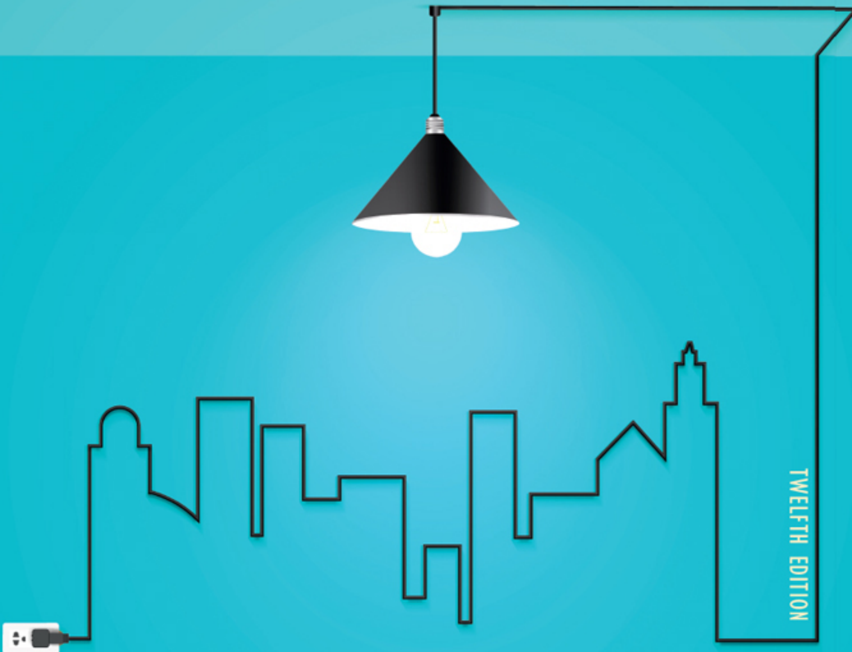


TWELFTH EDITION

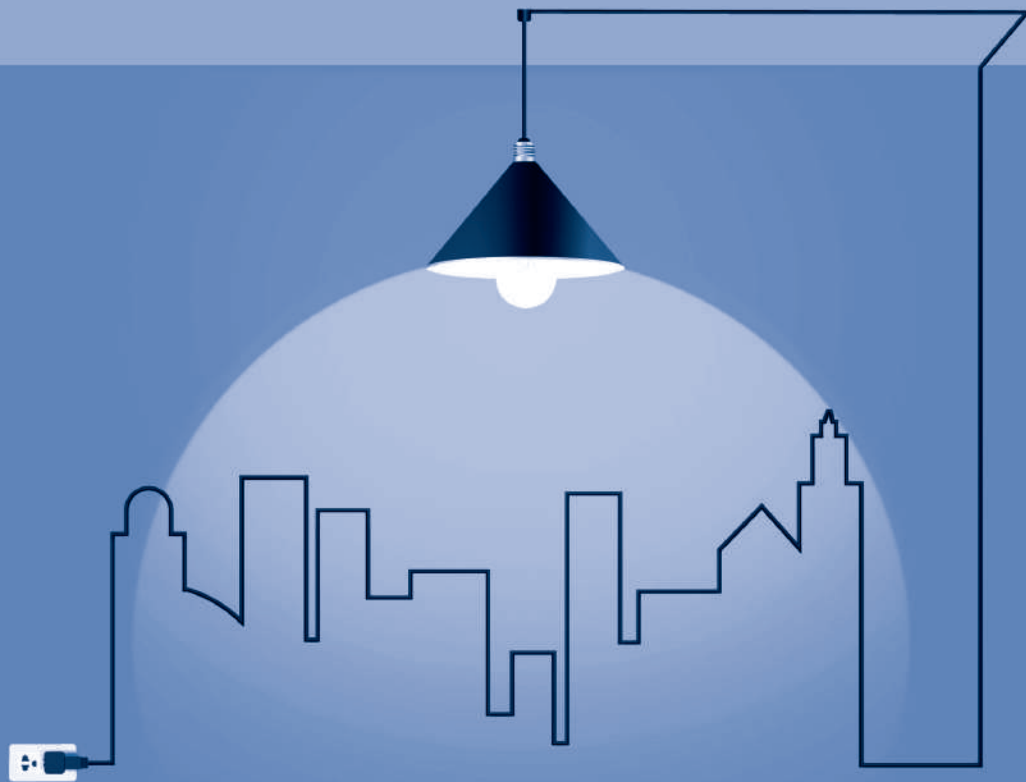
HUGHES

ELECTRICAL & ELECTRONIC TECHNOLOGY

REVISED BY JOHN HILEY, KEITH BROWN & IAN McKENZIE SMITH

ALWAYS LEARNING

PEARSON



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Summary of important formulae continued

and $\phi = \tan^{-1}\dfrac{X_1 + X_2}{R_1 + R_2}$

For a circuit having R and L in *parallel*

$$\mathbf{Y} = \frac{1}{R} - \frac{j}{X_L} = G - jB_L = Y\angle-\phi \quad [12.12]$$

For a circuit having R and C in *parallel*

$$\mathbf{Y} = \frac{1}{R} + \frac{j}{X_C} = G + jB_C = Y\angle\phi \quad [12.13]$$

For admittances

$$\mathbf{Y}_1 = G_1 + jB_1 \quad \text{and} \quad \mathbf{Y}_2 = G_2 + jB_2$$

in *parallel*, total admittance is

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 = (G_1 + G_2) + j(B_1 + B_2)$$
$$= Y\angle\phi$$

where $Y = \sqrt{(G_1 + G_2)^2 + (B_1 + B_2)^2}$

and $\phi = \tan^{-1}\dfrac{B_1 + B_2}{G_1 + G_2}$

If $\mathbf{V} = a + jb$

and $\mathbf{I} = c + jd$

Active power $= ac + bd$ [12.16]

and Reactive power $= bc - ad$ [12.17]

$S = P + jQ$ [12.18]

Terms and concepts

A **complex number** is one which represents the horizontal and vertical components of a polar number separately. The horizontal component is the **real component**, and the vertical component is the **imaginary component**.

Voltages, currents and impedances can all be represented by complex numbers.

However, care should be taken that complex voltages and complex currents contain time information, whereas complex impedances are merely independent operators.

Terms and concepts continued

Complex notation is especially useful when dealing with parallel networks since it simplifies both the addition (and subtraction) of the branch currents, and also the manipulation of the impedance, which is difficult if expressed in polar notation.

Power can be expressed in complex form, but if we wish to obtain the power from a voltage and current we need to use the conjugate of the current: this removes the time information, which otherwise distorts the solution.

Exercises 12

1. Express in rectangular and polar notations the phasors for the following quantities: (a) $i = 10 \sin \omega t$; (b) $i = 5 \sin(\omega t - \pi/3)$; (c) $v = 40 \sin(\omega t + \pi/6)$.

Draw a phasor diagram representing the above voltage and currents.

2. With the aid of a simple diagram, explain the j-notation method of phasor quantities.

Four single-phase generators whose e.m.f.s can be represented by: $e_1 = 20 \sin \omega t$; $e_2 = 40 \sin(\omega t + \pi/2)$; $e_3 = 30 \sin(\omega t - \pi/6)$; $e_4 = 10 \sin(\omega t - \pi/3)$; are connected in series so that their resultant e.m.f. is given by $e = e_1 + e_2 + e_3 + e_4$. Express each e.m.f. and the resultant in the form $a \pm jb$. Hence find the maximum value of e and its phase angle relative to e_1.

3. Express each of the following phasors in polar notation and draw the phasor diagram: (a) $10 + j5$; (b) $3 - j8$.

4. Express each of the following phasors in rectangular notation and draw the phasor diagram: (a) $20\angle 60°$; (b) $40\angle -45°$.

5. Add the two phasors of Q. 3 and express the result in: (a) rectangular notation; (b) polar notation. Check the values by drawing a phasor diagram to scale.

6. Subtract the second phasor of Q. 3 from the first phasor, and express the result in: (a) rectangular notation; (b) polar notation. Check the values by means of a phasor diagram drawn to scale.

7. Add the two phasors of Q. 4 and express the result in: (a) rectangular notation; (b) polar notation. Check the values by means of a phasor diagram drawn to scale.

8. Subtract the second phasor of Q. 4 from the first phasor and express the result in: (a) rectangular notation; (b) polar notation. Check the values by a phasor diagram drawn to scale.

9. Calculate the resistance and inductance or capacitance in *series* for each of the following impedances, assuming the frequency to be 50 Hz: (a) $50 + j30\ \Omega$; (b) $30 - j50\ \Omega$; (c) $100\angle 40°\ \Omega$; (d) $40\angle 60°\ \Omega$.

10. Derive expressions, in rectangular and polar notations, for the admittances of the following impedances: (a) $10 + j15\ \Omega$; (b) $20 - j10\ \Omega$; (c) $50\angle 20°\ \Omega$; (d) $10\angle -70°\ \Omega$.

11. Derive expressions, in rectangular and polar notations, for the impedances of the following admittances; (a) $0.2 + j0.5$ siemens; (b) $0.08\angle -30°$ siemens.

12. Calculate the resistance and inductance or capacitance in *parallel* for each of the following admittances, assuming the frequency to be 50 Hz: (a) $0.25 + j0.06$ S; (b) $0.05 - j0.1$ S; (c) $0.8\angle 30°$ S; (d) $0.5\angle -50°$ S.

13. A voltage, $v = 150 \sin(314t + 30°)$ volts, is maintained across a coil having a resistance of 20 Ω and an inductance of 0.1 H. Derive expressions for the r.m.s. values of the voltage and current phasors in: (a) rectangular notation; (b) polar notation. Draw the phasor diagram.

14. A voltage, $v = 150 \sin(314t + 30°)$ volts, is maintained across a circuit consisting of a 20 Ω non-reactive resistor in series with a loss-free 100 μF capacitor. Derive an expression for the r.m.s. value of the current phasor in: (a) rectangular notation; (b) polar notation. Draw the phasor diagram.

15. Calculate the values of resistance and reactance which, when in parallel, are equivalent to a coil having a resistance of 20 Ω and a reactance of 10 Ω.

16. The impedance of two parallel branches can be represented by $(24 + j18)\ \Omega$ and $(12 - j22)\ \Omega$ respectively. If the supply frequency is 50 Hz, find the resistance and inductance or capacitance of each circuit. Also, derive a symbolic expression in polar form for the admittance of the combined circuits, and thence find the phase angle between the applied voltage and the resultant current.

17. A coil of resistance 25 Ω and inductance 0.044 H is connected in parallel with a branch made up of a 50 μF capacitor in series with a 40 Ω resistor, and the whole is connected to a 200 V, 50 Hz supply. Calculate,

Exercises 8 continued

using symbolic notation, the total current taken from the supply and its phase angle, and draw the complete phasor diagram.

18. The current in a circuit is given by 4.5 + j12 A when the applied voltage is 100 + j150 V. Determine: (a) the complex expression for the impedance, stating whether it is inductive or capacitive; (b) the active power; (c) the phase angle between voltage and current.

19. Explain how alternating quantities can be represented by complex numbers.

If the potential difference across a circuit is represented by 40 + j25 V, and the circuit consists of a coil having a resistance of 20 Ω and an inductance of 0.06 H and the frequency is 79.5 Hz, find the complex number representing the current in amperes.

20. The impedances of two parallel branches can be represented by (20 + j15) Ω and (10 − j60) Ω respectively. If the supply frequency is 50 Hz, find the resistance and the inductance or capacitance of each branch. Also, derive a complex expression for the admittance of the combined network, and thence find the phase angle between the applied voltage and the resultant current. State whether this current is leading or lagging relative to the voltage.

21. An alternating e.m.f. of 100 V is induced in a coil of impedance 10 + j25 Ω. To the terminals of this coil there is joined a circuit consisting of two parallel impedances, one of 30 − j20 Ω and the other of 50 + j0 Ω. Calculate the current in the coil in magnitude and phase with respect to the induced voltage.

22. A circuit consists of a 30 Ω non-reactive resistor in series with a coil having an inductance of 0.1 H and a resistance of 10 Ω. A 60 μF loss-free capacitor is connected in parallel with the *coil*. The network is connected across a 200 V, 50 Hz supply. Calculate the value of the current in each branch and its phase relative to the supply voltage.

23. An impedance of 2 + j6 Ω is connected in series with two impedances of 10 + j4 Ω and 12 − j8 Ω, which are in parallel. Calculate the magnitude and power factor of the main current when the combined circuit is supplied at 200 V.

Chapter thirteen

Power in AC Circuits

Objectives

When you have studied this chapter, you should

- **have an understanding of active and reactive powers**
- **be capable of analysing the power in an a.c. circuit containing one component**
- **be aware that active power is dissipated**
- **be aware that reactive power is not dissipated**
- **have an understanding of the powers associated with an a.c. series circuit**
- **be familiar with power factor**
- **be capable of analysing the power factor of an a.c. circuit**
- **recognize the importance of power factor in practice**
- **be familiar with a technique of measuring the power in a single-phase circuit**

Contents

AC circuits deliver power to resistive and reactive loads. We find that in the case of resistive loads the energy is dissipated in the same way as a direct current dissipates energy in a resistor. However, we find a completely different situation with reactive loads – here the energy is first delivered to the load and then it is returned to the source, and then it is returned to the load and so on. It is like watching an unending rally in tennis as the ball of energy flies to and fro.

The power which gives rise to energy dissipation is the active power. The power describing the rate of energy moving in and out of reactances is reactive power and is an essential part of the energy transfer system.

We find therefore that we have to mix active and reactive powers and this leads us to talk about power factors. Most of us eventually meet with power factors when it comes to paying commercial electricity bills so it is a good idea to know for what we are paying.

13.1 The impossible power

When alternating current systems were first introduced, learned scientists claimed that it was impossible to deliver energy by such a means. Their argument was that power transfer would take place during the first half of the cycle – and then it would transfer back during the second half.

Curiously, there was some truth in what they claimed, but they had overlooked the basic relationship $p = i^2R$. The square of the current means that the power is positive no matter whether the current has a positive or a negative value. But it is only the resistive element that dissipates energy from the circuit. Inductors and capacitors do not dissipate energy, which supports the theory of the impossible power.

Let us therefore examine in more detail the energy transfer process which takes place first in resistive circuits and then in reactive circuits.

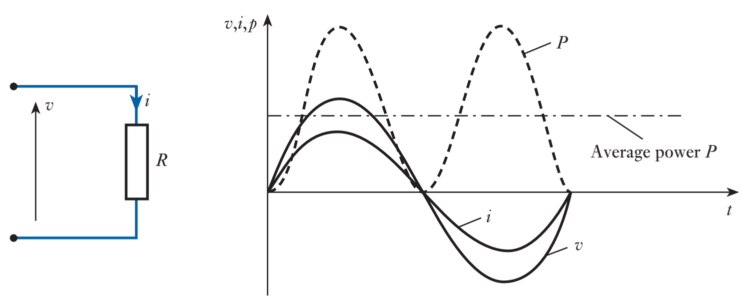
13.2 Power in a resistive circuit

In section 9.6 it was explained that when an alternating current flows through a resistor of R ohms, the average heating effect over a complete cycle is I^2R watts, where I is the r.m.s. value of the current in amperes.

If V volts is the r.m.s. value of the applied voltage, then for a non-reactive circuit having constant resistance R ohms, $V = IR$.

The waveform diagrams for resistance are shown in Fig. 13.1. To the current and voltage waves, there have been added the waves of the product vi. Since the instantaneous values of vi represent the instantaneous power p, it follows that these waves are the power waves. Because the power is continually fluctuating, the power in an a.c. circuit is taken to be the average value of the wave.

Fig. 13.1 Waveform diagrams for a resistive circuit



In the case of the pure resistance, the average power can be most easily obtained from the definition of the r.m.s. current in the circuit, i.e.

$$P = I^2R \qquad [13.1]$$

This relation can also be expressed as

$$P = VI \qquad [13.2]$$

Hence the power in a non-reactive circuit is given by the product of the ammeter and voltmeter readings, exactly as in a d.c. circuit.

The power associated with energy transfer from the electrical system to another system, such as heat, light or mechanical drives, is termed active power, thus the average given by I^2R is the active power of the arrangement.

Alternatively, the average power can be derived from a formal analysis of the power waveform.

$$P = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} (V_\mathrm{m}\sin\omega t \cdot I_\mathrm{m}\sin\omega t)\,\mathrm{d}t$$

$$= V_\mathrm{m}I_\mathrm{m}\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} (\sin^2\omega t)\,\mathrm{d}t$$

$$= V_\mathrm{m}I_\mathrm{m}\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \left(\frac{1-\cos 2\omega t}{2}\right)\mathrm{d}t$$

From this relation it can be seen that the wave has a frequency double that of the component voltage and current waves. This can be seen in Fig. 13.1; however, it also confirms that the wave is sinusoidal although it has been displaced from the horizontal axis.

$$P = V_\mathrm{m}I_\mathrm{m}\frac{\omega}{2\pi}\left[\frac{t}{2} - \frac{\sin 2\omega t}{4\omega}\right]_0^{\frac{2\pi}{\omega}}$$

$$= V_\mathrm{m}I_\mathrm{m}\frac{\omega}{2\pi}\cdot\frac{2\pi}{2\omega}$$

$$P = \frac{V_\mathrm{m}I_\mathrm{m}}{2} \qquad [13.3]$$

$$P = VI$$

13.3 Power in a purely inductive circuit

Consider a coil wound with such thick wire that the resistance is negligible in comparison with the inductive reactance X_L ohms. If such a coil is connected across a supply voltage V, the current is given by $I = V/X_L$ amperes. Since the resistance is very small, the heating effect and therefore the active power are also very small, even though the voltage and the current are large. Such a curious conclusion – so different from anything we have experienced in d.c. circuits – requires fuller explanation if its significance is to be properly understood. Let us therefore consider Fig. 13.2, which shows the applied voltage and the current for a purely inductive circuit, the current lagging the voltage by a quarter of a cycle.

The power at any instant is given by the product of the voltage and the current at that instant; thus at instant L, the applied voltage is LN volts and the current is LM amperes, so that the power at that instant is LN × LM watts and is represented to scale by LP.