

Calculus

AND ITS APPLICATIONS

ELEVENTH EDITION
GLOBAL EDITION

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Let's see if we can model the path of a baseball. Consider the following data.

HORIZONTAL DISTANCE, x (In feet)	VERTICAL DISTANCE, y (In feet)
0	4.5
50	43
100	82
200	130
285	142
300	134
360	100
400	60

Assume for the given data that (0, 4.5) is the point at home plate at which the ball is hit, roughly 4.5 ft above the ground. Also, assume the ball hits a billboard 60 ft above the ground and 400 ft from home plate.



- **1.** Plot the points and connect them with line segments. This can be done on many calculators by pressing STAT PLOT, turning on PLOT, and selecting the appropriate TYPE.
- 2. a) Use regression to find a cubic function

$$y = ax^3 + bx^2 + cx + d$$

that fits the data.

- **b)** Graph the function over the interval [0, 500].
- c) Does the function closely model the given data?
- **d)** Predict the horizontal distance from home plate at which the ball would have hit the ground had it not hit the billboard.
- **e)** Find the rate of change of the ball's height with respect to its horizontal distance from home plate.
- **f)** Find the point(s) at which the graph has a horizontal tangent line. Explain the significance of the point(s).
- 3. a) Use regression to find a quartic function

$$y = ax^4 + bx^3 + cx^2 + dx + e$$

that fits the data.

b) Graph the function over the interval [0, 500].



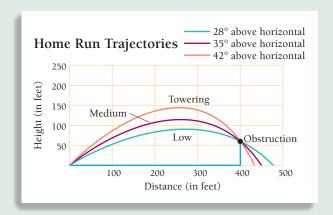
- c) Does the function closely model the given data?
- **d)** Predict the horizontal distance from home plate at which the ball would have hit the ground had it not hit the billboard.
- **e)** Find the rate of change of the ball's height with respect to its horizontal distance from home plate.
- **f)** Find the point(s) at which the graph has a horizontal tangent line. Explain the significance of the point(s).
- **4. a)** Although most calculators cannot fit such a function to the data, assume that the equation

$$y = 0.0015x\sqrt{202,500 - x^2}$$

has been found using a curve-fitting technique. Graph the function over the interval [0, 500].

- **b)** Predict the horizontal distance from home plate at which the ball would have hit the ground had it not hit the billboard.
- **c)** Find the rate of change of the ball's height with respect to its horizontal distance from home plate.
- **d)** Find the point(s) at which the graph has a horizontal tangent line. Explain the significance of the point(s).
- **5.** Look at the answers in Exercises 2(d), 3(d), and 4(b). Compare the merits of the quartic model in Exercise 3 to those of the model in Exercise 4.

Tale of the tape. Actually, scoreboard operators in the major leagues use different models to predict the distance that a home run ball would have traveled. The models are linear and are related to the trajectory of the ball, that is, how high the ball is hit. See the following graph.



Suppose a ball hits an obstruction d feet horizontally from home plate at a height of H feet. Then the estimated horizontal distance D that the ball would have traveled, depending on its trajectory type, is

Low trajectory: D = 1.1H + d, Medium trajectory: D = 0.7H + d, Towering trajectory: D = 0.5H + d.



Exercises

- **6.** For a ball striking an obstacle at d = 400 ft and H = 60 ft, estimate how far the ball would have traveled if it were following a low trajectory, a medium trajectory, or a towering trajectory.
- 7. In 1953, Hall-of-Famer Mickey Mantle hit a towering home run in old Griffith Stadium in Washington, D.C., that hit an obstruction 60 ft high and 460 ft from home plate. Reporters asserted at the time that the ball would have traveled 565 ft. Is this estimate valid?
- **8.** Use the appropriate formula to estimate the distance *D* for each of the following famous long home runs.
 - a) Ted Williams (Boston Red Sox, June 9, 1946):
 Purportedly the longest home run ball ever hit to right
 field at Boston's Fenway Park, Williams's ball landed in
 the stands 502 feet from home plate, 30 feet above the
 ground. Assume a medium trajectory.
 - b) Reggie Jackson (Oakland Athletics, July 13, 1971): Jackson's mighty blast hit an electrical transformer on top of the right-field roof at old Tiger Stadium in the 1971 All-Star Game. The transformer was 380 feet from home plate, 100 feet up. Find the distance the ball would have traveled, assuming a low trajectory and then a medium trajectory. (Jackson's home-run ball left the bat at an estimated 31.5° angle, about halfway between the low and medium range.)
 - c) Richie Sexson (Arizona Diamondbacks, April 26, 2004): Sexson hit a drive that caromed off the center-field scoreboard at Bank One Ballpark in Phoenix. The scoreboard is 414 feet from home plate and 75 feet high. Assume a medium trajectory.

The reported distances these balls would have traveled are 527 feet for Williams's home run, 530 feet for Jackson's, and 469 feet for Sexson's. (*Source*: www.hittrackeronline.com.) How close are your estimates?

Many thanks to Robert K. Adair, professor of physics at Yale University, for many of the ideas presented in this application.



What You'll Learn

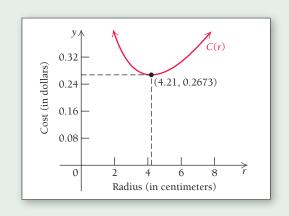
- **2.1** Using First Derivatives to Classify Maximum and Minimum Values and Sketch Graphs
- **2.2** Using Second Derivatives to Classify Maximum and Minimum Values and Sketch Graphs
- **2.3** Graph Sketching: Asymptotes and Rational Functions
- **2.4** Using Derivatives to Find Absolute Maximum and Minimum Values
- **2.5** Maximum–Minimum Problems; Business, Economics, and General Applications
- **2.6** Marginals and Differentials
- **2.7** Elasticity of Demand
- **2.8** Implicit Differentiation and Related Rates

Why It's Important

In this chapter, we explore many applications of differentiation. We learn to find maximum and minimum values of functions, and that skill allows us to solve many kinds of problems in which we need to find the largest and/or smallest value in a real-world situation. Our differentiation skills will be applied to graphing, and we will use differentials to approximate function values. We will also use differentiation to explore elasticity of demand and related rates.

Where It's Used

Minimizing Cost: Minimizing cost is a common goal in manufacturing. For example, cylindrical food cans come in a variety of sizes. Suppose a soup can is to have a volume of 250 cm³. The cost of material for the two circular ends is \$0.0008/cm², and the cost of material for the side is \$0.0015/cm². What dimensions minimize the cost of material for the soup can?(*This problem appears as Example 3 in Section 2.5.*)

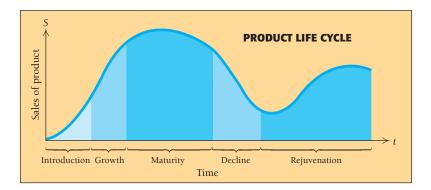


2.1

- Find relative extrema of a continuous function using the First-Derivative Test.
- Sketch graphs of continuous functions.

Using First Derivatives to Classify Maximum and Minimum Values and Sketch Graphs

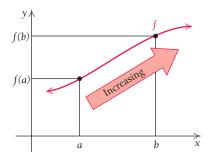
The graph below shows a typical life cycle of a retail product and is similar to graphs we will consider in this chapter. Note that the number of items sold varies with respect to time. Sales begin at a small level and increase to a point of maximum sales, after which they taper off to a low level, possibly because of new competitive products. The company then rejuvenates the product by making improvements. Think about versions of certain products: televisions can be traditional, flat-screen, or high-definition; music recordings have been produced as phonograph (vinyl) records, audiotapes, compact discs, and MP3 files. Where might each of these products be in a typical product life cycle? Is the curve below appropriate for each product?



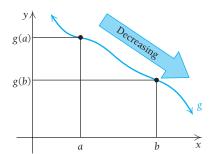
Finding the largest and smallest values of a function—that is, the maximum and minimum values—has extensive applications. The first and second derivatives of a function can provide information for graphing functions and finding maximum and minimum values.

Increasing and Decreasing Functions

If the graph of a function rises from left to right over an interval *I*, the function is said to be **increasing** on, or over, *I*. If the graph drops from left to right, the function is said to be **decreasing** on, or over, *I*.



If the input a is less than the input b, then the output for a is less than the output for b.



If the input *a* is less than the input *b*, then the output for *a* is greater than the output for *b*.

TECHNOLOGY CONNECTION



Exploratory

Graph the function

$$f(x) = -\frac{1}{3}x^3 + 6x^2 - 11x - 50$$

and its derivative

$$f'(x) = -x^2 + 12x - 11$$

using the window [-10, 25,-100, 150], with Xscl = 5 and Yscl = 25. Then TRACE from left to right along each graph. Moving the cursor from left to right, note that the *x*-coordinate always increases. If the function is increasing, the y-coordinate will increase as well. If a function is decreasing, the y-coordinate will decrease.

Over what intervals is *f* increasing? Over what intervals is f decreasing? Over what intervals is f'positive? Over what intervals is f'negative? What rules might relate the sign of f' to the behavior of f?

We can define these concepts as follows.

DEFINITIONS

A function *f* is **increasing** over *I* if, for every *a* and *b* in *I*,

if
$$a < b$$
, then $f(a) < f(b)$.

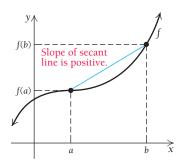
A function *f* is **decreasing** over *I* if, for every *a* and *b* in *I*,

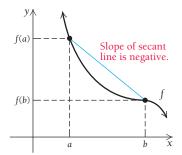
if
$$a < b$$
, then $f(a) > f(b)$.

The above definitions can be restated in terms of secant lines as shown below.

Increasing:
$$\frac{f(b) - f(a)}{b - a} > 0$$

Increasing:
$$\frac{f(b) - f(a)}{b - a} > 0$$
. Decreasing: $\frac{f(b) - f(a)}{b - a} < 0$.





The following theorem shows how we can use the derivative (the slope of a tangent line) to determine whether a function is increasing or decreasing.

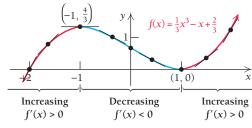
THEOREM 1

Let *f* be differentiable over an open interval *I*.

If f'(x) > 0 for all x in I, then f is increasing over I.

If f'(x) < 0 for all x in I, then f is decreasing over I.

Theorem 1 is illustrated in the following graph of $f(x) = \frac{1}{3}x^3 - x + \frac{2}{3}$.



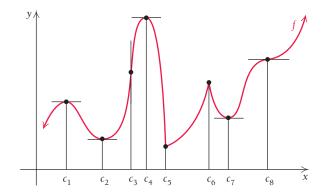
f is increasing over the intervals $(-\infty, -1)$ and $(1, \infty)$; slopes of tangent lines are positive.

f is decreasing over the interval (-1, 1); slopes of tangent lines are negative.

Note in the graph above that x = -1 and x = 1 are not included in any interval over which the function is increasing or decreasing. These values are examples of critical values.

Critical Values

Consider the following graph of a continuous function *f*.



Note the following:

- 1. f'(x) = 0 at $x = c_1, c_2, c_4, c_7$, and c_8 . That is, the tangent line to the graph is horizontal for these values.
- 2. f'(x) does not exist at $x = c_3, c_5$, and c_6 . The tangent line is vertical at c_3 , and there are corners at both c_5 and c_6 . (See also the discussion at the end of Section 1.4.)

DEFINITION

A **critical value** of a function f is any number c in the domain of f for which the tangent line at (c, f(c)) is horizontal or for which the derivative does not exist. That is, c is a critical value if f(c) exists and

$$f'(c) = 0$$
 or $f'(c)$ does not exist.

Thus, in the graph of *f* above:

- c_1 , c_2 , c_4 , c_7 , and c_8 are critical values because f'(c) = 0 for each value.
- c_3 , c_5 , and c_6 are critical values because f'(c) does not exist for each value.

A continuous function can change from increasing to decreasing or from decreasing to increasing *only* at a critical value. In the above graph, c_1 , c_2 , c_4 , c_5 , c_6 , and c_7 separate intervals over which the function f increases from those in which it decreases or intervals in which the function decreases from those in which it increases. Although c_3 and c_8 are critical values, they do not separate intervals over which the function changes from increasing to decreasing or from decreasing to increasing.