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James H. McClellan
Ronald W. Schafer
Mark A. Yoder

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James H. McClellan
Georgia Institute of Technology

Ronald W. Schafer
Stanford University

Mark A. Yoder
Rose-Hulman Institute of Technology

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Figure 5-4 also shows what happens when the input signal is finite length with a beginning and end. In this case, the input is zero for $\ell < 0$ and for $N \le \ell$ so it can be nonzero only when $0 \le \ell \le N-1$. This possibly nonzero region is called the *support* of the signal $x[n]$. When the input signal has finite support of length N, there is an interval of M samples at the beginning, where the computation involves fewer than $M+1$ nonzero samples as the sliding window of the filter engages with the input, and another interval of M samples at the end where the sliding window of the filter disengages from the input sequence. Furthermore, Fig. 5-4 shows that when the filter runs off the input signal, additional output values are created, so the output sequence can have a support that is as much as M samples longer than the input support. Thus, the length of the output sequence would be $N+M$ samples, where N is the length of the input signal. These general statements about the input and output of an FIR filter are illustrated by the following example.

EXAMPLE 5-1 Pulse Input to 3-Point Running Average

Consider a 3-point running average system expressed in the sliding window form of (5.6)

$$y[n] = \sum_{\ell=n-2}^{n} \tfrac{1}{3} x[\ell]$$

with input

$$x[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$$

as plotted in Fig. 5-5(a). We refer to this $x[n]$ as a "pulse input" because it is nonzero and constant over a short interval of length 11. The shaded regions in Fig. 5-5(a) highlight

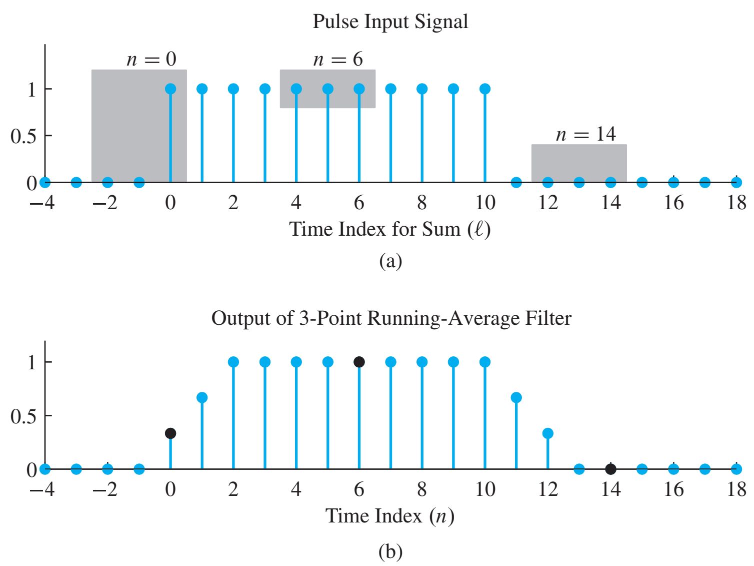

Figure 5-5 Computing the output of 3-point running-average filter. (a) Input signal $x[n]$ is a length-11 pulse. Shown in gray are three positions of the length-3 averaging interval which is the sliding window of this FIR filter. (b) Corresponding output of 3-point running averager with values at $n = 0, 6, 13$ highlighted (black dots).

the three samples of $x[\ell]$ involved in the computation of the output at $n = 0$, $n = 6$, and $n = 14$. These specific cases illustrate how the causal *sliding window* interval of length three scans across the input sequence moving sample-by-sample to produce the output signal. Note the three highlighted samples of the output in Fig. 5-5(b) which correspond to the three positions of the averaging interval in Fig. 5-5(a). Finally, note that the output is zero when $n < 0$ and $n > 12$ because for these values of n, the averaging interval includes only zero values of the input. Also note that the output for $n = 0, 1, 2$ follows a straight line from 0 to 1 because the averaging interval includes an additional unit sample as it moves to the right over the interval $0 \le n \le 2$. A similar taper occurs at the end of the pulse interval as the averaging interval includes fewer nonzero samples as it moves to the right. The output is constant and equal to one in the interval $2 \le n \le 10$, where the averaging interval includes three samples of $x[n]$ where $x[n] = 1$.

Example 5-1 illustrates the previous assertion about the support of the input and output signals: If $x[\ell]$ is nonzero only in the interval $0 \le \ell \le N - 1$, then for a causal FIR filter having $M + 1$ coefficients $\{b_k\}$ for $k = 0, 1, \ldots, M$, the corresponding output $y[n]$ can be nonzero only when $n \ge 0$ and $n - M \le N - 1$. Thus the support of the output signal is the interval $0 \le n \le N + M - 1$, and the length of the output support is $N + M$ samples. This is true regardless of the values of the $M + 1$ coefficients. In Example 5-1, $N = 11$ and $M = 2$ so the output is nonzero in the interval $0 \le n \le 12$ and the length of the output sequence is 13 samples. Problem P-5.6 considers the general case where the signal $x[n]$ begins at $n = N_1$ and ends at $n = N_2$.

EXAMPLE 5-2 FIR Filter Coefficients

In general, the FIR filter is completely defined once the set of filter coefficients $\{b_k\}$ is known. For example, if the coefficients of a causal filter are

$$\{b_k\} = \{3,\ -1,\ 2,\ 1\}$$

then we have a length-4 filter with $M = 3$, and (5.5) expands into a 4-point difference equation:

$$y[n] = \sum_{k=0}^{3} b_k x[n-k] = 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

The parameter M is the ***order*** of the FIR filter. The number of filter coefficients is also called the filter ***length*** (L). Usually, the length is one greater than the order, i.e., $L = M+1$. The terminology "order" will make more sense after we have introduced the z-transform in Chapter 9.

EXERCISE 5.2

Compute the output $y[n]$ for the length-4 filter whose coefficients are $\{b_k\} = \{3,\ -1,\ 2,\ 1\}$. Use the input signal given in Fig. 5-2(a). Verify that the partial answer tabulated here is correct, then fill in the missing values.

n	$n<0$	0	1	2	3	4	5	6	7	8	$n>8$
$x[n]$	0	2	4	6	4	2	0	0	0	0	0
$y[n]$	0	6	10	18	?	?	?	8	2	0	0

5-3.1 An Illustration of FIR Filtering

To illustrate some of the things that we have learned so far, and to show how FIR filters can modify sequences that vary in interesting ways, consider the signal

$$x[n] = \begin{cases} (1.02)^n + \frac{1}{2}\cos(2\pi n/8 + \pi/4) & 0 \le n \le 40 \\ 0 & \text{otherwise} \end{cases}$$

This signal, which is the sum of two components, is shown as the orange dots in Fig. 5-6(a). We often have real signals of this form; where one component is the signal of interest which, in this case, is the slowly varying exponential component $(1.02)^n$ and the other component is often considered *noise* or *interference* that degrades the observation of the desired signal. In this case, we consider the sinusoidal component $\frac{1}{2}\cos(2\pi n/8+\pi/4)$ to be an interfering signal that we wish to remove. The solid, exponentially growing curve shown superimposed in each of the plots in Fig. 5-6 simply connects the sample values of the desired signal $(1.02)^n$ by straight lines for reference in all three plots.

Now suppose that $x[n]$ is the input to a causal 3-point running averager, that is,

$$y_3[n] = \sum_{k=0}^{2} \tfrac{1}{3} x[n-k] \tag{5.7}$$

In this case, $M = 2$ and all the coefficients are equal to $1/3$. The output of this filter is shown in Fig. 5-6(b). We can notice several things about these plots.

(a) Observe that the input sequence $x[n]$ is zero prior to $n = 0$, and from (5.7) it follows that the output must be zero for $n < 0$.

(b) The output becomes nonzero at $n = 0$, and the shaded interval[9] of length $M = 2$ samples at the beginning of the nonzero part of the output sequence is the interval where the 3-point averager "runs onto" the input sequence. For $2 \le n \le 40$, the input samples within the 3-point averaging window are all nonzero.

[9]In Example 5-1, shading was used to highlight the moving averaging interval relative to the input sequence. In Fig. 5-6(b) and (c) shading is used on the output sequence to highlight the fixed intervals at the beginning and end of the output where the averaging interval engages with and disengages from the finite-length input sequence.

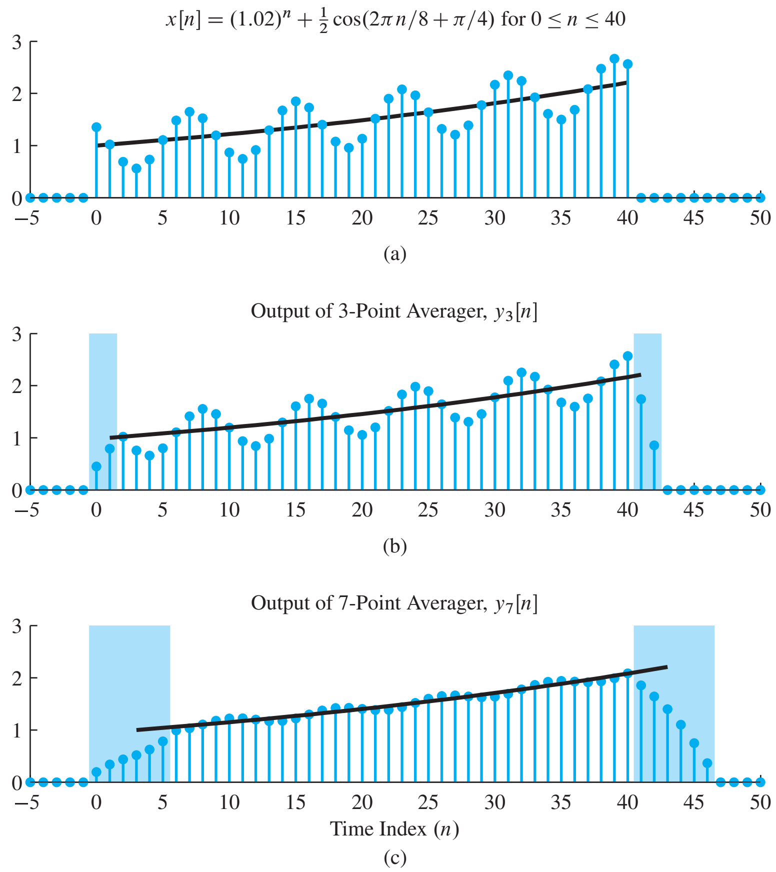

Figure 5-6 Illustration of running-average filtering. (a) Input signal; (b) output of 3-point averager; (c) output of 7-point averager.

(c) There is another shaded interval of length $M = 2$ samples at the end (after sample 40), where the filter window "runs off of" the input sequence.

(d) Observe that the size of the sinusoidal component in $y_3[n]$ has been reduced slightly, but that the component is not eliminated by the filter. The solid line showing the values of the exponential component has been shifted to the right by $M/2 = 1$ sample to account for the shift introduced by the causal filter.

The 3-point running averager has barely reduced the size of the fluctuations in the input signal, so we have not recovered the desired component. Intuitively, we might think that averaging over a longer interval might produce better results. The plot in Fig. 5-6(c) shows the output of a 7-point running averager as defined by

$$y_7[n] = \sum_{k=0}^{6} \tfrac{1}{7} x[n-k] \tag{5.8}$$

In this case, since $M = 6$ and all the coefficients are equal to $1/7$, we observe the following:

(a) The shaded regions at the beginning and end of the output are now $M = 6$ samples long.

(b) Now the size of the sinusoidal component is greatly reduced relative to the input sinusoid, and the component of the output is very close to the exponential component of the input (after a shift of $M/2 = 3$ samples).

What can we conclude from this example? First, it appears that FIR filtering can modify signals in ways that may be useful. Second, the length of the averaging interval seems to have a big effect on the resulting output. Third, the running-average filters appear to introduce a shift equal to $M/2$ samples. All of these observations can be shown to apply to more general FIR filters defined by (5.5). However, before we can fully appreciate the details of this example, we must explore the properties of FIR filters in greater detail. We will gain full appreciation of this example only upon the completion of Chapter 6.

EXERCISE 5.3

Use MATLAB to implement an 8-point averager and process the composite signal in Fig. 5-6(a). The output should have no sinusoidal interference in the region where the length-8 sliding window completely overlaps the input signal. Verify that this is true in the MATLAB output and then explain why this is true by considering the effect of the 8-point averager on the sinusoid alone.

5-4 The Unit Impulse Response and Convolution

In this section, we introduce three new ideas: the unit impulse sequence, the unit impulse response, and the convolution sum. We show that the *impulse response provides a complete characterization* of the FIR filter, because the convolution sum gives a formula for computing the output from the input when the unit impulse response is known.

5-4.1 Unit Impulse Sequence

The ***unit impulse*** is perhaps the simplest sequence because it has only one nonzero value, which occurs at $n = 0$. The mathematical notation is that of the Kronecker ***delta function***

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{5.9}$$

It is tabulated in the second row of this table:

n	...	−2	−1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	**1**	0	0	0	0	0	0	0
$\delta[n-2]$	0	0	0	0	0	**1**	0	0	0	0	0

A shifted impulse such as $\delta[n-2]$ is nonzero when its argument is zero, that is, when $n - 2 = 0$, or equivalently $n = 2$. The third row of the previous table gives the values of the shifted impulse $\delta[n-2]$, and Fig. 5-7 shows a plot of that sequence.

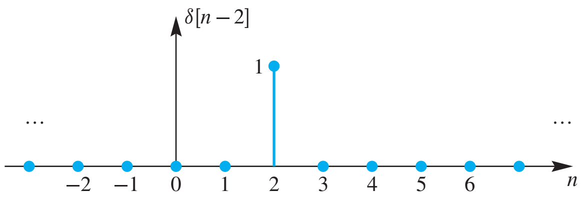

Figure 5-7 Shifted impulse sequence, $\delta[n-2]$.

The shifted impulse is a concept that is very useful in representing signals and systems. For example, we can show that the formula

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4] \tag{5.10}$$

is equivalent to defining $x[n]$ by tabulating its five nonzero values. To interpret (5.10), we must observe that the appropriate definition of multiplying a sequence by a number is to multiply each value of the sequence by that number; likewise, adding two or more sequences is defined as adding the sequence values at corresponding positions (times). The following table shows the individual sequences in (5.10) and their sum:

n	...	−2	−1	0	1	2	3	4	5	6	...
$2\delta[n]$	0	0	0	**2**	0	0	0	0	0	0	0
$4\delta[n-1]$	0	0	0	0	**4**	0	0	0	0	0	0
$6\delta[n-2]$	0	0	0	0	0	**6**	0	0	0	0	0
$4\delta[n-3]$	0	0	0	0	0	0	**4**	0	0	0	0
$2\delta[n-4]$	0	0	0	0	0	0	0	**2**	0	0	0
$x[n]$	0	0	0	**2**	**4**	**6**	**4**	**2**	0	0	0

Equation (5.10) is a compact mathematical representation of the signal in Fig. 5-2(a). It turns out that any sequence can be represented in this way. The equation

$$\begin{aligned} x[n] &= \sum_k x[k]\delta[n-k] \\ &= \cdots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \cdots \end{aligned} \tag{5.11}$$

is true if k ranges over all the nonzero values of the sequence $x[n]$. Equation (5.11) states the obvious: The sequence is formed by using scaled shifted impulses to place samples of the right size at the correct index positions.

5-4.2 Unit Impulse Response Sequence

The output from a filter is often called the *response* to the input, so when the input is the unit impulse, $\delta[n]$, the output is called the ***unit impulse response***.[10] We reserve the notation $h[n]$ for this special output signal. To emphasize the impulse response in

[10]We usually shorten this to ***impulse response***, with *unit* being understood.