GLOBAL EDITION

Fluid Mechanics for Engineers

in SI Units

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FLUID MECHANICS
FOR ENGINEERS
IN SI UNITS

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Because the difference in water surface elevations in the reservoirs is 20 m, $z_{s2} - z_{s1} = 20$ m, and Equation 4.156 gives

$$z_{s1} + h_{pump} = z_{s2} + h_{turb} + h_\ell \quad \rightarrow$$
$$h_{pump} = (z_{s2} - z_{s1}) + h_{turb} + h_\ell = 20\text{ m} + 0 + 3.97\text{ m} = 23.97\text{ m}$$

Therefore, the power, P_{pump}, that must be supplied to the pump to deliver the required flow (6.3 L/s) is given by Equation 4.155 as

$$P_{pump} = \frac{\gamma Q h_{pump}}{\eta_{pump}} = \frac{(9.79)(0.0063)(23.97)}{0.80} = \mathbf{1.85\ kW}$$

The theoretical power required to drive the pump is 1.85 kW. This requirement will likely need to be refined because a commercially available pump might not be able to deliver exactly 6.3 L/s when the head difference between the reservoirs is 20 m.

Forms of the energy equation useful for gases and vapors. For gases and vapors, heat exchange, temperature changes, and compressibility are usually processes of concern, and elevation changes usually have a negligible effect on the energy balance (due to the relatively small density of air). To facilitate analyses of gas and vapor flows, the energy equation, Equation 4.149, is commonly expressed in one of the following forms:

$$\frac{p_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2 + \left[\frac{\dot{W}_s}{\dot{m}}\right] + \left[(u_2 - u_1) - \frac{\dot{Q}}{\dot{m}}\right] \tag{4.157}$$

or

$$h_1 + \frac{1}{2}V_1^2 + gz_1 = h_2 + \frac{1}{2}V_2^2 + gz_2 + \left[\frac{\dot{W}_s}{\dot{m}}\right] - \left[\frac{\dot{Q}}{\dot{m}}\right] \tag{4.158}$$

where h_i is the enthalpy at section i as defined by Equation 4.132. The energy equation in the forms given by Equations 4.157 and 4.158 are valid for any gas or vapor and for any process. A term that is also commonly used is the *stagnation enthalpy*, H, defined as

$$H = h + \frac{1}{2}V^2 + gz \tag{4.159}$$

So the form of the energy equation given by Equation 4.158 can be expressed as

$$H_2 - H_1 = \frac{\dot{Q}}{\dot{m}} - \frac{\dot{W}_s}{\dot{m}} \tag{4.160}$$

where H_1 and H_2 are the stagnation enthalpies at sections 1 and 2, respectively. Therefore, the energy equation in the form given by Equation 4.160 states that the change in stagnation enthalpy is equal to the heat transfer into the control volume minus the shaft work done by the fluid in the control volume. Some knowledge of thermodynamics is necessary to evaluate enthalpies, and in the case of vapors, vapor tables or charts are commonly required because vapor properties cannot be expressed by the simple ideal gas law equation. By using the enthalpy instead of the internal energy to represent the energy of a flowing fluid, the (pressure) energy associated with pushing the fluid is automatically taken into account by the enthalpy, which is one of the main reasons for defining enthalpy as a property.

EXAMPLE 4.25

A turbine is used to extract energy from steam as illustrated in Figure 4.38, where the inflow conduit has a diameter of 300 mm and the outflow conduit has a diameter of 500 mm. On the inflow side, steam enters at a velocity of 35 m/s, with a density of 0.6 kg/m^3 and an enthalpy of 5000 kJ/kg. At the outflow side, steam exits with a density of 0.1 kg/m^3 and an enthalpy of 3000 kJ/kg. The centerlines of the inflow and outflow conduits are at approximately the same elevation, and the system is sufficiently well insulated that adiabatic conditions can be assumed. Estimate the power produced by the turbine.

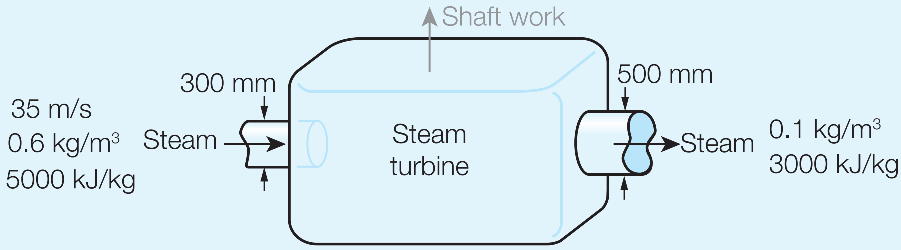

Figure 4.38: Power generation by a steam turbine

SOLUTION

From the given data: $D_1 = 300$ mm, $D_2 = 500$ mm, $V_1 = 35$ m/s, $\rho_1 = 0.6$ kg/m^3, $h_1 = 5000$ kJ/kg, $\rho_2 = 0.1$ kg/m^3, $h_2 = 3000$ kJ/kg, $z_1 = z_2$, and $\dot{Q} = 0$. The inflow area, A_1, and the mass flow rate through the system, $\dot{m}$, can be calculated from the given data as follows:

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.07069\,\text{m}^2, \qquad \dot{m} = \rho_1 V_1 A_1 = (0.6)(35)(0.07069) = 1.484\,\text{kg/s}$$

The outflow velocity can be estimated by applying the steady-state conservation of mass equation, which requires that

$$\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \quad \rightarrow \quad V_2 = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{D_1}{D_2}\right)^2 V_1 = \left(\frac{0.6}{0.1}\right)\left(\frac{300}{500}\right)^2 (35) = 75.6 \text{ m/s}$$

The inflow and outflow stagnation enthalpies, H_1 and H_2, respectively, are given by Equation 4.159 as follows:

$$H_1 = h_1 + \frac{1}{2}V_1^2 + gz_1 = 5000 + \frac{1}{2}(35)^2\ [\times 10^{-3}\ \text{kJ/J}] + 0 = 5000 \text{ kJ/kg}$$

$$H_2 = h_2 + \frac{1}{2}V_2^2 + gz_2 = 3000 + \frac{1}{2}(75.6)^2\ [\times 10^{-3}\ \text{kJ/J}] + 0 = 3000 \text{ kJ/kg}$$

It is apparent from these results that the kinetic energy contributes a negligible amount to the stagnation enthalpy. Applying the energy equation in the form of Equation!4.160 gives

$$H_2 - H_1 = \frac{\dot{Q}}{\dot{m}} - \frac{\dot{W}_\text{s}}{\dot{m}} \quad \rightarrow \quad 3000 - 5000 = 0 - \frac{\dot{W}_\text{s}}{1.484} \quad \rightarrow \quad \dot{W}_\text{s} = 2.97 \times 10^3 \text{ kW} = 2.97 \text{ MW}$$

The theoretical power output generated by the steam turbine is **2.97 MW**. The usable power output will be less than 2.97 MW due to inefficiencies in power transmission and generator operation.

Forms of the energy equation useful for ideal gases. In many applications, gases are sufficiently far removed from the liquid state that they can be treated as ideal gases. Differential and finite changes in the internal energy and enthalpy of an ideal gas can be expressed in terms of specific heats as

$$\mathrm{d}u = c_v\,\mathrm{d}T, \qquad \mathrm{d}h = c_p\,\mathrm{d}T \tag{4.161}$$

where c_v and c_p are the constant volume and constant pressure specific heats of an ideal gas. Assuming that specific heats remain constant within the temperature range encountered by a gas, the energy equation in the forms of Equations 4.157 and 4.158 can be conveniently expressed in terms of temperature changes as

$$\frac{p_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2 + c_v(T_2 - T_1) + \left[\frac{\dot{W}_\mathrm{s}}{\dot{m}}\right] - \left[\frac{\dot{Q}}{\dot{m}}\right] \tag{4.162}$$

or

$$\frac{1}{2}V_1^2 + gz_1 = \frac{1}{2}V_2^2 + gz_2 + c_p(T_2 - T_1) + \left[\frac{\dot{W}_\mathrm{s}}{\dot{m}}\right] - \left[\frac{\dot{Q}}{\dot{m}}\right] \tag{4.163}$$

EXAMPLE 4.26

Air flows into a compressor through a 200-mm-diameter tube and out of the compressor through a 100-mm-diameter tube. The inflow air has a velocity of 60 m/s, a temperature of 25°C, and a pressure of 101 kPa. The outflow air has a temperature of 80°C and a pressure of 250 kPa. Cooling water within the compressor assembly removes heat at a rate of 20 kJ per kg of air that passes through the compressor. Estimate the power consumption of the compressor.

SOLUTION

From the given data: D_1 = 200 mm, D_2 = 100 mm, V_1 = 60 m/s, T_1 = 25°C = 298.15 K, p_1 = 101 kPa, T_2 = 80°C = 353.15 K, p_2 = 250 kPa, and $\dot{Q} = -20$ kJ/kg. For standard air, R = 287.1 J/kg·K and c_p = 1003 J/kg·K. The following preliminary calculations are useful:

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.2)^2}{4} = 3.142 \times 10^{-2}\ \mathrm{m}^2, \qquad A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(0.1)^2}{4} = 7.854 \times 10^{-3}\ \mathrm{m}^2$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{101 \times 10^3}{(287.1)(298.15)} = 1.180\ \mathrm{kg/m^3}, \quad \rho_2 = \frac{p_2}{RT_2} = \frac{250 \times 10^3}{(287.1)(353.15)} = 2.466\ \mathrm{kg/m^3}$$

$$\dot{m} = \rho_1 V_1 A_1 = (1.180)(60)(3.142 \times 10^{-2}) = 2.224\ \mathrm{kg/s},$$

$$V_2 = \frac{\dot{m}}{\rho_2 A_2} = \frac{2.224}{(2.466)(7.854 \times 10^{-3})} = 114.8\ \mathrm{m/s}$$

The energy equation in the form of Equation 4.163 can be applied directly. Gravitational effects associated with the difference in elevation (if any) between the inflow and outflow tubes can be assumed negligible in comparison with the magnitude of the other terms in the energy equation. Applying Equation 4.163 gives

$$\frac{1}{2}V_1^2 + \cancel{gz_1} = \frac{1}{2}V_2^2 + \cancel{gz_2} + c_p(T_2 - T_1) + \left[\frac{\dot{W}_s}{\dot{m}}\right] - \left[\frac{\dot{Q}}{\dot{m}}\right]$$

$$\frac{1}{2}(60)^2 = \frac{1}{2}(114.8)^2 + 1003(353.15 - 298.15) + \left[\frac{\dot{W}_s}{2.224}\right] - \left[\frac{-20 \times 10^3}{2.224}\right] \quad \rightarrow$$

$$\dot{W}_s = -1.354 \times 10^5 \text{ W}$$

Therefore, the power consumption of the compressor is approximately **135 kW**.

Practical application. The energy equation is commonly applied in the analysis of airflow through ducts that are used to distribute air in heating, ventilating, and air conditioning (HVAC) systems. Circulating air in these ducts is usually driven by fans that do shaft work on the air in the duct.

EXAMPLE 4.27

A 1-kW intake fan is used to pull in air from a room and push it through a 1 m × 0.5 m duct as shown in Figure 4.39(a). The air pressure on the downstream side of the fan is approximately equal to the room air pressure, with the energy input of the fan going toward increasing the velocity of the air. (a) Estimate the air velocity in the duct if the fan is 100% efficient in transferring energy to the air. (b) What would be the air velocity in the duct if the energy transfer was 70% efficient?

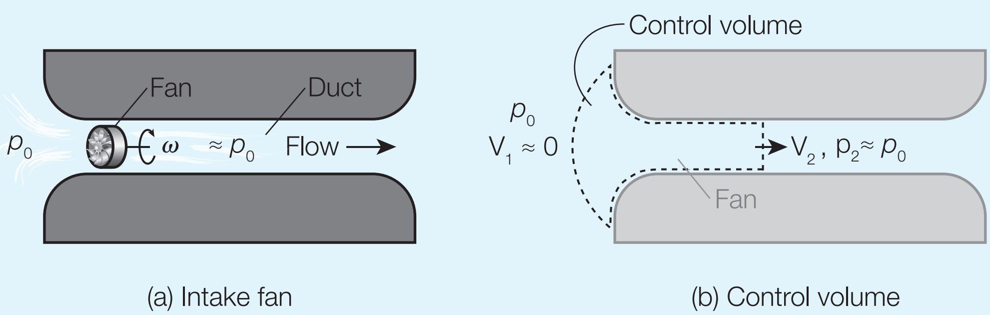

Figure 4.39: Intake fan for a ventilating system

SOLUTION

From the given data: $\dot{W}_s = -1$ kW $= -10^3$ W and $A = (1)(0.5) = 0.5$ m^2. For standard air, $\rho =$ 1.225 kg/m^3.

(a) If the fan is 100% efficient in transferring energy to the air, then the energy loss is equal to zero and the energy equation (Equation 4.157) gives

$$\frac{p_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2 + \left[\frac{\dot{W}_s}{\dot{m}}\right] \tag{4.164}$$

where sections 1 and 2 are the upstream and downstream sections, respectively, of the control volume as shown in Figure 4.39(b). From the given conditions, $p_1 \approx p_2$, $V_1 \approx 0$, and elevation changes can be neglected such that $z_1 \approx z_2$. It can be further assumed that the flow is incompressible, such that $\rho_1 \approx \rho_2 \approx \rho$. Substituting the given data and approximations into Equation 4.164 gives

$$\cancel{\frac{p_1}{\rho_1}} + 0 + 0 = \cancel{\frac{p_2}{\rho_2}} + \frac{1}{2}V_2^2 + 0 + \left[\frac{-10^3}{\rho V_2 A}\right] \;\rightarrow\; 0 = \frac{1}{2}V_2^2 + \left[\frac{-10^3}{(1.225)V_2(0.5)}\right] \;\rightarrow\; V_2 = \mathbf{14.8\ m/s}$$

(b) If the fan is not 100% efficient in transferring energy to the air, then the energy equation (Equation 4.157) can be expressed as

$$\frac{p_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2 - E_s + E_\ell \tag{4.165}$$

where E_s is the shaft work done by the fan on the air per unit mass of air and E_ℓ is the energy loss per unit mass of air. Representing the efficiency as η and taking $\eta = 0.7$ gives

$$\eta = \frac{E_s - E_\ell}{E_s} \quad\rightarrow\quad 0.7 = \frac{E_s - E_\ell}{E_s} \quad\rightarrow\quad E_\ell = 0.3E_s \tag{4.166}$$

Substituting Equation 4.166 into Equation 4.165 and implementing the aforementioned flow approximations gives

$$0 = \frac{1}{2}V_2^2 - (1-0.3)E_s \;\rightarrow\; 0 = \frac{1}{2}V_2^2 - (1-0.3)\left[\frac{10^3}{(1.225)V_2(0.5)}\right] \;\rightarrow\; V_2 = \mathbf{13.2\ m/s}$$

Hence, when the efficiency of the fan is reduced from 100% to 70%, the air velocity induced by the fan is reduced from 14.8 m/s to 13.2 m/s, a reduction of approximately 11%. The difference between the energy input of the fan and the change in mechanical energy of the air as it passes through the fan is due to frictional effects in the airflow.

4.6.3 Unsteady-State Energy Equation

The unsteady-state energy equation can be derived in a similar manner to the steady-state energy equation, with the main difference being that the term accounting for the storage of energy within the control volume is retained. The resulting unsteady-state energy equation can be expressed in the following useful form:

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{cv} e\rho\, d\mathcal{V} + \dot{m}_2\left(h_2 + gz_2 + \alpha_2\frac{V_2^2}{2}\right) - \dot{m}_1\left(h_1 + gz_1 + \alpha_1\frac{V_1^2}{2}\right) \tag{4.167}$$

where e is the energy unit mass as defined by Equation 4.127, "cv" represents the control volume, and $d\mathcal{V}$ is a volume element within the control volume.

EXAMPLE 4.28

The 0.2-m^3 insulated tank shown in Figure 4.40 is to be filled with air from a high-pressure supply line. The tank initially contains air at a temperature of 20°C and an absolute pressure of 120 kPa. The supply line maintains air at a temperature of 15°C and an absolute pressure of 1.8 MPa. At the instant the valve connecting the supply line to the tank is opened, a flow meter measures an airflow rate of 5.08×10^{-4} m^3/s. Estimate the rate of change of temperature in the tank. The velocities in the supply line and the tank are sufficiently small that they may be neglected in the analysis.

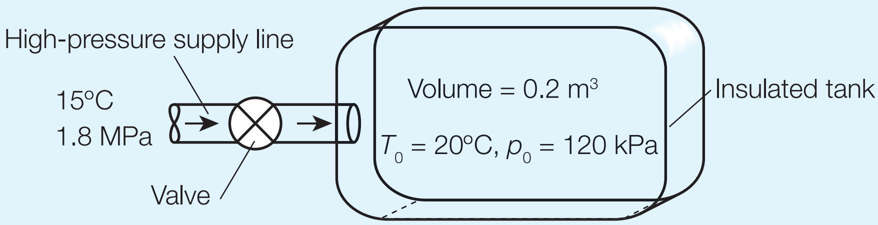

Figure 4.40: Tank connected to a supply line

SOLUTION

From the given data: $\mathcal{V} = 0.2$ m^3, $T_0 = 20°\text{C} = 293$ K, $p_0 = 120$ kPa, $T_1 = 15°\text{C} = 288$ K, $p_1 = 1.8$ MPa, and $Q_0 = 5.08 \times 10^{-4}$ m^3/s. For standard air: $R = 287.1$ J/kg·K and $c_v = 716$ J/kg·K. Applying the energy equation, Equation 4.167, with the tank as the control volume and recalling that $e = V^2/2 + gz + u$ gives

$$\underbrace{\dot{Q}}_{=0} - \underbrace{\dot{W}_\text{s}}_{=0} = \frac{\text{d}}{\text{d}t}\int_\text{cv}\left(\underbrace{\frac{1}{2}V^2}_{\approx 0} + \underbrace{gz}_{=0} + u\right)\rho\,\text{d}\mathcal{V}$$
$$+ \underbrace{\dot{m}_2}_{=0}\left(h_2 + gz_2 + \alpha_2\frac{1}{2}V_2^2\right) - \dot{m}_1\left(h_1 + \underbrace{gz_1}_{=0} + \underbrace{\alpha_1\frac{1}{2}V_1^2}_{\approx 0}\right)$$

where the conditions of an insulated tank ($\dot{Q} = 0$), no shaft work on the air in the tank ($\dot{W}_\text{s} = 0$), negligible velocities in the tank and the supply line ($\frac{1}{2}V^2$ and $\frac{1}{2}V_1^2 \approx 0$), zero datum and negligible gravitational effect (gz and $gz_1 = 0$), and no outflow ($\dot{m}_2 = 0$) are indicated. Taking $h_1 = p_1/\rho_1 + u_1$, the energy equation simplifies to

$$0 = \frac{\text{d}}{\text{d}t}\int_\text{cv} u\rho\,\text{d}\mathcal{V} - \dot{m}\left(\frac{p_1}{\rho_1} + u_1\right) \tag{4.168}$$

where $\dot{m}$ is the mass flow rate into the tank, previously represented by $\dot{m}_1$. Representing the mass of air in the tank as M and noting that $p_1/\rho_1 = RT_1$ and $\text{d}u = c_v\,\text{d}T$, Equation 4.168 gives

$$0 = \frac{\text{d}}{\text{d}t}(uM) - \dot{m}(RT_1 + u_1) \quad\rightarrow\quad 0 = u\frac{\text{d}M}{\text{d}t} + M\frac{\text{d}u}{\text{d}t} - \dot{m}(RT_1 + u_1)$$
$$\rightarrow\quad 0 = u\dot{m} + Mc_v\frac{\text{d}T}{\text{d}t} - \dot{m}(RT_1 + u_1) \quad\rightarrow\quad \frac{\text{d}T}{\text{d}t} = \frac{\dot{m}[RT_1 + u_1 - u]}{Mc_v}$$
$$\rightarrow\quad \frac{\text{d}T}{\text{d}t} = \frac{\dot{m}[RT_1 + c_v(T_1 - T)]}{Mc_v} \tag{4.169}$$