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Figure 5.92 To verify the existence of the blind spot, close one eye and, at a distance of about 10 inches, look directly at the X—the 2 will disappear. Moving closer will cause the 2 to reappear while the 1 vanishes.

The normal wavelength range of human vision is roughly 390 nm to 780 nm (Table 3.4). However, studies have extended these limits down to about 310 nm in the ultraviolet and up to roughly 1050 nm in the infrared. Indeed, people have reported "seeing" X-radiation. The limitation on ultraviolet transmission in the eye is set by the crystalline lens, which absorbs in the UV. People who have had a lens removed surgically have greatly improved UV sensitivity.

The area of exit of the optic nerve from the eye contains no receptors and is insensitive to light; accordingly, it is known as the **blind spot** (see Fig. 5.92). The optic nerve spreads out over the back of the interior of the eye in the form of the retina.

Just about at the center of the retina is a small depression from 2.5 to 3 mm in diameter known as the yellow spot, or **macula**. It is composed of more than twice as many cones as rods. There is a tiny rod-free region about 0.3 mm in diameter at the center of the macula called the **fovea centralis**. (In comparison, the image of the full Moon on the retina is about 0.2 mm in diameter—Problem 5.101.) Here the cones are thinner (with diameters of 0.0030 mm to 0.0015 mm) and more densely packed than anywhere else in the retina. Since the fovea provides the sharpest and most detailed information, the eyeball is continuously moving, so that light coming from the area on the object of primary interest falls on this region. An image is constantly shifted across different receptor cells by these normal eye movements. If such movements did not occur and the image was kept stationary on a given set of photoreceptors, it would actually tend to fade out. Without the fovea the eye would lose 90 to 95% of its capability, retaining only peripheral vision.

Another fact that indicates the complexity of the sensing system is that the rods are multiply connected to nerve fibers, and a single such fiber can be activated by any one of about a hundred rods. By contrast, cones in the fovea are individually connected to nerve fibers. The actual perception of a scene is constructed by the eye–brain system in a continuous analysis of the time-varying retinal image. Just think how little trouble the blind spot causes, even with one eye closed.

Between the nerve-fiber layer of the retina and the humor is a network of large retinal blood vessels, which can be observed entoptically. One way is to close your eye and place a bright small source against the lid. You'll "see" a pattern of shadows (*Purkinje figures*) cast by the blood vessels on the sensitive retinal layer.

Accommodation

The fine focusing, or **accommodation**, of the human eye is a function performed by the crystalline lens. The lens is suspended in position behind the iris by ligaments that are connected to a circular yoke composed of the **ciliary muscles**. Ordinarily, these muscles are relaxed, and in that state they pull outward radially on the network of fine fibers holding the rim of the lens. This draws the pliable lens into a fairly flat configuration, increasing its radii, which in turn increases its focal length [Eq. (5.16)]. With the muscles completely relaxed, the light from an object at infinity will be focused on the retina (Fig. 5.93). As the object moves closer to the eye, the ciliary muscles contract, relieving the external tension on the periphery of the lens, which then bulges slightly under its own elastic forces. In so doing, the focal length decreases such that s_i is kept constant. As the object comes still closer, the yoke of ciliary muscles becomes more tensely contracted, the circular region they encompass gets still smaller, and the lens surfaces take on even smaller radii. The closest point on which the eye can focus is known as the **near point**. In a normal eye it might be about 7 cm for a teenager, 12 cm or so for a young adult, roughly 28 to 40 cm in the middle-aged, and about 100 cm by 60 years of age. Visual instruments are designed with this in mind, so that the eye need not strain unnecessarily. Clearly, the eye cannot focus on two different objects at once. This will be made obvious if, while looking through a piece of glass, you try to focus on it and the scene beyond at the same time.

Mammals generally accommodate by varying the lens curvature, but there are other means. Fish move only the lens itself toward or away from the retina, just as the camera lens is moved to focus. Some mollusks accomplish the same thing by contracting or expanding the whole eye, thus altering the relative distance between lens and retina. For birds of prey, which

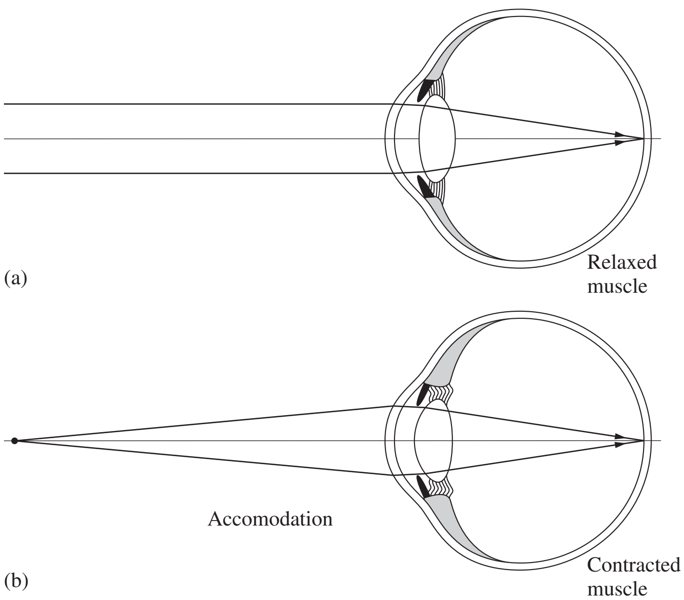


Figure 5.93 Accommodation—changes in the lens configuration.

must keep a rapidly moving object in constant focus over a wide range of distances as a matter of survival, the accommodation mechanism is quite different. They accommodate by greatly changing the curvature of the cornea.

5.7.2 Eyeglasses

Spectacles were probably invented some time in the late thirteenth century, possibly in Italy. A Florentine manuscript of the period (1299), which no longer exists, spoke of "spectacles recently invented for the convenience of old men whose sight has begun to fail." These were biconvex lenses, little more than variations on the handheld magnifying or reading glasses, and polished gemstones were no doubt employed as lorgnettes long before that. Roger Bacon (ca. 1267) wrote about negative lenses rather early on, but it was almost another two hundred years before Nicholas Cusa first discussed their use in eyeglasses and a hundred years more before such glasses ceased to be a novelty, in the late 1500s. Amusingly, it was considered improper to wear spectacles in public even as late as the eighteenth century, and we see few users in the paintings up until that time. In 1804 Wollaston, recognizing that traditional (fairly flat, biconvex, and concave) eyeglasses provided good vision only while one looked through their centers, patented a new, deeply curved lens. This was the forerunner of modern-day meniscus (from the Greek *meniskos*, the diminutive for moon, i.e., crescent) lenses, which allow the turning eyeball to see through them from center to margin without significant distortion.

The earliest known picture (ca. 1352) of someone wearing eyeglasses. This is a portrait of Cardinal Ugo di Provenza, who died in 1262, painted by Tomasso da Modena. (Cardinal Ugo of Provenza (1351), Tomaso da Modena. Fresco in the Capitol Room in the Church of San Nicolò, Treviso. Photo from collection of author.)

It is customary and quite convenient in physiological optics to speak about the **dioptric power**, $\mathscr{D}$, of a lens, which is simply the reciprocal of the focal length. When f is in meters, the unit of power is the inverse meter, or *diopter*, symbolized by D: 1 $\text{m}^{-1} = 1$ D. For example, if a converging lens has a focal length of $+1$ m, its power is $+1$ D; with a focal length of -2 m (a diverging lens), $\mathscr{D} = -\frac{1}{2}$ D; for $f = +10$ cm, $\mathscr{D} = 10$ D. Since a thin lens of index n_l in air has a focal length given by

$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad [5.16]$$

its power is

$$\mathscr{D} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (5.69)$$

You can get a sense of the direction in which we are moving by considering, in rather loose terms, that each surface of a lens bends the incoming rays—the more bending, the stronger the surface. A convex lens that strongly bends the rays at both surfaces has a short focal length and a large dioptric power. We already know that the focal length for two thin lenses in contact is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad [5.38]$$

This means that the combined power is the sum of the individual powers, that is,

$$\mathscr{D} = \mathscr{D}_1 + \mathscr{D}_2$$

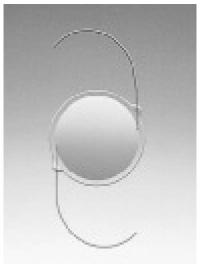
Thus a convex lens with $\mathscr{D}_1 = +10$ D in contact with a negative lens of $\mathscr{D}_2 = -10$ D results in $\mathscr{D} = 0$; the combination behaves like a parallel sheet of glass. Furthermore, we can imagine a lens, for example, a double convex lens, as being composed of two planar-convex lenses in intimate contact, back to back. The power of each of these follows from Eq. (5.69); thus for the first planar-convex lens ($R_2 = \infty$),

$$\mathscr{D}_1 = \frac{(n_l - 1)}{R_1} \qquad (5.70)$$

and for the second,

$$\mathscr{D}_2 = \frac{(n_l - 1)}{-R_2} \qquad (5.71)$$

These expressions may be equally well defined as giving the *powers of the respective surfaces* of the initial double convex lens. In other words, *the power of any thin lens is equal to the sum of the powers of its surfaces*. Because R_2 for a convex lens is a negative number, both $\mathscr{D}_1$ and $\mathscr{D}_2$ will be positive in that case. The power of a surface, defined in this way, is not generally the



When the normally clear lens in the eye becomes cloudy, the condition is referred to as a **cataract**. The resulting haziness can have a devastating effect on vision. In extreme cases the crystalline lens is usually surgically removed. A small convex plastic lens (an **intraocular lens implant**) is then inserted in the eye to enhance its convergence. (The photo shows an enlarged image of this type of converging spherical lens; it's actually only about 6 mm in diameter.) Its use has all but eliminated the need for the thick "cataract eyeglasses" that were once required after surgery. (E.H.)

reciprocal of its focal length, although it is when immersed in air. Relating this terminology to the commonly used model for the human eye, we note that the power of the crystalline lens *surrounded by air* is about +19 D. The cornea provides roughly +43 of the total +58.6 D of the intact unaccommodated eye.

A normal eye, despite the connotation of the word, is not as common as one might expect. By the term *normal*, or its synonym *emmetropic*, we mean an eye that is capable of focusing parallel rays on the retina while in a relaxed condition—that is, one whose second focal point lies on the retina. For the unaccommodated eye, we define the object point whose image lies on the retina to be the **far point**. Thus for the normal eye the most distant point that can be brought to a focus on the retina, the far point, is located at infinity (which for all practical purposes is anywhere beyond about 5 m). In contrast, when the focal point does not lie on the retina, the eye is *ametropic* (e.g., it suffers hyperopia, myopia, or astigmatism). This can arise either because of abnormal changes in the refracting mechanism (cornea, lens, etc.) or because of alterations in the length of the eyeball that change the distance between the lens and the retina. The latter is by far the more common cause. Just to put things in proper perspective, note that about 25% of young adults require ± 0.5 D or less of eyeglass correction, and perhaps as many as 65% need only ± 1.0 D or less.

Nearsightedness—Negative Lenses

Myopia is the condition in which parallel rays are brought to focus in front of the retina; the power of the lens system as configured is too large for the anterior-posterior axial length of the eye. Images of distant objects fall in front of the retina, the far point is closer in than infinity, and all points beyond it will appear blurred. This is why myopia is often called **nearsightedness**; an eye with this defect sees nearby objects clearly (Fig. 5.94). To correct the condition, or at least its symptoms, we place an additional lens in front of the eye such that the combined spectacle–eye lens system has its focal point on the retina. Since the myopic eye can clearly see objects closer than the far point, the spectacle lens must cast relatively nearby images of distant objects. Hence we introduce a negative lens that

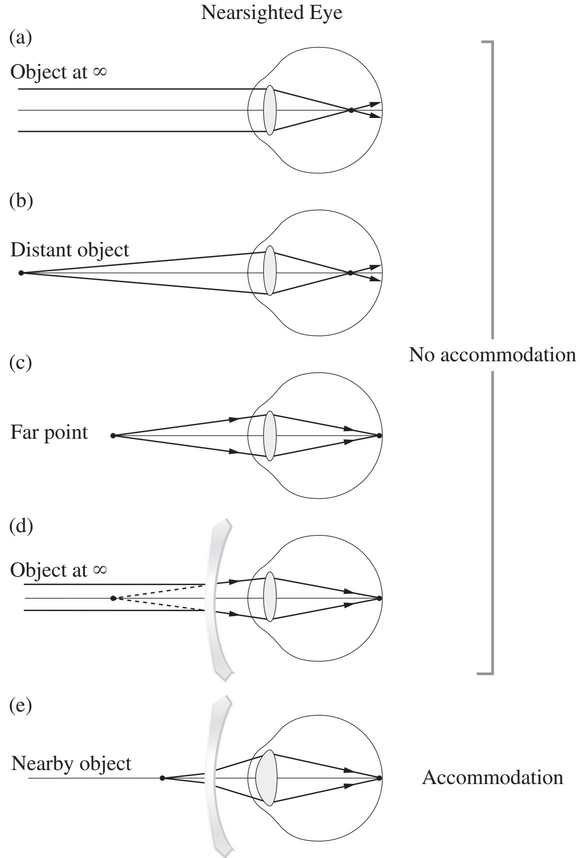


Figure 5.94 Correction of the nearsighted eye.

will diverge the rays a bit. Resist the temptation to suppose that we are merely reducing the power of the system. In point of fact, the power of the lens–eye combination is most often made to equal that of the unaided eye. If you are wearing glasses to correct myopia, take them off; the world gets blurry, but it doesn't change size. Try casting a real image on a piece of paper using your glasses—it can't be done.

Example 5.14

Suppose an eye has a far point of 2 m. All would be well if a spectacle lens appeared to bring more distant objects in closer than 2 m. If the virtual image of an object at infinity is formed by a concave lens at 2 m, the eye will see the object clearly with an unaccommodated lens. Find the needed focal length.

SOLUTION

Using the thin-lens approximation (eyeglasses are generally thin to reduce weight and bulk), we have

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{\infty} + \frac{1}{-2} \qquad [5.17]$$

and $f = -2$ m while $\mathscr{D} = -\frac{1}{2}$ D.

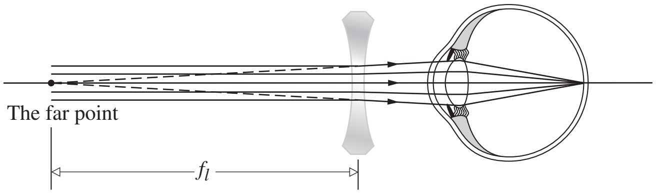


Figure 5.95 The far-point distance equals the focal length of the correction lens.

Notice in the above example that the far-point distance, measured from the correction lens, equals its focal length (Fig. 5.95). The eye views the right-side-up virtual images of all objects formed by the correction lens, and those images are located between its far and near points. Incidentally, the near point also moves away a little, which is why myopes often prefer to remove their spectacles when threading needles or reading small print; they can then bring the material closer to the eye, thereby increasing the magnification.

The calculation we have just performed overlooks the separation between the correction lens and the eye—in effect, it applies to contact lenses more than to spectacles. The separation is usually made equal to the distance of the first focal point of the eye ($\approx$16 mm) from the cornea, so that no magnification of the image over that of the unaided eye occurs. Many people have unequal eyes, yet both yield the same magnification. A change in M_T for one and not the other would be a disaster. Placing the correcting lens at the eye's first focal point avoids the problem completely, regardless of the power of that lens [take a look at Eq. (6.8)]. To see this, just draw a ray from the top of some object through that focal point. The ray will enter the eye and traverse it parallel to the optic axis, thus establishing the height of the image. Yet, since this ray is unaffected by the presence of the spectacle lens, whose center is at the focal point, the image's location may change on insertion of such a lens, but its height and therefore M_T will not [see Eq. (5.24)].

The question now becomes: What is the equivalent power of a spectacle lens at some distance d from the eye (i.e., equivalent to that of a contact lens with a focal length f_c that equals the far-point distance)? It will do for our purposes to approximate the eye by a single lens and take d from that eye-lens to the spectacle as roughly equal to the cornea–eyeglass distance, around 16 mm. Given that the focal length of the correction lens is f_l and the focal length of the eye is f_e, the combination has a focal length provided by Eq. (5.36), that is,

$$\text{b.f.l.} = \frac{f_e(d - f_l)}{d - (f_l + f_e)} \tag{5.72}$$

This is the distance from the eye-lens to the retina. Similarly, the equivalent contact lens combined with the eye-lens has a focal length given by Eq. (5.38):

$$\frac{1}{f} = \frac{1}{f_c} + \frac{1}{f_e} \tag{5.73}$$

where f = b.f.l. Inverting Eq. (5.72), setting it equal to Eq. (5.73), and simplifying, we obtain the result $1/f_c = 1/(f_l - d)$, independent of the eye itself. In terms of power,

$$\mathscr{D}_c = \frac{\mathscr{D}_l}{1 - \mathscr{D}_l d} \tag{5.74}$$

A spectacle lens of power $\mathscr{D}_l$ a distance d from the eye-lens has an effective power the same as that of a contact lens of power $\mathscr{D}_c$. Notice that since d is measured in meters and thus is quite small, unless $\mathscr{D}_l$ is large, as it often is, $\mathscr{D}_c \approx \mathscr{D}_l$. Usually, the point on your nose where you choose to rest your eyeglasses has little effect, but that's certainly not always the case; an improper value of d has resulted in many a headache.

It is common, though not universal, to say that a person whose vision is corrected by a contact lens of power −6 D is a 6 D myope.

EXAMPLE 5.15

Describe the spectacle lenses that would correct the vision of a 6 D myope. The person wants to wear the lenses 12 mm from each eye.

SOLUTION

A 6 D myope has too much convergence and needs −6 D contact lenses. Using Eq. (5.74) the power of the spectacle lenses can be calculated from

$$\mathscr{D}_c = \frac{\mathscr{D}_l}{1 - \mathscr{D}_l d}$$

where

$$\mathscr{D}_c - \mathscr{D}_c\mathscr{D}_l d = \mathscr{D}_l$$

$$\mathscr{D}_c = \mathscr{D}_l(1 + \mathscr{D}_c d)$$

$$\mathscr{D}_l = \frac{\mathscr{D}_c}{1 + \mathscr{D}_c d} = \frac{-6}{1 + (-6)(0.012)}$$

and

$$\mathscr{D}_l = -6.47\,\text{D}$$

Farsightedness—Positive Lenses

Hyperopia (or *hypermetropia*) is the defect that causes the second focal point of the unaccommodated eye to lie behind the retina (Fig. 5.96). **Farsightedness**, as you might have guessed it would be called, is often due to a shortening of the anteroposterior axis of the eye—the lens is too close to the retina. To increase the bending of the rays, a positive spectacle lens is placed in front of the eye. The hyperopic eye can and must accommodate to see distant objects distinctly, but it will be at its limit to do so for a near point, which is much farther away than it would be normally (this we take as 254 mm, or just 25 cm). It will consequently be unable to see nearby objects clearly. A converging corrective lens with positive power will effectively move a close object out beyond the near point where the eye has

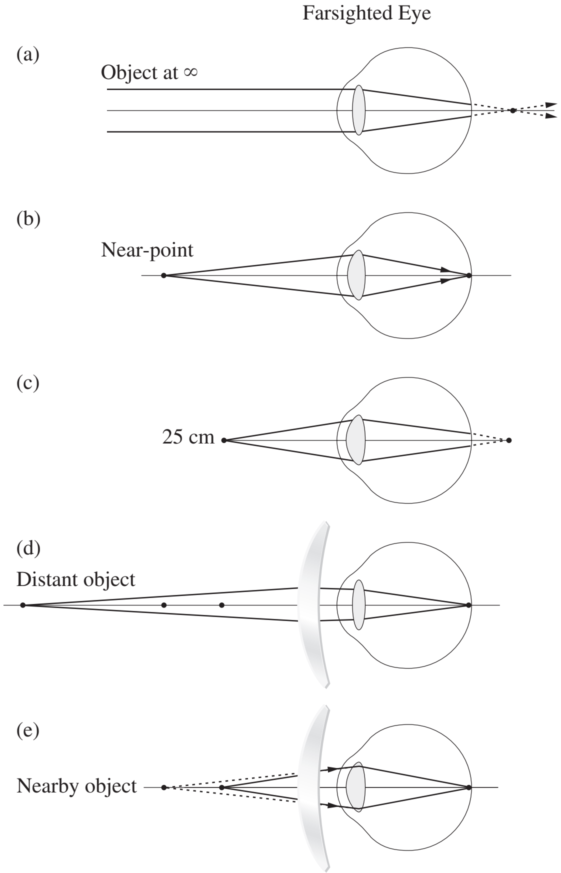


Figure 5.96 Correction of the farsighted eye.

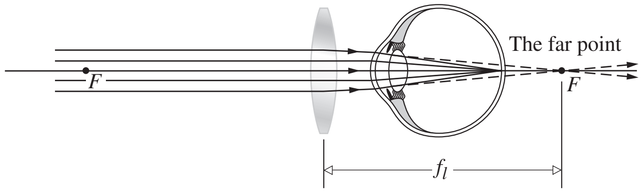


Figure 5.97 Again the far-point distance equals the focal length of the correction lens.

adequate acuity; that is, it will form a distant virtual image, which the eye can then see clearly.

Example 5.16

Suppose that a hyperopic eye has a near point of 125 cm. Find the needed corrective lens.

SOLUTION

For an object at +25 cm to have its image at $s_i = -125$ cm so that it can be seen as if through a normal eye, the focal length must be

$$\frac{1}{f} = \frac{1}{(-1.25)} + \frac{1}{0.25} = \frac{1}{0.31}$$

or $f = 0.31$ m and $\mathcal{D} = +3.2$ D. This is in accord with Table 5.3, where $s_o < f$. These spectacles will cast real images—try it if you're hyperopic.

As shown in Fig. 5.97, the correcting lens allows the relaxed eye to view objects at infinity. In effect, it creates an image on its focal "plane" (passing through F), which then serves as a virtual object for the eye. The point (whose image lies on the retina) is once again the *far point*, and it's a distance f_l *behind* the lens. The hyperope can comfortably "see" the far point, and any lens located anywhere in front of the eye that has an appropriate focal length will serve that purpose.

Very gentle finger pressure on the lids above and below the cornea will temporarily distort it, changing your vision from blurred to clear and vice versa.

Astigmatism—Anamorphic Lenses

Perhaps the most common eye defect is **astigmatism**. It arises from an uneven curvature of the cornea. In other words, the cornea is asymmetric. Suppose we pass two meridional planes (one containing the optical axis) through the eye such that the (curvature or) power is maximal on one and minimal on the other. If these planes are perpendicular, the *astigmatism* is *regular* and correctable; if not, it is *irregular* and not easily corrected. Regular astigmatism can take different forms; the eye can be emmetropic, myopic, or hyperopic in various combinations and degrees on the two perpendicular meridional planes. Thus, as a simple example, the columns of a checkerboard might be well focused, while the rows are blurred due to myopia or hyperopia. Obviously, these meridional planes need not be horizontal and vertical (Fig. 5.98).

The great astronomer Sir George B. Airy used a concave sphero-cylindrical lens to ameliorate his own myopic astigmatism in 1825. This was probably the first time astigmatism had been corrected. But it was not until the publication in 1862 of a treatise on cylindrical lenses and astigmatism by the Dutchman Franciscus Cornelius Donders (1818–1889) that ophthalmologists were moved to adopt the method on a large scale.

Any optical system that has a different value of M_T or $\mathcal{D}$ in two principal meridians is said to be *anamorphic*. Thus, for example, if we rebuilt the system depicted in Fig. 5.41, this time

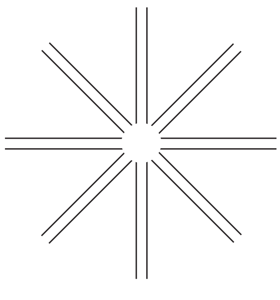
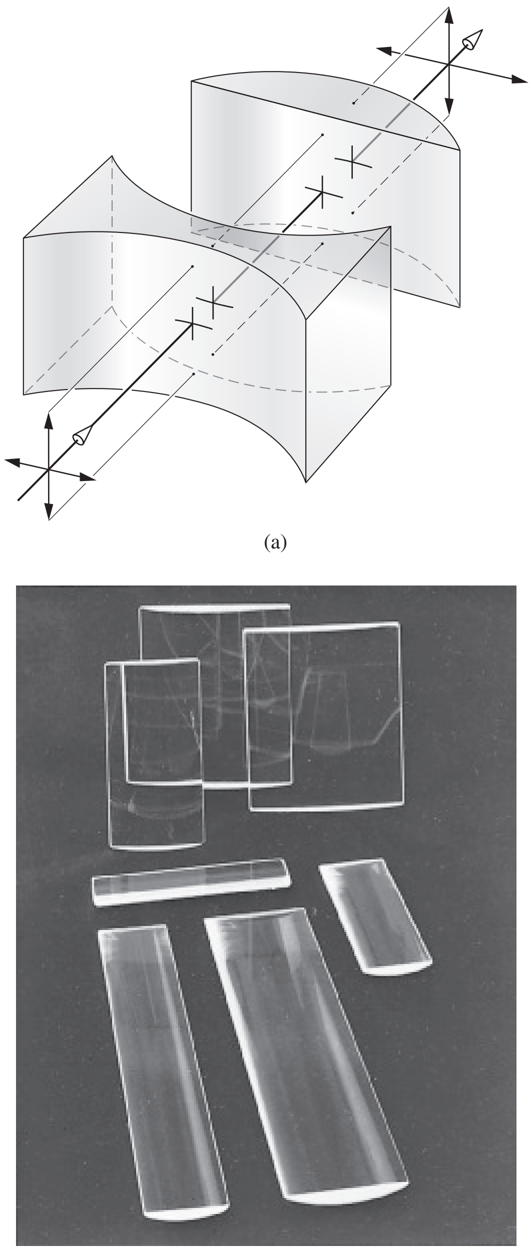


Figure 5.98 A test for astigmatism of the eye. View this figure through one unaided eye. If one set of lines appears bolder than the others, you have astigmatism. Hold the figure close to your eye; move it away slowly and note which set of lines comes into focus first. If two sets seem to be equally clear, rotate the figure until only one set is in focus. If all sets are clear you don't have astigmatism.



(a)

(b)

Figure 5.99 (*a*) An anamorphic system. (*b*) Cylindrical lenses (Melles Griot)

using cylindrical lenses (Fig. 5.99), the image would be distorted, having been magnified in only one plane. This is just the sort of distortion needed to correct for astigmatism when a defect exists in only one meridian. An appropriate planar cylindrical spectacle lens, either positive or negative, would restore essentially normal vision. When both perpendicular meridians require correction, the lens may be *sphero-cylindrical* or even *toric* as in Fig. 5.100.

Just as an aside, we note that anamorphic lenses are used in other areas, as, for example, in the making of wide-screen motion pictures, where an extra-large horizontal field of view

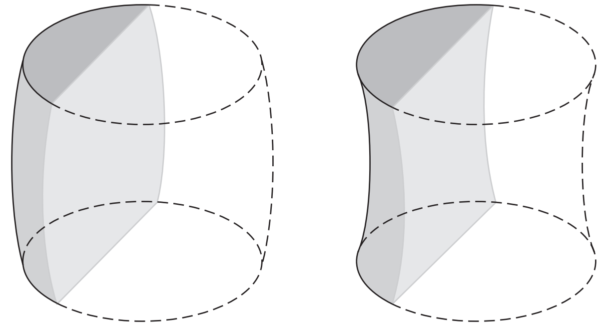


Figure 5.100 Toric surfaces.

is compacted onto the regular film format. When shown through a special lens, the distorted picture spreads out again. On occasion a television station will show short excerpts without the special lens—you may have seen the weirdly elongated result.

5.7.3 The Magnifying Glass

An observer can cause an object to appear larger, for the purpose of examining it in detail, by simply bringing it closer to her eye. As the object is brought nearer and nearer, its retinal image increases, remaining in focus until the crystalline lens can no longer provide adequate accommodation. Should the object come closer than this *near point*, the image will blur (Fig. 5.101). A single positive lens can be used, in effect to add refractive power to the eye, so that the object can be brought still closer and yet be in focus. The lens so used is referred to variously as a **magnifying glass**, a *simple magnifier*, or a *simple microscope*. In any event, its function is *to provide an image of a nearby object that is larger than the image seen by the unaided eye* (Fig. 5.102). Devices of this sort have been around for a long time. In fact, a quartz convex lens ($f \approx 10$ cm), which may have served as a

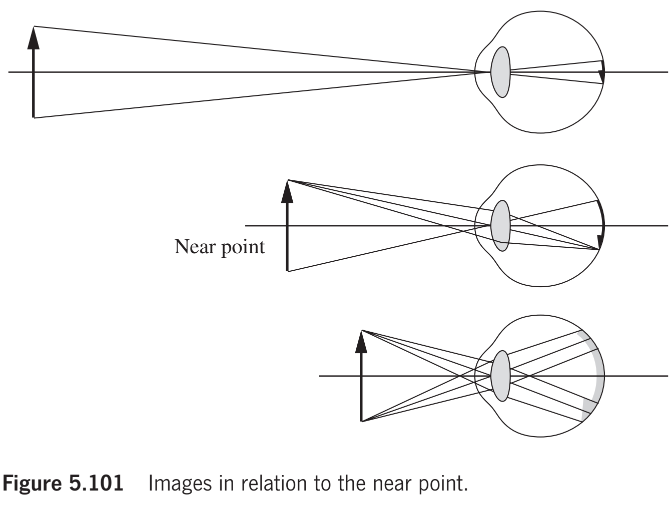


Figure 5.101 Images in relation to the near point.