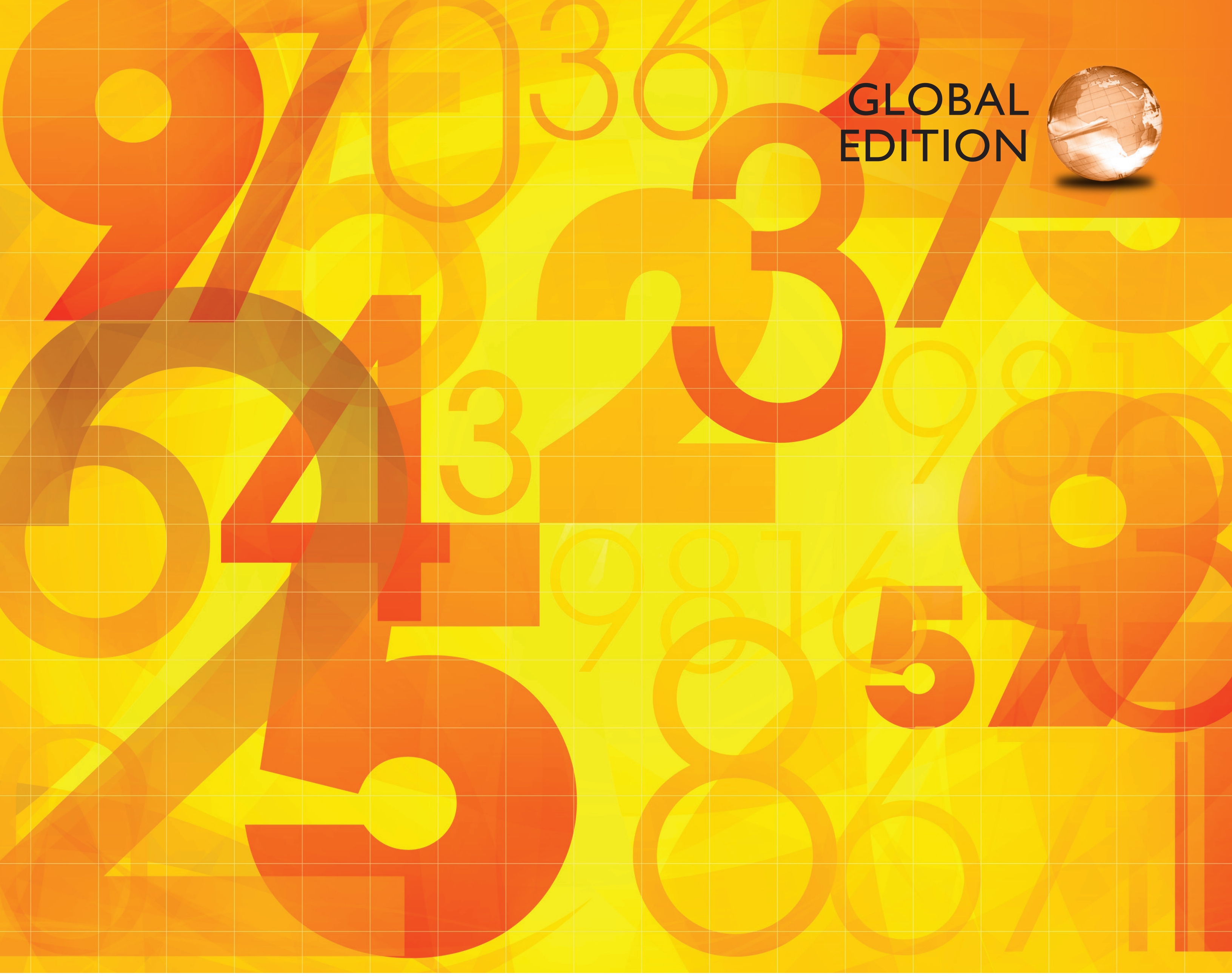


GLOBAL EDITION

College Algebra and Trigonometry

THIRD EDITION

J. S. Ratti • Marcus McWaters

ALWAYS LEARNING

PEARSON

College Algebra and Trigonometry

37. **Chemistry.** **Boyle's Law** states that at a constant temperature, the pressure P of a compressed gas is inversely proportional to its volume V. If the pressure is 20 pounds per square inch when the volume of the gas is 300 cubic inches, what is the pressure when the gas is compressed to 100 cubic inches?

38. **Chemistry.** In the Kelvin temperature scale, the lowest possible temperature (called absolute zero) is 0 K, where K denotes degrees Kelvin. The relationship between Kelvin temperature (T_K) and Celsius temperature (T_C) is given by $T_K = T_C + 273$.

    The pressure P exerted by a gas varies directly as its temperature T_K and inversely as its volume V. Assume that at a temperature of 260 K, a gas occupies 13 cubic inches at a pressure of 36 pounds per square inch.
    a. Find the volume of the gas when the temperature is 300 K and the pressure is 40 pounds per square inch.
    b. Find the pressure when the temperature is 280 K and the volume is 39 cubic inches.

39. **Weight.** The weight of an object varies inversely as the square of the object's distance from the center of Earth. The radius of Earth is 3960 miles.
    a. If an astronaut weighs 120 pounds on the surface of Earth, how much does she weigh 6000 miles above the surface of Earth?
    b. If a miner weighs 200 pounds on the surface of Earth, how much does he weigh 10 miles below the surface of Earth?

40. **Making a profit.** Suppose you wanted to make a profit by buying gold by weight at one altitude and selling at another altitude for the same price per unit weight. Should you buy or sell at the higher altitude? (Use the information from Exercise 39.)

In Exercises 41 and 42, use Newton's Law of Universal Gravitation. (See Example 6.)

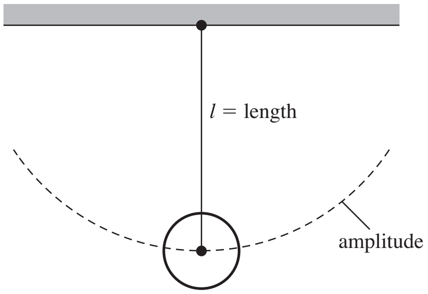
41. **Gravity on the Moon.** The mass of the Moon is about 7.4×10^{22} kilograms, and its radius is about 1740 kilometers. How much is the acceleration due to gravity on the surface of the Moon?

42. **Gravity on the Sun.** The mass of the Sun is about 2×10^{30} kilograms, and its radius is 696,000 kilometers. How much is the acceleration due to gravity on the surface of the Sun?

43. **Illumination.** The intensity I of illumination from a light source is inversely proportional to the square of the distance d from the source. Suppose the intensity is 320 candela at a distance of 10 feet from a light source.
    a. What is the intensity at 5 feet from the source?
    b. How far away from the source will the intensity be 400 candela?

44. **Speed and skid marks.** Police estimate that the speed s of a car in miles per hour varies directly as the square root of d, where d in feet is the length of the skid marks left by a car traveling on a dry concrete pavement. A car traveling 48 miles per hour leaves skid marks of 96 feet.
    a. Write an equation relating s and d.
    b. Use the equation in part (a) to estimate the speed of a car whose skid marks stretched (i) 60 feet, (ii) 150 feet, and (iii) 200 feet.
    c. Suppose you are driving 70 miles per hour and slam on your brakes. How long will your skid marks be?

45. **Simple pendulum.** *Periodic motion* is motion that repeats itself over successive equal intervals of time. The time required for one complete repetition of the motion is called the *period*. The period of a simple pendulum varies directly as the square root of its length. What is the effect on the length if the period is doubled?



(*Note:* The amplitude of a pendulum has no effect on its period. This is what makes pendulums such good timekeepers. Because they invariably lose energy due to friction, their amplitude decreases but their period remains constant.)

46. **Biology.** The volume V of a lung is directly proportional to its internal surface area A. A lung with volume 400 cubic centimeters from a certain species has an average internal surface area of 100 square centimeters. Find the volume of a lung of a member of this species if the lung's internal surface area is 120 square centimeters.

47. **Horsepower.** The horsepower H of an automobile engine varies directly as the square of the piston radius R and the number N of pistons.
    a. Write the given information in equation form.
    b. What is the effect on the horsepower if the piston radius is doubled?
    c. What is the effect on the horsepower if the number of pistons is doubled?
    d. What is the effect on the horsepower if the radius of the pistons is cut in half and the number of pistons is doubled?

48. **Safe load.** The safe load that a rectangular beam can support varies jointly as the width and square of the depth of the beam and inversely as its length. A beam 4 inches wide, 6 inches deep, and 25 feet long can support a safe load of 576 pounds. Find the safe load for a beam that is of the same material but is 6 inches wide, 10 inches deep, and 20 feet long.

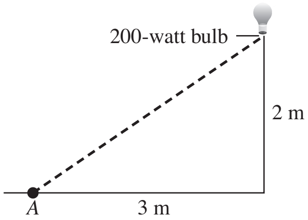
Beyond the Basics

49. **Energy from a windmill.** The energy E from a windmill varies jointly as the square of the length l of the blades and the cube of the wind velocity .
    a. Express the given information as an equation with k as the constant of variation.
    b. If blades of length 10 feet and wind velocity 8 miles per hour generate 1920 watts of electric power, find k.
    c. How much electric power would be generated if the blades were 8 feet long and the wind velocity was 25 miles per hour?
    d. If the velocity of the wind doubles, what happens to E?
    e. If the length of blades doubles, what happens to E?
    f. What happens to E if both the length of the blades and the wind velocity are doubled?

50. Intensity of light. The intensity of light, I_d, at a distance d from the source of light varies directly as the intensity I of the source and inversely as d^2. It is known that at a distance of 2 meters, a 100-watt bulb produces an intensity of approximately 2 watts per square meter.

a. Find the constant of variation, k.

b. Suppose a 200-watt bulb is located on a wall 2 meters above the floor. What is the intensity of light at a point A that is 3 meters from the wall?



c. What is the intensity of illumination at the point A in the figure if the bulb is raised by 1 meter?

51. Metabolic rate. Metabolism is the sum total of all physical and chemical changes that take place within an organism. According to the laws of thermodynamics, all of these changes will ultimately release heat, so the metabolic rate is a measure of heat production by an animal. Biologists have found that the normal resting metabolic rate of a mammal is directly proportional to the $\frac{3}{4}$ power of its body weight.

The resting metabolic rate of a person weighing 75 kilograms is 75 watts.

a. Find the constant of proportionality, k.

b. Estimate the resting metabolic rate of a brown bear weighing 450 kilograms.

c. What is the effect on metabolic rate if the body weight is multiplied by 4?

d. What is the approximate weight of an animal with a metabolic rate of 250 watts?

52. Comparing gravitational forces. The masses of the Sun, Earth, and Moon are 2×10^{30} kilograms, 6×10^{24} kilograms, and 7.4×10^{22} kilograms, respectively. The Earth–Sun distance is about 400 times the Earth–Moon distance. Use Newton's Law of Universal Gravitation to compare the gravitational attraction between the Sun and Earth with that between Earth and the Moon.

53. Kepler's Third Law. Suppose an object of mass M_1 orbits around an object of mass M_2. Let r be the average distance in meters between the centers of the two objects and let T be the *orbital period* (the time in seconds the object completes one orbit). Kepler's Third Law states that T^2 is directly proportional to r^3 and inversely proportional to $M_1 + M_2$. The constant of proportionality is $\frac{4\pi^2}{G}$.

a. Write the equation that expresses Kepler's Third Law.

b. Earth orbits the Sun once each year at a distance of about 1.5×10^8 kilometers. Find the mass of the Sun. Use these estimates:
Mass of Earth + Mass of Sun $\approx$ Mass of Sun,
$G = 6.67 \times 10^{-11}$, 1 year $= 3.15 \times 10^7$ seconds

54. Use Kepler's Third Law. The Moon orbits Earth in about 27.3 days at an average distance of about 384,000 kilometers. Find the mass of Earth. [*Hint:* Convert the orbital period, defined in Exercise 53, to seconds; also use Mass of Earth + Mass of Moon $\approx$ Mass of Earth.]

55. Spreading a rumor. Let P be the population of a community. The rate R (per day) at which a rumor spreads in the community is jointly proportional to the number N of people who have already heard the rumor and the number $(P - N)$ of people who have not yet heard the rumor.

A rumor about the president of a college is spread in his college community of 10,000 people. Five days after the rumor started, 1000 people had heard it, and it was spreading at the rate of 45 additional people per day.

a. Write an equation relating R, P, and N.

b. Find the constant k of variation.

c. Find the rate at which the rumor was spreading when one-half the college community had heard it.

d. How many people had heard the rumor when it was spreading at the rate of 100 people per day?

Critical Thinking/Discussion/Writing

56. Electricity. The current I in an electric circuit varies directly as the voltage V and inversely as the resistance R. If the resistance is increased by 20%, what percent increase must occur in the voltage to increase the current by 30%?

57. Precious stones. The value of a precious stone is proportional to the square of its weight.

a. Calculate the loss incurred by cutting a diamond worth \$1000 into two pieces whose weights are in the ratio 2:3.

b. A precious stone worth \$25,000 is accidentally dropped and broken into three pieces, the weights of which are in the ratio 5:9:11. Calculate the loss incurred due to breakage.

c. A diamond breaks into five pieces, the weights of which are in the ratio 1:2:3:4:5. If the resulting loss is \$85,000, find the value of the original diamond. Also calculate the value of a diamond whose weight is twice that of the original diamond.

58. Bus service. The profit earned in running a bus service is jointly proportional to the distance and the number of passengers over a certain fixed number. The profit is \$80 when 30 passengers are carried a distance of 40 km and is \$180 when 35 passengers are carried 60 km. What is the minimum number of passengers that results in no loss?

59. Weight of a sphere. The weight of a sphere is directly proportional to the cube of its radius. A metal sphere has a hollow space about its center in the form of a concentric sphere, and its weight is $\frac{7}{8}$ times the weight of a solid sphere of the same radius and material. Find the ratio of the inner to the outer radius of the hollow sphere.

60. Train speed. A locomotive engine can go 24 miles per hour, and its speed is reduced by a quantity that varies directly as the square root of the number of cars it pulls. Pulling four cars, its speed is 20 miles per hour. Find the greatest number of cars the engine can pull.

Maintaining Skills

In Exercises 61–70, simplify each expression.

61. 5^0

62. 2^3

63. 3^{-2}

64. $\left(\frac{1}{2}\right)^3$

65. $\left(\frac{1}{2}\right)^{-4}$

66. $2^{x-1} \cdot 2^{3-x}$

67. $5^{2x-3} \cdot 5^{3-x}$

68. $\dfrac{2^{3x-2}}{2^{x-5}}$

69. $(2^{x-1})^x$

70. $\sqrt[3]{\sqrt[4]{2}}$

In Exercises 71–76, solve each equation for the requested variable.

71. $y = mx + b$ for m

72. $ax + by = 3$ for x

73. $A = B(1 + C)$ for C

74. $A = B(1 + C)^3$ for C

75. $A = B \cdot 10^{-n}$ for B

76. $A = B \cdot 10^m + C$ for B

SUMMARY Definitions, Concepts, and Formulas

3.1 Quadratic Functions

i. A **quadratic function** f is a function of the form

$$f(x) = ax^2 + bx + c, a \neq 0.$$

ii. The **standard form** of a quadratic function is

$$f(x) = a(x - h)^2 + k, a \neq 0.$$

iii. The graph of a quadratic function is a transformation of the graph of $y = x^2$.

iv. The graph of a quadratic function is a parabola with vertex

$$(h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

v. The maximum $(\text{if } a < 0)$ or minimum $(\text{if } a > 0)$ value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at the y-coordinate of the vertex of the parabola.

3.2 Polynomial Functions

i. A function f of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_2x^2 + a_1x + a_0, a_n \neq 0$$

is a polynomial function of degree n.

ii. The graph of a polynomial function is smooth and continuous.

iii. The end behavior of the graph of a polynomial function depends on the sign of the leading coefficient and the degree (even-odd) of the polynomial.

iv. A real number c is a **zero** of a function f if $f(c) = 0$. Geometrically, c is an x-intercept of the graph of $y = f(x)$.

v. If in the factorization of a polynomial function $f(x)$ the factor $(x - a)$ occurs exactly m times, then a is a zero of **multiplicity** m. If m is odd, the graph of $y = f(x)$ crosses the x-axis at a; if m is even, the graph touches but does not cross the x-axis at a.

vi. If the degree of a polynomial function $f(x)$ is n, then $f(x)$ has, at most, n real zeros and the graph of $f(x)$ has, at most, $(n - 1)$ turning points.

vii. **Intermediate Value Theorem:** Let $f(x)$ be a polynomial function and a and b be two numbers such that $a < b$. If $f(a)$ and $f(b)$ have opposite signs, then there is at least one number c, with $a < c < b$, for which $f(c) = 0$.

viii. See page 339 for graphing a polynomial function.

3.3 Dividing Polynomials

i. **Division algorithm:** If a polynomial $F(x)$ is divided by a polynomial $D(x) \neq 0$, there are unique polynomials $Q(x)$ and $R(x)$ such that $F(x) = D(x)Q(x) + R(x)$, where either $R(x) = 0$ or $\deg R(x) < \deg D(x)$. In words, "The dividend equals the product of the divisor and the quotient plus the remainder."

ii. Synthetic division is a shortcut for dividing a polynomial $F(x)$ by $x - a$.

iii. **Remainder Theorem:** If a polynomial $F(x)$ is divided by $(x - a)$, the remainder is $F(a)$.

iv. **Factor Theorem:** A polynomial function $F(x)$ has $(x - a)$ as a factor if and only if $F(a) = 0$.

3.4 The Real Zeros of a Polynomial Function

i. **Rational Zeros Theorem:** If $\frac{p}{q}$ is a rational zero in lowest terms for a polynomial function with integer coefficients, then p is a factor of the constant term and q is a factor of the leading coefficient.

ii. **Descartes's Rule of Signs.** Let $F(x)$ be a polynomial function with real coefficients.

a. The number of positive zeros of F is equal to the number of variations of signs of $F(x)$ or is less than that number by an even integer.

b. The number of negative zeros of F is equal to the number of variations of sign of $F(-x)$ or is less than that number by an even integer.

iii. **Rules for Bounds on the Zeros.** Suppose a polynomial $F(x)$ is synthetically divided by $x - k$.

a. If $k > 0$ and each number in the last row is zero or positive, then k is an upper bound on the zeros of $F(x)$.

b. If $k < 0$ and the numbers in the last row alternate in sign, then k is a lower bound on the zeros of $F(x)$.

3.5 The Complex Zeros of a Polynomial Function

i. **Fundamental Theorem of Algebra.** An nth-degree polynomial equation has at least one complex zero.

ii. **Factorization Theorem for Polynomials.** If $P(x)$ is a polynomial of degree $n \geq 1$, it can be factored into

n (not necessarily distinct) linear factors of the form $P(x) = a(x - r_1)(x - r_2) \cdots (x - r_n)$, where a, $r_1, r_2, \ldots, r_n$ are complex numbers.

iii. **Number of Zeros Theorem.** A polynomial of degree n has exactly n complex zeros, provided a zero of multiplicity k is counted k times.

iv. **Conjugate Pairs Theorem.** If $a + bi$ is a zero of the polynomial function P (with real coefficients), then $a - bi$ is also a zero of P.

3.6 Rational Functions

i. A function $f(x) = \dfrac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomials and $D(x) \neq 0$, is called a rational function. The domain of f is the set of all real numbers except the real zeros of $D(x)$.

ii. The line $x = a$ is a vertical asymptote of the graph of f if $|f(x)| \to \infty$ as $x \to a^+$ or as $x \to a^-$.

iii. If $\dfrac{N(x)}{D(x)}$ is in lowest terms, then the graph of $F(x)$ has vertical asymptotes at the real zeros of $D(x)$.

iv. The line $y = k$ is a horizontal asymptote of the graph of f if $f(x) \to k$ as $x \to \infty$ or as $x \to -\infty$.

v. A procedure for graphing rational functions is given on page 387.

3.7 Variation

k is a nonzero constant called the constant of variation.

Variation	Equation
y varies directly with x.	$y = kx$
y varies with the nth power of x.	$y = kx^n$
y varies inversely with x.	$y = \dfrac{k}{x}$
y varies inversely with the nth power of x.	$y = \dfrac{k}{x^n}$
z varies jointly with the nth power of x and the mth power of y.	$z = kx^n y^m$
z varies directly with the nth power of x and inversely with the mth power of y.	$z = \dfrac{kx^n}{y^m}$

REVIEW EXERCISES

Basic Skills and Concepts

In Exercises 1–10, graph each quadratic function by finding (i) whether the parabola opens up or down and by finding (ii) its vertex, (iii) its axis, (iv) its x-intercepts, (v) its y-intercept, and (vi) the intervals over which the function is increasing and decreasing.

1. $y = (x - 5)^2 + 4$ **2.** $y = (x + 2)^2 - 3$

3. $y = -2(x - 3)^2 + 4$ **4.** $y = -\dfrac{1}{2}(x + 1)^2 + 2$

5. $y = -2x^2 + 3$ **6.** $y = 2x^2 + 4x - 1$

7. $y = 3x^2 - 6x + 7$ **8.** $y = -2x^2 - x + 3$

9. $y = 3x^2 - 2x + 1$ **10.** $y = 3x^2 - 5x + 4$

In Exercises 11–14, determine whether the given quadratic function has a maximum or a minimum value and then find that value.

11. $f(x) = -x^2 + 2x + 4$ **12.** $f(x) = 8x - 4x^2 - 3$

13. $f(x) = x^2 - 4x + 1$ **14.** $f(x) = \dfrac{1}{2}x^2 - \dfrac{3}{4}x + 2$

In Exercises 15–18, graph each polynomial function by using transformations on the appropriate function $y = x^n$.

15. $f(x) = (x + 1)^3 - 2$ **16.** $f(x) = (x + 1)^4 + 2$

17. $f(x) = (1 - x)^3 + 1$ **18.** $f(x) = x^4 + 3$

In Exercises 19–24, for each polynomial function f,

(i) Determine the end behavior of f.
(ii) Determine the zeros of f. State the multiplicity of each zero. Determine whether the graph of f crosses or only touches the axis at each x-intercept.
(iii) Find the x- and y-intercepts of the graph of f.
(iv) Use test numbers to find the intervals over which the graph of f is above or below the x-axis.
(v) Sketch the graph of $y = f(x)$.

19. $f(x) = x(x - 1)(x + 2)$ **20.** $f(x) = x^3 - x$

21. $f(x) = -x^2(x - 1)^2$ **22.** $f(x) = -x^3(x - 2)^2$

23. $f(x) = -x^2(x^2 - 1)$

24. $f(x) = -(x - 1)^2(x^2 + 1)$

In Exercises 25–28, divide by using long division.

25. $\dfrac{x^3 + x^2 - 11x + 2}{x - 3}$ **26.** $\dfrac{8x^2 - 14x + 15}{2x - 3}$

27. $\dfrac{8x^3 + 26x^2 - x + 10}{2x + 7}$

28. $\dfrac{x^3 - 3x^2 + 4x + 7}{x^2 - 2x + 6}$

In Exercises 29–32, divide by using synthetic division.

29. $\dfrac{3x^5 - 5x^4 - 12x^3 + 2x - 1}{x - 3}$

30. $\dfrac{-4x^3 + 3x^2 - 5x}{x - 6}$

31. $\dfrac{x^4 - x^3 - 19x^2 + 6x + 8}{x + 4}$

32. $\dfrac{3x^5 - 2x^4 + x^2 - 16x - 132}{x + 2}$

In Exercises 33–36, a polynomial function $f(x)$ and a constant c are given. Find $f(c)$ by (i) evaluating the function and (ii) using synthetic division and the Remainder Theorem.

33. $f(x) = x^3 + 4x^2 - 11x - 26; c = 3$

34. $f(x) = 2x^3 + x^2 - 15x - 2; c = -2$

35. $f(x) = x^3 + 5x^2 - 4x + 1; c = -5$

36. $f(x) = x^5 + 2; c = 1$

In Exercises 37–40, a polynomial function $f(x)$ and a constant c are given. Use synthetic division to show that c is a zero of $f(x)$. Use the result to final all zeros of $f(x)$.

37. $f(x) = x^3 - x^2 - 9x + 9; c = -3$

38. $f(x) = 2x^3 - 3x^2 - 12x + 4; c = -2$

39. $f(x) = 6x^3 - 35x^2 - 7x + 6; c = \dfrac{1}{3}$

40. $f(x) = 4x^3 + 19x^2 - 13x + 2; c = \dfrac{1}{4}$

In Exercises 41 and 42, use the Rational Zeros Theorem to list all possible rational zeros of $f(x)$.

41. $f(x) = 3x^4 - 9x^2 + 7x^3 - 7x + 6$

42. $f(x) = 9x^3 - 36x^2 - 4x + 16$

In Exercises 43–50, use Descartes's Rule of Signs and the Rational Zeros Theorem to find all real zeros of each polynomial function.

43. $f(x) = x^3 - 6x^2 + 11x - 6$

44. $f(x) = x^3 + 2x^2 - 5x - 6$

45. $f(x) = x^4 - 4x^3 - x^2 + 16x - 12$

46. $f(x) = 2x^3 - 9x^2 + 12x - 5$

47. $x^3 - 4x^2 - 5x + 14$

48. $x^3 - 5x^2 + 3x + 1$

49. $2x^3 - 5x^2 - 2x + 2$

50. $x^4 - 3x^2 + 2$

In Exercises 51–56, find all of the zeros of $f(x)$, real and nonreal.

51. $f(x) = x^3 - 7x + 6$; one zero is 2.

52. $f(x) = x^4 + x^3 - 3x^2 - x + 2$; 1 is a zero of multiplicity 2.

53. $f(x) = x^4 - 2x^3 + 6x^2 - 18x - 27$; two zeros are -1 and 3.

54. $f(x) = 4x^3 - 19x^2 + 32x - 15$; one zero is $2 - i$.

55. $f(x) = x^4 + 2x^3 + 9x^2 + 8x + 20$; one zero is $-1 + 2i$.

56. $f(x) = x^5 - 7x^4 + 24x^3 - 32x^2 + 64$; $2 + 2i$ is a zero of multiplicity 2.

In Exercises 57–68, solve each equation in the complex number system.

57. $x^3 - x^2 - 4x + 4 = 0$

58. $2x^3 + x^2 - 12x - 6 = 0$

59. $4x^3 - 7x - 3 = 0$

60. $x^4 - 3x^2 - 4$

61. $x^3 - 8x^2 + 23x - 22 = 0$

62. $x^3 - 3x^2 + 8x + 12 = 0$

63. $3x^3 - 5x^2 + 16x + 6 = 0$

64. $2x^3 - 9x^2 + 18x - 7 = 0$

65. $x^4 + x^3 - 2x^2 + 4x - 24$

66. $x^4 - x^3 - 13x^2 + x + 12 = 0$

67. $2x^4 - x^3 - 2x^2 + 13x - 6 = 0$

68. $3x^4 - 14x^3 + 28x^2 - 10x - 7 = 0$

In Exercises 69 and 70, show that the given equation has no rational roots.

69. $x^3 + 13x^2 - 6x - 2 = 0$

70. $3x^4 - 9x^3 - 2x^2 - 15x - 5 = 0$

In Exercises 71 and 72, use the Intermediate Value Theorem to find the value of the real root between 1 and 2 of each equation to two decimal places.

71. $x^3 + 6x^2 - 28 = 0$

72. $x^3 + 3x^2 - 3x - 7 = 0$

In Exercises 73–80, graph each rational function by following the five-step procedure outlined in Section 3.6.

73. $f(x) = 1 + \dfrac{1}{x}$

74. $f(x) = \dfrac{2 - x}{x}$

75. $f(x) = \dfrac{x}{x^2 - 1}$

76. $f(x) = \dfrac{x^2 - 9}{x^2 - 4}$

77. $f(x) = \dfrac{x^3}{x^2 - 9}$

78. $f(x) = \dfrac{x + 1}{x^2 - 2x - 8}$

79. $f(x) = \dfrac{x^4}{x^2 - 4}$

80. $f(x) = \dfrac{x^2 + x - 6}{x^2 - x - 12}$

Applying the Concepts

81. **Variation.** Assuming that y varies directly as x and $y = 12$ when $x = 4$, find y when $x = 5$.

82. **Variation.** Assuming that p varies inversely as q and $p = 4$ when $q = 3$, find p when $q = 4$.

83. **Variation.** Assuming that s varies directly as the square of t and $s = 20$ when $t = 2$, find s when $t = 3$.

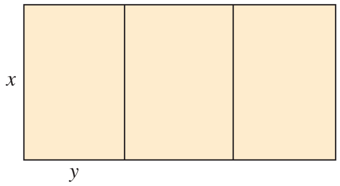
84. **Variation.** Assuming that y varies inversely as x^2 and $y = 4$ when $x = 5$, find x when $y = 25$.

85. **Missile path.** A missile fired from the origin of a coordinate system follows a path described by the equation

$y = -\frac{1}{10}x^2 + 20x$, where x is in yards and the x-axis is on the ground. Sketch the missile's path and determine what its maximum altitude is and where it hits the ground.

86. **Minimizing area.** Suppose a wire 20 centimeters long is to be cut into two pieces, each of which will be formed into a square. Find the size of each piece that minimizes the total area.

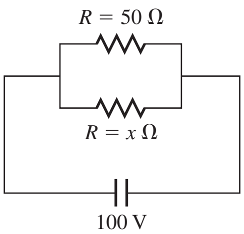
87. A farmer wants to fence off three identical adjoining rectangular pens, each 400 square feet in area. See the figure. What should the width and length of each pen be so that the least amount of fence is used?



88. Suppose the outer boundary of the pens in Exercise 87 requires heavy fence that costs \$5 per foot and two internal partitions cost \$3 per foot. What dimensions x and y will minimize the cost?

89. **Electric circuit.** In the circuit shown in the figure, the voltage $V = 100$ volts and the resistance $R = 50$ ohms. We want to determine the size of the remaining resistor (x ohms). The power absorbed by the circuit is given by

$$p(x) = \frac{V^2 x}{(R + x)^2}.$$



a. Graph the function $y = p(x)$.
b. Use a graphing calculator to find the value of x that maximizes the power absorbed.

90. **Maximizing area.** A sheet of paper for a poster is 18 square feet in area. The margins at the top and bottom are 9 inches each, and the margin on each side is 6 inches. What should the dimensions of the paper be if the printed area is to be a maximum?

91. **Maximizing profit.** A manufacturer makes and sells printers to retailers at \$24 per unit. The total daily cost C in dollars of producing x printers is given by

$$C(x) = 150 + 3.9x + \frac{3}{1000}x^2.$$

a. Write the profit P as a function of x.
b. Find the number of printers the manufacturer should produce and sell to achieve maximum profit.
c. Find the average cost $\overline{C}(x) = \frac{C(x)}{x}$. Graph $y = \overline{C}(x)$.

92. **Meteorology.** The function $p = \frac{69.1}{a + 2.3}$ relates the atmospheric pressure p in inches of mercury to the altitude a in miles from the surface of Earth.
a. Find the pressure on Mount Kilimanjaro at an altitude of 19,340 feet.
b. Is there an altitude at which the pressure is 0?

93. **Wages.** An employee's wages are directly proportional to the time he or she has worked. Sam earned \$280 for 40 hours. How much would Sam earn if he worked 35 hours?

94. **Car's stopping distance.** The distance required for a car to come to a stop after its brakes are applied is directly proportional to the square of its speed. If the stopping distance for a car traveling 30 miles per hour is 25 feet, what is the stopping distance for a car traveling 66 miles per hour?

95. **Illumination.** The amount of illumination from a source of light varies directly as the intensity of the source and inversely as the square of the distance from the source. At what distance from a light source of intensity 300 candlepower will the illumination be one-half the illumination 6 inches from the source?

96. **Chemistry.** *Charles's Law* states that at a constant pressure, the volume V of a gas is directly proportional to its temperature T (in Kelvin degrees). If a bicycle tube is filled with 1.2 cubic feet of air at a temperature of 295 K, what will the volume of the air in the tube be if the temperature rises to 310 K while the pressure stays the same?

97. **Electric circuits.** The current I (measured in amperes) in an electric circuit varies inversely as the resistance R (measured in ohms) when the voltage is held constant. The current in a certain circuit is 30 amperes when the resistance is 300 ohms.
a. Find the current in the circuit if the resistance is decreased to 250 ohms.
b. What resistance will yield a current of 60 amperes?

98. **Safe load.** The safe load that a circular column can support varies directly as the square root of its radius and inversely as the square of its length. A pillar with radius 4 inches and length 12 feet can safely support a 20-ton load. Find the load that a pillar of the same material with diameter 6 inches and length 10 feet can safely support.

99. **Spread of disease.** An infectious cold virus spreads in a community at a rate R (per day) that is jointly proportional to the number of people who are infected with the virus and the number of people in the community who are not infected yet. After the tenth day of the start of a certain infection, 15% of the total population of 20,000 people of Pollutville had been infected and the virus was spreading at the rate of 255 people per day.
a. Find the constant of proportionality, k.
b. At what rate is the disease spreading when one-half of the population is infected?
c. Find the number of people infected when the rate of infection reached 95 people per day.

100. **Coulomb's Law.** The electric charge is measured in coulombs. (The charges on an electron and a proton, which are equal and opposite, are approximately 1.602×10^{-19} coulomb.) Coulomb's Law states that the force F between two particles is jointly proportional to their charges q_1 and q_2 and inversely proportional to the square of the distance between the two particles. Two charges are acted upon by a repulsive force of 96 units. What is the force if the distance between the particles is quadrupled?