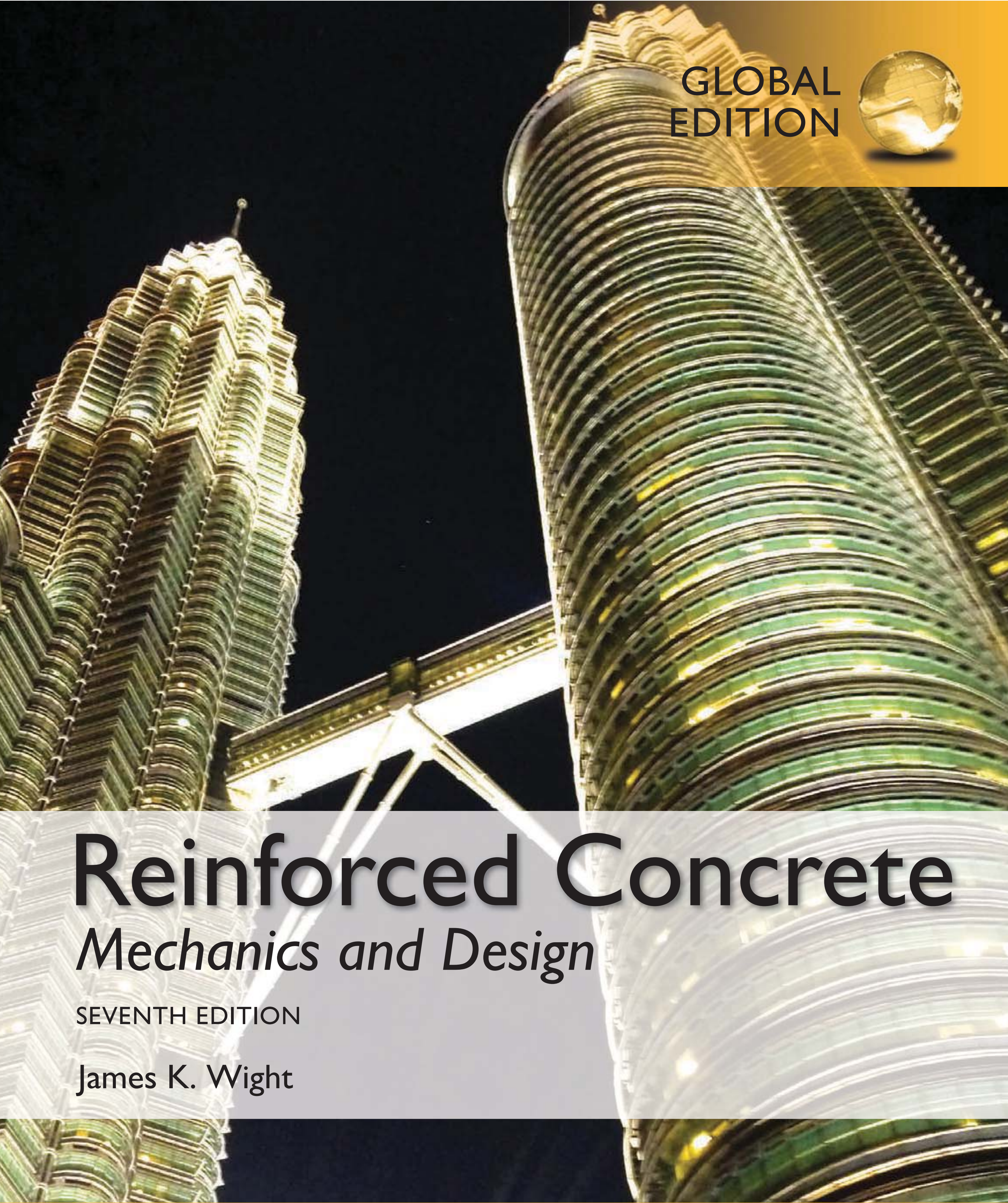

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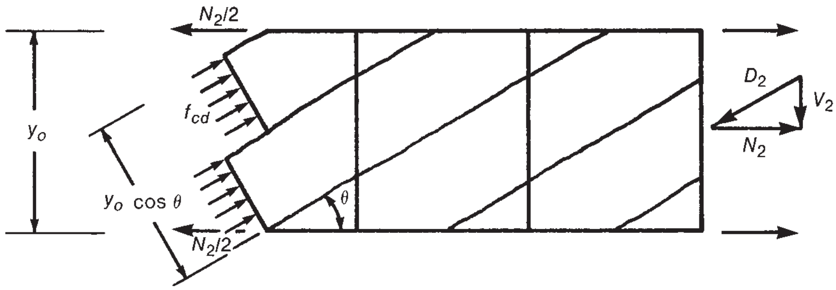


Fig. 7-27
Side of space truss—replacement of shear force V_2.

force V_2 can be replaced with a diagonal compression force, D_2, parallel to the concrete struts and an axial tension force, N_2, where D_2 and N_2 are respectively given by

$$D_2 = \frac{V_2}{\sin\theta} \tag{7-31}$$

and

$$N_2 = V_2 \cot\theta \tag{7-32}$$

Because the shear flow, q, is constant from point to point along side 2, the force N_2 acts along the centroidal axis of side 2. For a beam with longitudinal bars in the top and bottom corners of side 2, half of N_2 will be resisted by each corner bar. A similar resolution of forces occurs on each side of the truss (e.g. $N_1 = V_1 \cot\theta$). For a rectangular member, as shown in Fig. 7-20b, the total longitudinal force is

$$N = 2(N_1 + N_2) \tag{7-33}$$

Substituting Eqs. (7-25a and 7-25b) into (7-33) and taking T equal to T_n gives

$$N = \frac{T_n}{2A_o} 2(x_o + y_o)\cot\theta \tag{7-34}$$

where $2(x_o + y_o)$ is approximately equal to the perimeter of the closed stirrup, p_h. Longitudinal reinforcement with a total area of A_ℓ must be provided for the longitudinal force, N. Assuming that this reinforcement yields at nominal strength, with a yield strength of f_y, produces

$$A_\ell f_y = N$$

or

$$A_\ell = \frac{T_n p_h}{2A_o f_y}\cot\theta \tag{7-35}$$

and again taking $A_o = 0.85\,A_{oh}$ and T_u/ϕ for T_n gives

$$A_\ell = \frac{(T_u/\phi)p_h}{1.7A_{oh} f_y}\cot\theta \tag{7-36}$$

Alternatively, Eq. (7-35) can be solved for T_n, resulting in the second ACI Code equation for nominal torsional strength,

$$T_n = \frac{2A_o A_\ell f_y}{p_h} \tan\theta \tag{7-37}$$

(ACI Code Eq. 22.7.6.1b)

Because the individual wall tension forces N_1, N_2, N_3, and N_4 act along the centroidal axes of the side in question, the total force, N, acts along the centroidal axis of the member. For this reason, the required longitudinal torsional reinforcement from Eq. (7-36) must be distributed evenly around the perimeter of the cross section so that the centroid of the bar areas coincides approximately with the centroid of the member. One bar must be placed in each corner of the stirrups to anchor the compression struts where the compressive forces change direction around the corner.

Value of θ

The ACI Code allows θ to be taken as any value from 30° to 60°. ACI Code Section 22.7.6.1 requires that the value of θ used to calculate the required area of longitudinal steel, A_ℓ, be the same value as used to calculate the area, A_t, and spacing, s, of transverse reinforcement. This is because a reduction in θ leads to a reduction in the required area of stirrups at a given spacing, as indicated by Eq. (7-29), and an increase in the required area of longitudinal steel, as indicated by Eq. (7-36). ACI Code Section 22.7.6.1.2 suggests using a default value of $\theta = 45°$ for nonprestressed reinforced concrete members. This value will be used in the examples for this chapter.

Limits on Concrete and Steel Strengths for Design

Due to a lack of experimental data for beams constructed with high-strength concrete and tested in torsion, the ACI Code limits the value of $\sqrt{f'_c}$ to 100 psi in all torsion design calculations. Also, to control crack widths at service loads, the ACI Code limits the reinforcement yield strengths $(f_y \text{ and } f_{yt})$ *used in torsion design calculations* to 60 ksi.

Combined Shear and Torsion

In ACI Codes prior to 1995, a portion T_c of the torsion was carried by concrete and a portion V_c of the shear was carried by concrete. When both shear and torsion acted, an elliptical interaction diagram was assumed between T_c and V_c, and stirrups were provided for the rest of the torsion and shear. The derivation of Eqs. (7-30) and (7-36) from the space truss analogy assumed that all the torsion was carried by reinforcement, T_s, without any "torsion carried by concrete," T_c. When shear and torsion act together, the 1995 and subsequent ACI Codes assume that V_c remains constant and T_c remains equal to zero, so that

$$V_n = V_c + V_s \tag{6-9}$$

$$T_n = T_s \tag{7-38}$$

where V_c is normally given by Eq. (6-8b). The assumption that there is no interaction between V_c and T_c greatly simplifies the calculations, compared with those required by the

ACI Codes prior to 1995. Design comparisons carried out by ACI Committee 318 showed that, for combinations of low V_u and high T_u, with v_u less than about $0.8(\phi 2\sqrt{f'_c})$ psi, the current ACI Code method requires more stirrups than are required by previous ACI Codes. For v_u greater than this value, the thin-walled-tube method requires the same or marginally fewer stirrups than required by earlier editions of the ACI Code.

Amounts of Torsional and Shear Reinforcement

Torsional reinforcement consists of closed stirrups and longitudinal bars. According to ACI Code Section 9.5.4.3, these are added to the longitudinal bars and stirrups provided for flexure and shear.

In designing for shear, a given size of stirrup, with the area of the two outer legs being A_v, is chosen, and the required spacing, s, is computed. In considering combined shear and torsion, it is necessary to add the stirrups required for shear to those required for torsion. The area of stirrups required for shear and torsion will be computed in terms of A_v/s and A_t/s, both with units of in.^{2}/in. of length of beam. Because A_v refers to all legs of a stirrup (usually two), while A_t refers to only one perimeter leg, the total required stirrup area is

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s} \tag{7-39}$$

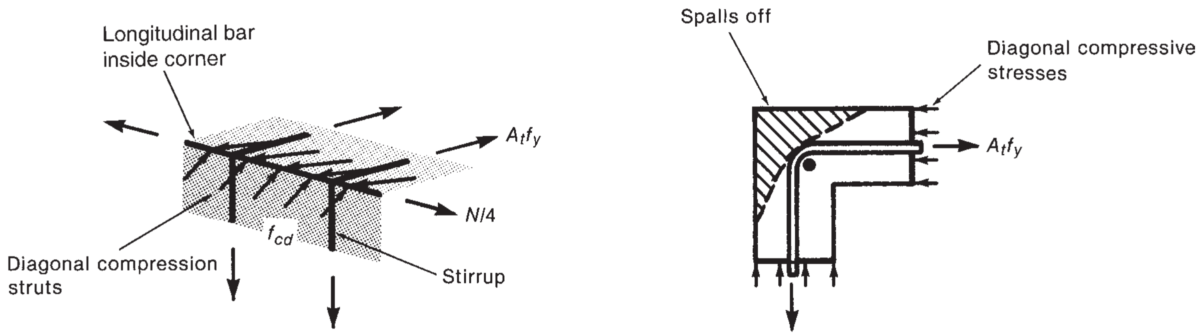
where A_{v+t} refers to the cross-sectional area of both legs of a stirrup. Using Eq. (7-39) it is possible to select a stirrup size and compute the required spacing s. If a stirrup in a wide beam has more than two legs for shear, only the outer legs should be included in the summation in Eq. (7-39).

Types of Torsional Reinforcement and Its Anchorage

Because the inclined cracks can spiral around the beam, as shown in Figs. 7-8c and 7-20, stirrups are required in all four faces of the beam. For this reason, ACI Code Section 9.7.6.3 requires the use of closed transverse reinforcement perpendicular to the axis of the member that satisfies detail requirements in ACI Code Section 25.7.1.6. These should extend as close to the perimeter of the member as cover requirements will allow, so as to make A_{oh} as large as possible.

Tests by Mitchell and Collins [7-10] have examined the types of stirrup anchorages required. Figure 7-28a shows one corner of the space truss model shown in Fig. 7-20b. The inclined compressive stresses in the concrete, f_{cd}, have components parallel to the top and side surfaces, as shown in Fig. 7-28b. The components acting toward the corner are balanced by tensions in the stirrups. The concrete outside the reinforcing cage is not well anchored, and the shaded region will spall off if the compression in the outer shell is large. For this reason, ACI Code Section 25.7.1.6 requires that stirrups be anchored with 135° hooks around a longitudinal bar if the corner can spall. If the concrete around the stirrup anchorage is restrained against spalling by a flange or slab or similar member, the ACI Code allows the use of the anchorage details shown in Fig. 7-29a.

ACI Code Section 9.7.5.4 requires that longitudinal reinforcement for torsion be developed at both ends of a beam. Because the maximum torsions generally act at the ends of a beam, it is generally necessary to anchor the longitudinal torsional reinforcement for its yield strength at the face of the support. This may require hooks or horizontal U-shaped bars lap spliced with the longitudinal torsion reinforcement. A common error is to extend the bottom reinforcement in spandrel beams loaded in torsion 6 in. into the support, as allowed in ACI Code Section 9.7.3.8.1 for positive moment reinforcement. Generally, this is not adequate to develop the longitudinal bars needed to resist torsion.



(a) Forces at a corner of the space truss. (b) Spalling at corner of space truss.

Fig. 7-28
Compressive strut forces at a corner of a torsional member.

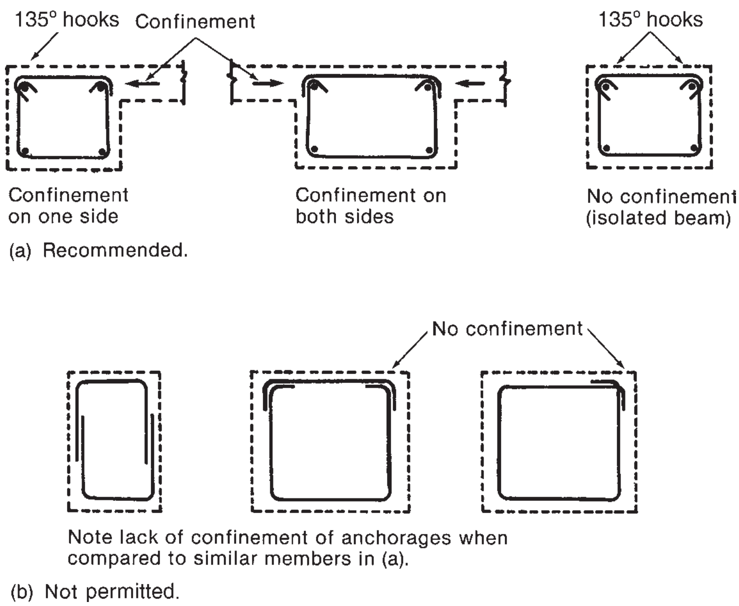


Fig. 7-29
Anchorage of closed stirrups.
(From [7-10].)

Minimum Torsional Reinforcement

When the factored torsional moment exceeds the threshold torsion the larger of: (a) the torsional reinforcement satisfying the strength requirements of ACI Code Section 22.7.6.1, and (b) the minimum reinforcement required by ACI Code Section 9.6.4 must be provided. ACI Code Section 9.6.4.2 specifies that the minimum area of closed stirrups shall be

$$A_v + 2A_t = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_{yt}}, \text{ and } \geq \frac{50 b_w s}{f_{yt}} \tag{7-40}$$

In SI units, this becomes

$$A_v + 2A_t = 0.062\sqrt{f'_c}\,\frac{b_w s}{f_{yt}}, \text{ and } \geq \frac{0.35 b_w s}{f_{yt}} \tag{7-40M}$$

In Hsu's tests [7-5] of rectangular reinforced concrete members subjected to pure torsion, two beams failed at the torsional cracking load. In these beams, the total ratio of the volume of the stirrups and longitudinal reinforcement to the volume of the concrete was 0.802 and 0.88 percent. A third beam, with a volumetric ratio of 1.07 percent, failed at 1.08 times the cracking torque. All the other beams tested by Hsu had volumetric ratios of 1.07 percent or greater and failed at torques in excess of 1.2 times the cracking torque. This suggests that beams with similar concrete and steel strengths loaded in pure torsion should have a minimum volumetric ratio of torsional reinforcement in the order of 0.9 to 1.0 percent. Thus, the minimum volumetric ratio should be set at about 1 percent; that is,

$$\frac{A_{\ell,\,\min} s}{A_{cp} s} + \frac{A_t p_h}{A_{cp} s} \geq 0.01 \tag{7-41a}$$

or

$$A_{\ell,\,\min} = 0.01 A_{cp} - \frac{A_t p_h}{s} \tag{7-41b}$$

If the constant 0.01 is assumed to be a function of the material strengths in the test specimens, the constant in the first term on the right-hand side of Eq. (7-41b) can be rewritten as $7.5\sqrt{f'_c}/f_y$. In the 1971 to 1989 ACI Codes, a transition was provided between the total volume of reinforcement required by the equation for $A_{\ell,\,\min}$ for pure torsion and the much smaller amount of minimum reinforcement required in beams subjected to shear without torsion. This was accomplished by multiplying the same term by $\tau/(\tau + v)$, giving

$$A_{\ell,\,\min} = \frac{7.5\sqrt{f'_c}}{f_y} A_{cp}\left(\frac{\tau}{\tau + v}\right) - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \tag{7-42}$$

During the development of the 1995 torsion provisions, it was assumed that a practical limit on $\tau/(\tau + v)$ was 2/3 for beams that satisfied Eq. (7-36). When this was introduced, Eq. (7-42) became the following from ACI Code Section 9.6.4.3.

$$A_{\ell,\,\min} = \frac{5\sqrt{f'_c}}{f_y} A_{cp} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \tag{7-43}$$

This equation was derived for the case of pure torsion. When it is applied to combined shear, moment, and torsion, it is not clear how much of the area of the stirrups should be included in A_t/s. In this book, we shall assume that A_t/s in Eq. (7-43) is the actual amount of transverse reinforcement provided for *torsion strength* in Eq. (7-30), where A_t is for one leg of a closed stirrup. The value of A_t/s should not be taken less than $25b_w/f_{yt}$, half of the minimum amount corresponding to Eq. (7-40).

In SI units, (7-43) becomes

$$A_{\ell,\,\min} = \frac{5\sqrt{f'_c}}{12 f_y} A_{cp} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \tag{7-43M}$$

Spacing of Torsional Reinforcement

Figure 7-28 shows that the corner longitudinal bars in a beam help to anchor the compressive forces in the struts between cracks. If the stirrups are too far apart, or if the longitudinal bars in the corners are too small in diameter, the compressive forces will tend to bend the longitudinal bars outward, weakening the beam. ACI Code Section 9.7.6.3.3 limits the stirrup spacing to the smaller of $p_h/8$ or 12 in., where p_h is the perimeter of the outermost closed stirrups.

As stated earlier, longitudinal torsional reinforcement should be distributed around the perimeter of the closed stirrups, with the centroid of the steel approximately at the centroid of the cross section. ACI Code Section 9.7.5.1 states that the maximum spacing between longitudinal bars is 12 in. The longitudinal reinforcement should be inside the stirrups, with a bar inside each corner of the stirrups. ACI Code Section 9.7.5.2 requires the diameter of the longitudinal bars to be at least 0.042 times the stirrup spacing, but not less than 0.375 in. In tests, [7-10] corner bars with a diameter of 1/31 of the stirrup spacing bent outward at failure.

ACI Code Section 9.7.5.3 requires that torsional reinforcement continue a distance $(b_t + d)$ past the point where the torque is less than the thereshold torsion, where b_t is the width of that part of the cross section containing the closed stirrups (Fig. 7-24). This length takes into account the fact that torsional cracks spiral around the beam.

Maximum Shear and Torsion

A member loaded by torsion or by combined shear and torsion may fail by yielding of the stirrups and longitudinal reinforcement, as assumed in the derivation of Eqs. (7-30) and (7-36), or by crushing of the concrete due to the diagonal compressive forces. Also, a serviceability failure may occur if the inclined cracks are too wide at service loads. The limit on combined shear and torsion in ACI Code Section 22.7.7.1 was derived to limit service-load crack widths, but as is shown later, it also gives a lower bound on the web's crushing capacity.

Crack Width Limit

As was explained in Section 6-4, ACI Code Section 22.5.1.2 attempts to guard against excessive crack widths by limiting the maximum shear, V_s, that can be transferred by stirrups to an upper limit of $8\sqrt{f'_c}b_w d$. In ACI Code Section 22.7.7.1, the same concept is used, expressed in terms of stresses. The shear stress, v, due to direct shear is $V_u/b_w d$. From Eq. (7-9), with A_o after torsional cracking taken as $0.85A_{oh}$ and $t = A_{oh}/p_h$, the shear stress, τ, due to torsion is $T_u p_h/(1.7A_{oh}{}^2)$. In a *hollow section*, these two shear stresses are additive on one side, at point A in Fig. 7-30a, and the limit is given by

$$\frac{V_u}{b_w d} + \frac{T_u p_h}{1.7A_{oh}{}^2} \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \tag{7-44}$$

(ACI Eq. 22.7.7.1b)

If the wall thickness varies around the cross section, as for example in Fig. 7-7, ACI Code Section 22.7.7.1.2 states that Eq. (7-44) is evaluated at the location where the left-hand side is the greatest.

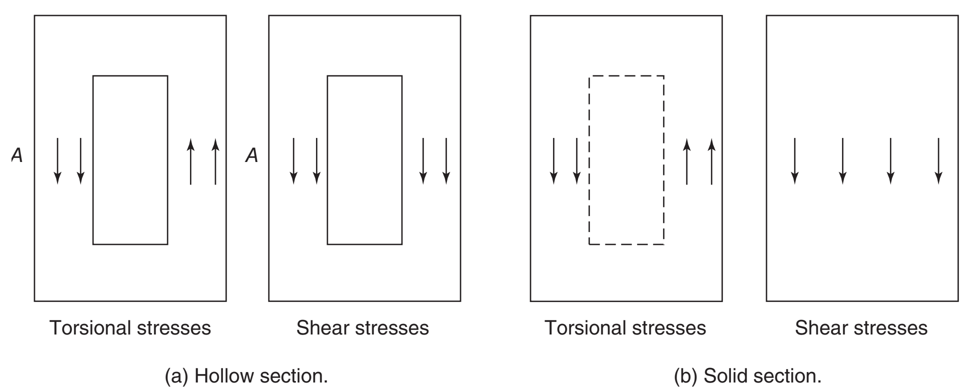


Fig. 7-30
Addition of shear stresses due to torsion and shear. (From [7-11].)