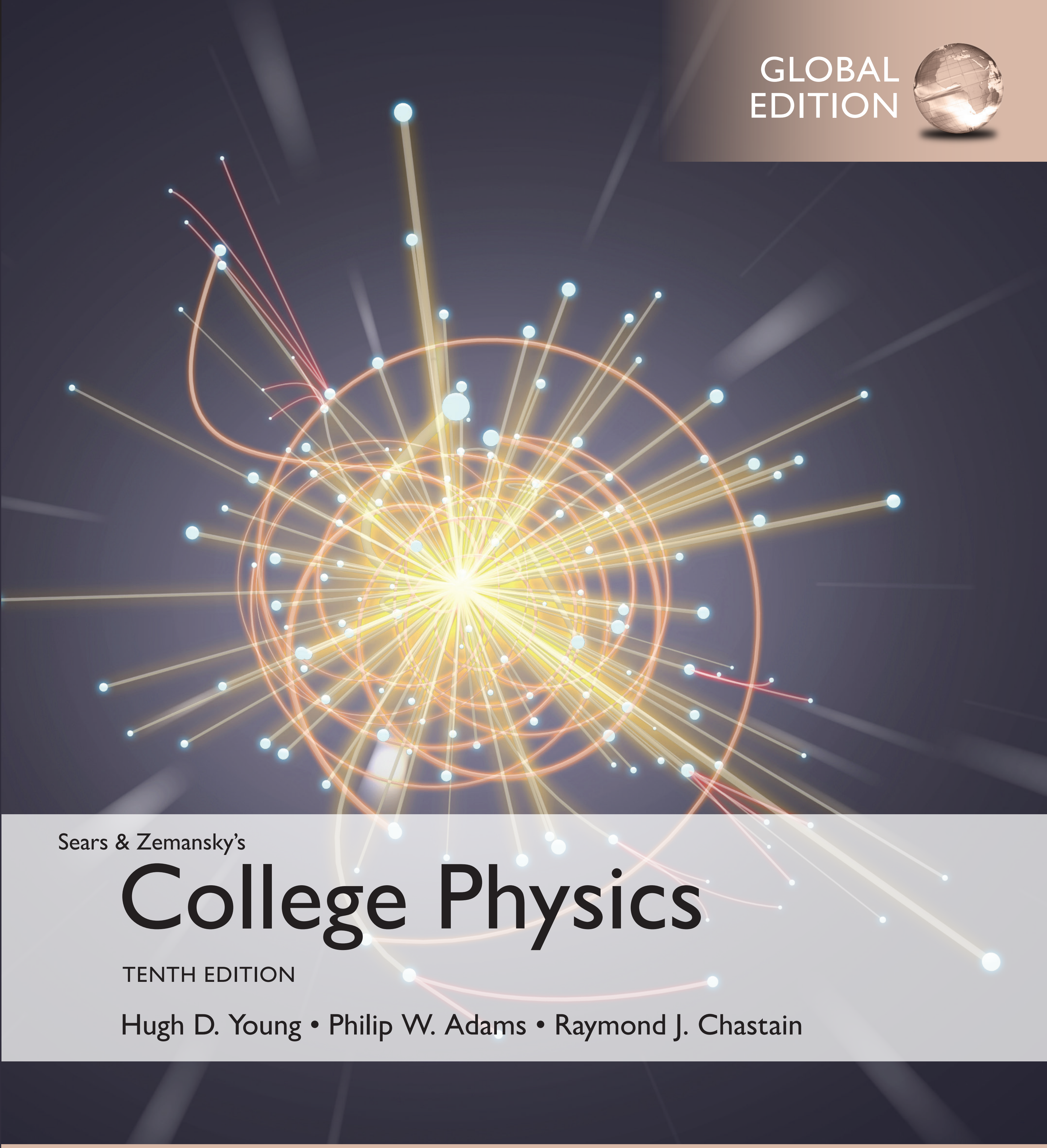

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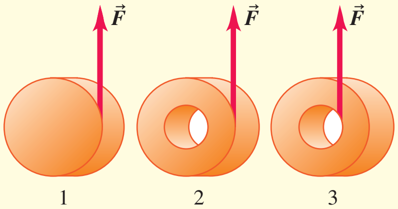
CONCEPTUAL ANALYSIS 10.2

Rotating cylinders

Three cylinders, each with a total mass of M and an outer radius of R, are shown in Figure 10.9. In cylinder 1, the mass is uniformly distributed. Cylinders 2 and 3 are identical hollow cylinders, with an inner radius of $R/2$. All three cylinders can rotate about their centers of mass. A force with a magnitude of F acts on cylinders 1 and 2 at the outer radius and on cylinder 3 at its inner radius. Which choice correctly ranks the magnitudes of the angular accelerations of the three cylinders, from smallest to largest?

A. $\alpha_3 > \alpha_2 > \alpha_1$ B. $\alpha_1 > \alpha_2 > \alpha_3$
C. $\alpha_2 > \alpha_1 > \alpha_3$ D. $\alpha_1 = \alpha_2 > \alpha_3$



▲ **FIGURE 10.9**

SOLUTION For this situation, we know that the magnitude of the angular acceleration of each cylinder about its center of mass is the ratio of the magnitude of the torque produced by the force F to the moment of inertia of each cylinder about its center. For cylinders 2 and 3, where the hollow cavity means that more of the mass of the cylinder has been pushed away from the axis of rotation, the moment of inertia must be larger than for the uniform cylinder. We also know that the torque produced by F is the same for cylinders 1 and 2, but only half as much for cylinder 3 because the moment arm is half as long. Cylinder 1, which has the largest torque and the smallest moment of inertia, must have the largest angular acceleration. Cylinder 3, which has the same moment of inertia and a smaller torque than cylinder 2, must have the smaller angular acceleration. Therefore, B is the correct answer.

PROBLEM-SOLVING STRATEGY 10.1 **Rotational dynamics**

Our strategy for solving problems in rotational dynamics is similar to Problem-Solving Strategy 5.2 on the use of Newton's second law.

SET UP

1. Sketch the situation and select a body or bodies to analyze. You will apply $\sum\vec{F} = m\vec{a}$ or $\sum\tau = I\alpha$, or sometimes both, to each object.
2. Draw a free-body diagram for each body. Because we're now dealing with extended bodies rather than objects that can be treated as particles, your diagram must show the shape of the body accurately, with all dimensions and angles that you'll need for calculations of torque. A body may have translational motion, rotational motion, or both. As always, include all the forces that act on each body (and no others), including the body's weight (taken to act at the center of mass).
3. Choose coordinate axes for each body. Remember that, by convention, the positive sense of rotation is counterclockwise; this is the positive sense for angular position, angular velocity, angular acceleration, and torque.
4. If more than one object is involved, carry out the preceding steps for each object. There may be geometric relationships between the motions of two or more objects or between the translational and rotational motions of the same object. Express these in algebraic form, usually as relationships between two linear accelerations or between a linear acceleration and an angular acceleration.

SOLVE

5. Write a separate equation of motion ($\sum\vec{F} = m\vec{a}$ or $\sum\tau = I\alpha$, or both) for each body, including separate equations for the x and y components of forces, if necessary. Then solve the equations to find the unknown quantities. This may involve solving a set of simultaneous equations.

REFLECT

6. Check particular cases or extreme values of quantities when possible, and compare the results for these particular cases or values with your intuitive expectations. Ask yourself whether the result makes sense.

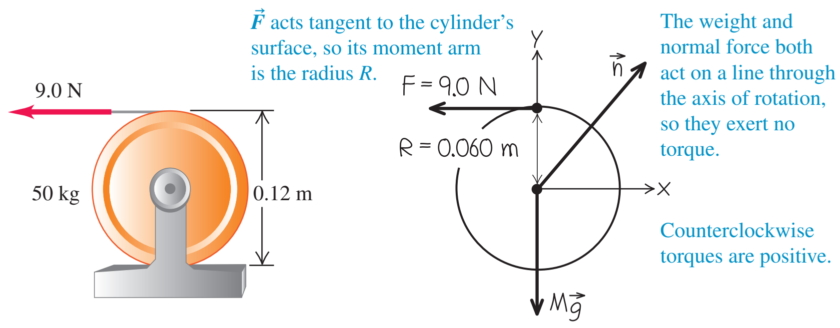
▲ **Application**
Don't fall for it.

To stay balanced while walking the tightrope, this acrobat must keep her center of mass directly above the rope. If the center of mass shifts to one side, gravity will exert a torque on the acrobat, tending to cause a rotation about the rope—a fall. To stay balanced, the acrobat must create a countertorque that has equal magnitude and opposite sign. That is why tightrope walkers carry a long pole. Because the torque due to a force is the product of the magnitude of the force and the moment arm, the gravitational forces on the ends of the pole exert significant torques. Thus, the acrobat controls the net torque acting on her center of gravity by manipulating the pole. Also, the acrobat holds the pole low, which moves her center of gravity closer to the rope.

EXAMPLE 10.2 Unwinding a winch (again)

Video Tutor Solution

Figure 10.10a shows the same situation that we analyzed in Example 9.7. A cable is wrapped several times around a uniform solid cylinder with diameter 0.12 m and mass 50 kg that can rotate freely about its axis. The cable is pulled by a force with magnitude 9.0 N. Assuming that the cable unwinds without stretching or slipping, find the magnitude of its acceleration.



(a) **(b)**

▲ **FIGURE 10.10**

SOLUTION

SET UP We draw a free-body diagram for the cylinder (Figure 10.10b). Because the force acts tangent to the outside of the cylinder, its moment arm relative to the axis of rotation is the cylinder's radius.

SOLVE The torque is $\tau = Fl = (9.0\ \text{N})(0.060\ \text{m}) = 0.54\ \text{N}\cdot\text{m}$. The moment of inertia is

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(50\ \text{kg})(0.060\ \text{m})^2 = 0.090\ \text{kg}\cdot\text{m}^2.$$

The angular acceleration α is given by Equation 10.5:

$$\alpha = \frac{\tau}{I} = \frac{0.54\ \text{N}\cdot\text{m}}{0.090\ \text{kg}\cdot\text{m}^2} = 6.0\ \text{rad/s}^2.$$

The magnitude a of the acceleration of the cable is given by Equation 9.14:

$$a = R\alpha = (0.060\ \text{m})(6.0\ \text{rad/s}^2) = 0.36\ \text{m/s}^2.$$

REFLECT In the equation $\alpha = \tau/I$, check the units to make sure they are consistent.

Practice Problem: To wind the winch back up, you exert a force that points straight down on its rightmost edge. What is the magnitude of the force you need to exert if you wind the winch with an angular acceleration of $5.0\ \text{rad/s}^2$? *Answer:* 7.5 N.

EXAMPLE 10.3 Dynamics of a bucket in a well

Video Tutor Solution

Now let's go back to the old-fashioned well in Example 9.8. In that example we were limited to a conservation-of-energy analysis of the system, which gave us only the speeds of the bucket and the winch at certain points in the motion. Now we can use Newton's second law for rotation to find the acceleration of the bucket (mass m) and the angular acceleration of the winch cylinder.

SOLUTION

SET UP We start by sketching the situation, using the information from Example 9.8 (Figure 10.11a). As before, the winch cylinder has mass M and radius R; the bucket has mass m and falls a distance h. We must treat the two objects separately, so we draw a free-body diagram for each (Figure 10.11b). We take the positive direction of the y coordinate for the bucket to be downward; the positive sense of rotation for the winch is counterclockwise. We assume that the rope is massless and that it unwinds without slipping.

SOLVE Newton's second law applied to the bucket (mass m) gives

$$\sum F_y = mg - T = ma_y.$$

The forces on the cylinder are its weight Mg, the upward normal force n exerted by the axle, and the rope tension T. The first two forces act along lines through the axis of rotation and thus have no torque with respect to that axis. The moment arm of the force T on the cylinder is R, and the torque is $\tau = TR$. Applying Equation 10.5 and the expression for the moment of inertia of a solid cylinder from Table 9.2 to the winch cylinder, we find that

$$\sum\tau = I\alpha \quad \text{and} \quad TR = \tfrac{1}{2}MR^2\alpha.$$

As in Example 10.2, the angular acceleration α of the cylinder is proportional to the magnitude of acceleration a_y of the unwinding rope:

CONTINUED

$a_y = R\alpha$. We use this equation to replace $(R\alpha)$ with a_y in the cylinder equation, and then we divide by R. The result is

$$T = \tfrac{1}{2}Ma_y.$$

Now we substitute this expression for T into the equation of motion for the bucket (mass m):

$$mg - \tfrac{1}{2}Ma_y = ma_y.$$

When we solve this equation for a_y, we get

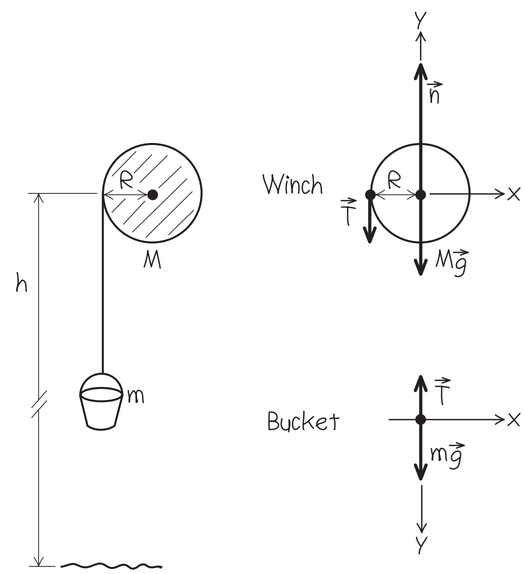
$$a_y = \frac{g}{1 + M/2m}.$$

Combining this result with the kinematic equation $a_y = R\alpha$, we obtain the angular acceleration α :

$$\alpha = \frac{g/R}{1 + M/2m}.$$

Finally, we can substitute the equation for a_y back into the equation $\sum\vec{F} = m\vec{a}$ for m to get an expression for the tension T in the rope:

$$T = mg - ma_y = mg - m\frac{g}{1 + M/2m} \quad \text{and} \quad T = \frac{mg}{1 + 2m/M}.$$



(a) Diagram of situation **(b)** Free-body diagrams

▲ **FIGURE 10.11** Our sketches for this problem.

REFLECT First, note that the tension in the rope is *less than* the weight mg of the bucket; if the two forces were equal (in magnitude), the bucket wouldn't accelerate downward.

We can check two particular cases. When M is much larger than m (a massive winch and a little bucket), the ratio m/M is much smaller than unity. Then the tension is nearly equal to mg, and the acceleration a_y is correspondingly much *less* than g. When M is zero, $T = 0$ and $a = g$; the bucket then falls freely. If it starts from rest at a height h above the water, its speed v when it strikes the water is given by $v^2 = 2a_yh$; thus,

$$v = \sqrt{2a_yh} = \sqrt{\frac{2gh}{1 + M/2m}}.$$

This is the same result that we obtained using energy considerations in Example 9.8. But note that there we couldn't find the tension in the rope or the accelerations of the winch cylinder and bucket.

Practice Problem: (a) What is the relationship between m and M if the bucket's acceleration is half the acceleration of free fall? (b) In this case, what is the rope tension T? *Answers:* (a) $m = M/2$, (b) $T = mg/2 = Mg/4$.

NOTE: In problems such as Example 10.3, in which an object is suspended by a rope, you may be tempted to assume at the start that the tension in the rope is equal to the object's weight. If the object is in equilibrium, this may be true, but when the object accelerates, the tension is usually *not* equal to the object's weight. Be careful!

Rotation about a moving axis

We have already discussed the kinetic energy of a rigid body rotating about a moving axis (in Section 9.5). We can extend our analysis of the dynamics of rotational motion to some cases in which the axis of rotation moves. In these cases, the body has both translational and rotational motions. Every possible motion of a rigid body can be represented as a combination of translational motion of the center of mass and rotation about an axis through the center of mass. A rolling wheel or ball and a yo-yo unwinding at the end of a string are familiar examples of such motion.

The key to this more general analysis, which we won't derive in detail, is that Equation 10.5 $(\sum\tau = I\alpha)$ is valid *even when the axis of rotation moves* if the following two conditions are met: (1) The axis must be an axis of symmetry and must pass through the center of mass of the body; (2) the axis must not change direction. The moment of inertia must be computed with respect to this center-of-mass axis; to emphasize this point, we'll denote it as I_{cm}.

PROBLEM-SOLVING STRATEGY 10.2 **Rotation about a moving axis**

The problem-solving strategy outlined in Section 10.2 is equally useful here. There is one new wrinkle: When a rigid body undergoes translational and rotational motions at the same time, we need two separate equations of motion *for the same body*. (The situation is reminiscent of our analysis of projectile motion in Chapter 3, where we used separate equations for the x and y coordinates of a projectile.)

For a rigid body, one of the equations of motion is based on $\sum \vec{F} = m\vec{a}$ for the translational motion of the center of mass of the body. We showed (in Equation 8.23) that for a body with total mass M, the acceleration $\vec{a}_{cm}$ of the center of mass is the same as that of a point mass M acted on by all the forces that act on the actual body. The other equation of motion is based on $\sum \tau = I_{cm}\alpha$ for the rotational motion of the body about the axis through the center of mass with moment of inertia I_{cm}. In addition, there is often a geometric relationship between the linear and angular accelerations, such as when a wheel rolls without slipping or a string rotates a pulley while passing over it.

EXAMPLE 10.4 Dynamics of a primitive yo-yo

In our first moving-axis example we will consider a yo-yo (like the one we analyzed in Example 9.9). As shown in Figure 10.12a, the yo-yo consists of a solid disk with radius R and mass M. In our earlier analysis, we used energy considerations to find the yo-yo's speed after it had dropped a certain distance. Now let's find the acceleration of the yo-yo and the tension in the string. Note that these quantities cannot be calculated using the conservation-of-energy analysis alone.

Video Tutor Solution

SOLUTION

SET UP We start with a free-body diagram, including a coordinate system (Figure 10.12b). If the string unwinds without slipping, the linear displacement of the center of mass of the cylindrical yo-yo in any time interval equals R times its angular displacement. This gives us the kinematic relationships $v_{cm,y} = R\omega$ and $a_{cm,y} = R\alpha$. (If this scenario isn't obvious, imagine moving along with the center of mass of the cylinder and watching the string unwind. From that point of view, the kinematic situation is just the same as in Examples 10.2 and 10.3.)

SOLVE The equation for the translational motion of the center of mass is

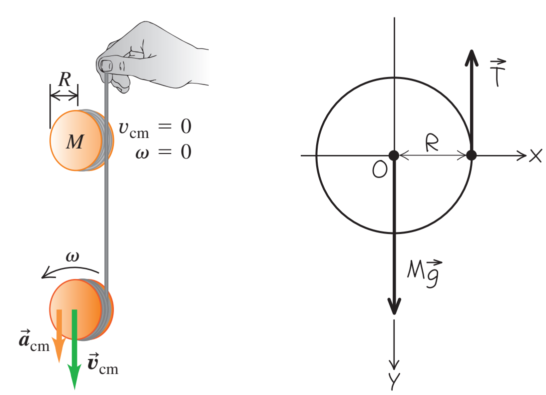
$$\sum F_y = Mg - T = Ma_{cm,y}.$$

The moment of inertia for an axis through the center of mass is $I_{cm} = \frac{1}{2}MR^2$, and the equation for rotational motion about the axis through the center of mass is

$$\sum \tau = TR = I_{cm}\alpha = \tfrac{1}{2}MR^2\alpha.$$

We can now combine this equation with the relationship $a_{cm,y} = R\alpha$ to eliminate α, obtaining $TR = \frac{1}{2}MRa_{cm,y}$, or $T = \frac{1}{2}Ma_{cm,y}$. Finally, we combine this result with the equation for the center-of-mass motion ($\sum F_y = Mg - T = Ma_{cm,y}$) to obtain expressions for T and $a_{cm,y}$. The results are amazingly simple:

$$a_{cm,y} = \tfrac{2}{3}g, \qquad T = \tfrac{1}{3}Mg.$$



(a) The yo-yo **(b)** Our free-body diagram

▲ **FIGURE 10.12**

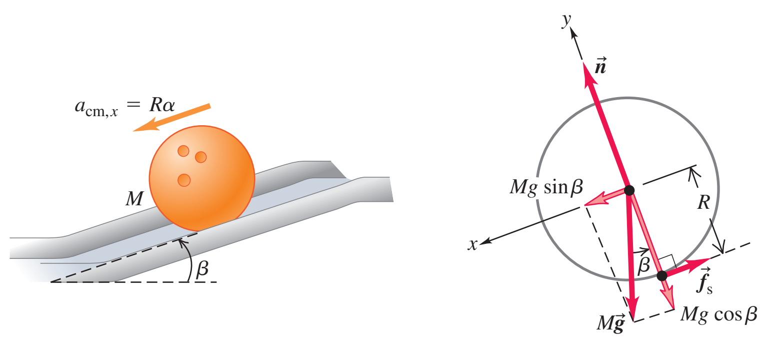
REFLECT The acceleration and tension are both proportional to g, as we should expect. The acceleration is independent of M and R, and the mass cancels out of the dynamic equations. For a yo-yo in the shape of a solid disk, the acceleration is exactly two-thirds the acceleration of free fall.

Practice Problem: If the solid cylinder is replaced by a thin cylindrical shell with the same mass and radius as before, find $a_{cm,y}$ and T. *Answers:* $a_{cm,y} = g/2$, $T = Mg/2$.

EXAMPLE 10.5 A rolling bowling ball

Now we will look at one of the most important examples of rotation about a moving axis. A bowling ball rolls without slipping down the return ramp at the side of the alley (Figure 10.13a). The ramp is inclined at an angle β to the horizontal. What is the ball's acceleration? What is the friction force acting on the ball? Treat the ball as a uniform solid sphere, ignoring the finger holes.

Video Tutor Solution



(a) (b)

▲ **FIGURE 10.13**

SOLUTION

SET UP Again, let's start with a free-body diagram, with the positive coordinate directions indicated (Figure 10.13b). Referring back to Table 9.2, we see that the moment of inertia of a solid sphere about an axis through its center of mass is $I_{cm} = \frac{2}{5}MR^2$. Because the ball rolls without slipping, the acceleration $a_{cm,x}$ of the center of mass is proportional to the ball's angular acceleration α ($a_{cm,x} = R\alpha$). We've represented the forces in terms of their components; it's most convenient to take the axes as parallel and perpendicular to the sloping ramp. The contact force of the ramp on the ball is represented in terms of its normal (n) and frictional (f_s) components. Note that f_s is a *static*-friction force because the ball doesn't slip on the ramp.

SOLVE The equations of motion for translational motion and for rotation about the axis through the center of mass are:

$$\text{Translation: } \sum F_x = Mg\sin\beta - f_s = Ma_{cm,x},$$
$$\text{Rotation: } \sum \tau = f_sR = I_{cm}\alpha = \left(\tfrac{2}{5}MR^2\right)\alpha.$$

We eliminate α from the second equation by using the kinematic relationship $a_{cm} = R\alpha$. Then we express I_{cm} in terms of M and R and divide through by R:

$$f_sR = I_{cm}\alpha = \left(\tfrac{2}{5}MR^2\right)\left(\frac{a_{cm,x}}{R}\right) \quad \text{and} \quad f_s = \tfrac{2}{5}Ma_{cm,x}.$$

Next, we insert this expression for f_s into the first equation above and solve for $a_{cm,x}$:

$$Mg\sin\beta - \tfrac{2}{5}Ma_{cm,x} = Ma_{cm,x},$$
$$a_{cm,x} = \tfrac{5}{7}g\sin\beta.$$

Finally, we substitute the right-hand side of this result back into the expression for f_s to obtain

$$f_s = \tfrac{2}{7}Mg\sin\beta.$$

REFLECT The acceleration is $\frac{5}{7}$ as large as it would be if the ball could *slide* without friction down the slope (like the toboggan in Example 5.12). That is why the friction force f_s is a static-friction force; it is needed to prevent slipping and to give the ball its angular acceleration. We can derive an expression for the minimum coefficient of static friction μ_s needed to prevent slipping. The magnitude of the normal force is $n = Mg\cos\beta$. To prevent slipping, the coefficient of friction must be at least as great as

$$\mu_s = \frac{\frac{2}{7}Mg\sin\beta}{Mg\cos\beta} = \tfrac{2}{7}\tan\beta.$$

If the plane is tilted only a little, β and $\tan\beta$ are small, and only a small value of μ_s is needed to prevent slipping. But as the angle increases, the required value of μ_s also increases, as we would expect intuitively.

Finally, suppose μ_s is *not* large enough to prevent slipping. Then we have a whole new ball game, so to speak. Our basic dynamic equations are still valid, but now the ball slides and rotates at the same time. There is no longer a definite relationship between $a_{cm,x}$ and α, and the friction force is now a *kinetic*-friction force, given by $f_k = \mu_k n = \mu_k Mg\cos\beta$. The acceleration of the center of mass is then

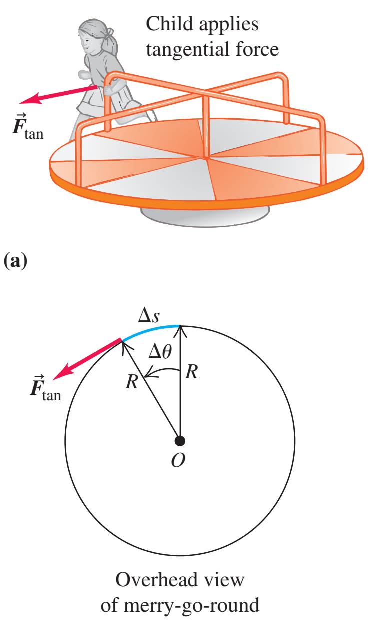
$$a_{cm,x} = g(\sin\beta - \mu_k\cos\beta).$$

(Note that this is the same result we found in Example 5.12, in which we hoped that the toboggan would slide, and not roll, down the hill.)

Practice Problem: Suppose we replace the bowling ball with a solid cylinder. Determine the acceleration of the cylinder and the minimum coefficient of static friction if the cylinder rolls down the ramp without slipping. *Answers:* $a_{cm,x} = \frac{2}{3}g\sin\beta$; $\mu_s = \frac{1}{3}\tan\beta$.

▲ **BIO Application**
Sensing angular acceleration.

How do we detect angular movement so that we can perform complex activities such as tumbling and diving? The mammalian inner ear has three semicircular canals filled with a viscous fluid. The canals are oriented such that they represent each of the three spatial axes. In response to angular acceleration, the canals and the sensory cells they contain rotate relative to the internal fluid, and this motion is detected by the cells and converted to electrical signals that are sent to the brain. Long before humans understood them, physical laws shaped evolution's design of life.



▲ **FIGURE 10.14** The work done by a tangential force acting on a body.

10.3 Work and Power in Rotational Motion

When you pedal a bicycle, you apply forces to a rotating body and do work on it. Similar things happen in many other real-life situations, such as a rotating motor shaft driving a power tool or a car engine propelling the vehicle. We can express this work in terms of torque and angular displacement.

Suppose a tangential force with constant magnitude F_{tan} acts at the rim of a merry-go-round platform with radius R (Figure 10.14) while the platform rotates through an angle $\Delta\theta$ about a fixed axis during a small time interval Δt. The displacement of the point at which the force acts is Δs. By definition, the work ΔW done by the force F_{tan} is $\Delta W = F_{tan}\Delta s$. If θ is measured in radians, $\Delta s = R\,\Delta\theta$, and it follows that

$$\Delta W = F_{tan} R\,\Delta\theta.$$

Now, $F_{tan}R$ is the *torque* τ due to the force F_{tan}, so

$$\Delta W = \tau\,\Delta\theta. \tag{10.6}$$

If the torque is *constant* while the angle changes by a finite amount $\Delta\theta = \theta_2 - \theta_1$, then the work done on the body by the torque is as follows:

Work done by a constant torque

The work done on a body by a constant torque equals the product of the torque and the angular displacement of the body:

$$W = \tau(\theta_2 - \theta_1) = \tau\,\Delta\theta. \tag{10.7}$$

Units: W is in joules, τ is in newton-meters (N · m), and θ is in radians.
Notes:

- This equation is the rotational analog of $W = F_{\|}s$ (Equation 7.2) for the work done by a constant force with component $F_{\|}$ in the direction of displacement.
- $\Delta\theta = \theta_f - \theta_i$.

The force in Figure 10.14b has no radial component. If there were such a component, it would do no work because the displacement of the point of application has no radial component. A radial component of force would also make no contribution to the torque about the axis of rotation, so Equations 10.6 and 10.7 are correct for any force, even if it does have a radial component. The same argument applies to an *axial* component of force—that is, a component parallel to the axis of rotation.

What about the *power* associated with work done by a torque acting on a rotating body? When we divide both sides of Equation 10.7 by the time interval Δt during which the angular displacement occurs, we find that

$$\frac{\Delta W}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t}.$$

But $\Delta W/\Delta t$ is the rate of doing work, or *power* P, and $\Delta\theta/\Delta t$ is the body's angular velocity ω, so we obtain the following relationship for power in rotational motion:

Power due to a constant torque

$$P = \tau\omega. \tag{10.8}$$

Units: P is in watts, τ is in newton-meters, and ω is in rad/s.