–39*c* must be superimposed on the one shown in Fig. 5–39*b*. In Fig. 5–39*c* the maximum shear stress or the modulus of rupture is found from the torsion formula

$$\tau_r = \frac{T_p c_o}{J} = \frac{[19.24\ (10^3)\text{N}\cdot\text{m}]\ (0.050\ \text{m})}{(\pi/2)[(0.050\ \text{m})^4 - (0.025\ \text{m})^4]} = 104.53(10^6)\ \text{N/m}^2$$

$$= 104.53\ \text{MPa}$$

Also, at the inner wall of the tube the shear stress is

$$\tau_i = (104.53\ \text{MPa})\left(\frac{25\ \text{mm}}{50\ \text{mm}}\right) = 52.27\ \text{MPa} \qquad \textit{Ans.}$$

The resultant residual shear-stress distribution is shown in Fig. 5–39*d*.

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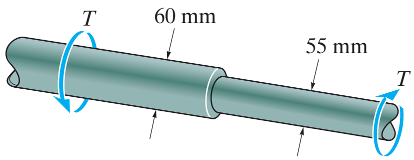
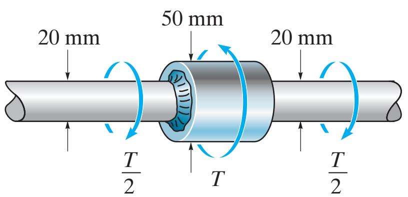
PROBLEMS

*5–120.** The stepped shaft is subjected to a torque **T** that produces yielding on the surface of the larger diameter segment. Determine the radius of the elastic core produced in the smaller diameter segment. Neglect the stress concentration at the fillet.



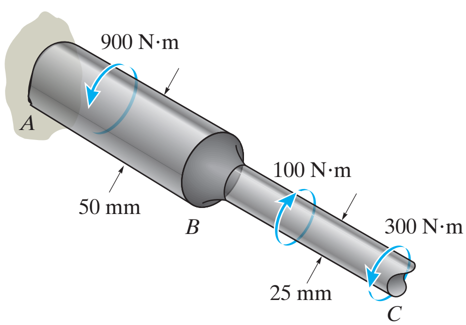
Prob. 5–120

5–121. The steel step shaft has an allowable shear stress of $\tau_{\text{allow}} = 8$ MPa If the transition between the cross sections has a radius $r = 4$ mm, determine the maximum torque T that can be applied.



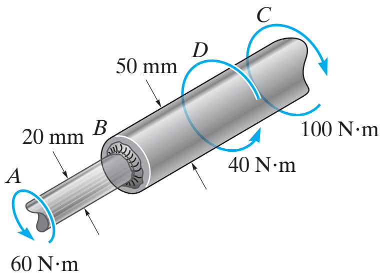
Prob. 5–121

5–122. The shaft is fixed to the wall at A and is subjected to the torques shown. Determine the maximum shear stress in the shaft. A fillet weld having a radius of 2.75 mm is used to connect the shafts at B.



Prob. 5–122

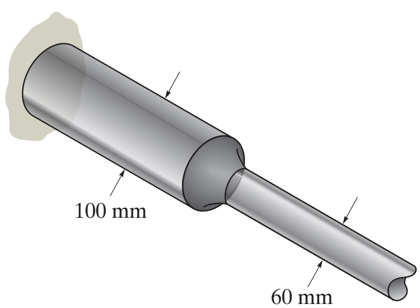
5–123. The steel shaft is made from two segments: AB and BC, which are connected using a fillet weld having a radius of 2.8 mm. Determine the maximum shear stress developed in the shaft.



Prob. 5–123

*5–124.** The built-up shaft is to be designed to rotate at 450 rpm while transmitting 230 kW of power. Is this possible? The allowable shear stress is $\tau_{\text{allow}} = 150$ MPa.

5–125. The built-up shaft is designed to rotate at 450 rpm. If the radius of the fillet weld connecting the shafts is $r = 13.2$ mm, and the allowable shear stress for the material is $\tau_{\text{allow}} = 150$ MPa, determine the maximum power the shaft can transmit.



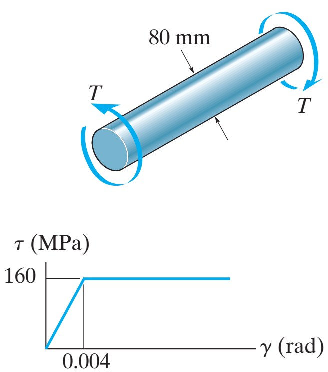
Probs. 5–124/125

5–126. A solid shaft has a diameter of 40 mm and length of 1 m. It is made from an elastic-plastic material having a yield stress of $\tau_Y = 100$ MPa. Determine the maximum elastic torque T_Y and the corresponding angle of twist. What is the angle of twist if the torque is increased to $T = 1.2T_Y$? $G = 80$ GPa.

5–127. Determine the torque needed to twist a short 2-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic perfectly plastic and having a yield stress of $\tau_Y = 50$ MPa. Assume that the material becomes fully plastic.

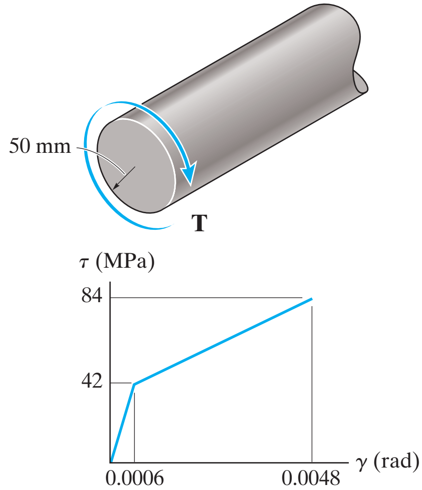
*__5–128.__ A solid shaft is subjected to the torque T, which causes the material to yield. If the material is elastic plastic, show that the torque can be expressed in terms of the angle of twist ϕ of the shaft as $T = \frac{4}{3} T_Y(1-\phi_Y^3/4\phi^3)$, where T_Y and ϕ_Y are the torque and angle of twist when the material begins to yield.

5–129. The solid shaft is made of an elastic perfectly plastic material. Determine the torque T needed to form an elastic core in the shaft having a radius of $\rho_Y = 20$ mm. If the shaft is 3 m long, through what angle does one end of the shaft twist with respect to the other end? When the torque is removed, determine the residual stress distribution in the shaft and the permanent angle of twist.



Prob. 5–129

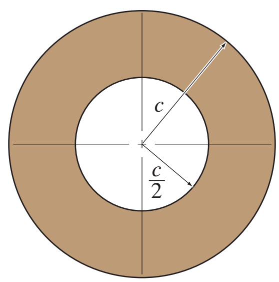
5–130. The shaft is subjected to a maximum shear strain of 0.0048 rad. Determine the torque applied to the shaft if the material has strain hardening as shown by the shear stress–strain diagram.



Prob. 5–130

5–131. A solid shaft having a diameter of 50 mm is made of elastic-plastic material having a yield stress of $\tau_Y = 112$ MPa and shear modulus of $G = 84$ GPa. Determine the torque required to develop an elastic core in the shaft having a diameter of 25 mm. Also, what is the plastic torque?

*__5–132.__ The hollow shaft has the cross section shown and is made of an elastic perfectly plastic material having a yield shear stress of τ_Y. Determine the ratio of the plastic torque T_p to the maximum elastic torque T_Y.



Prob. 5–132