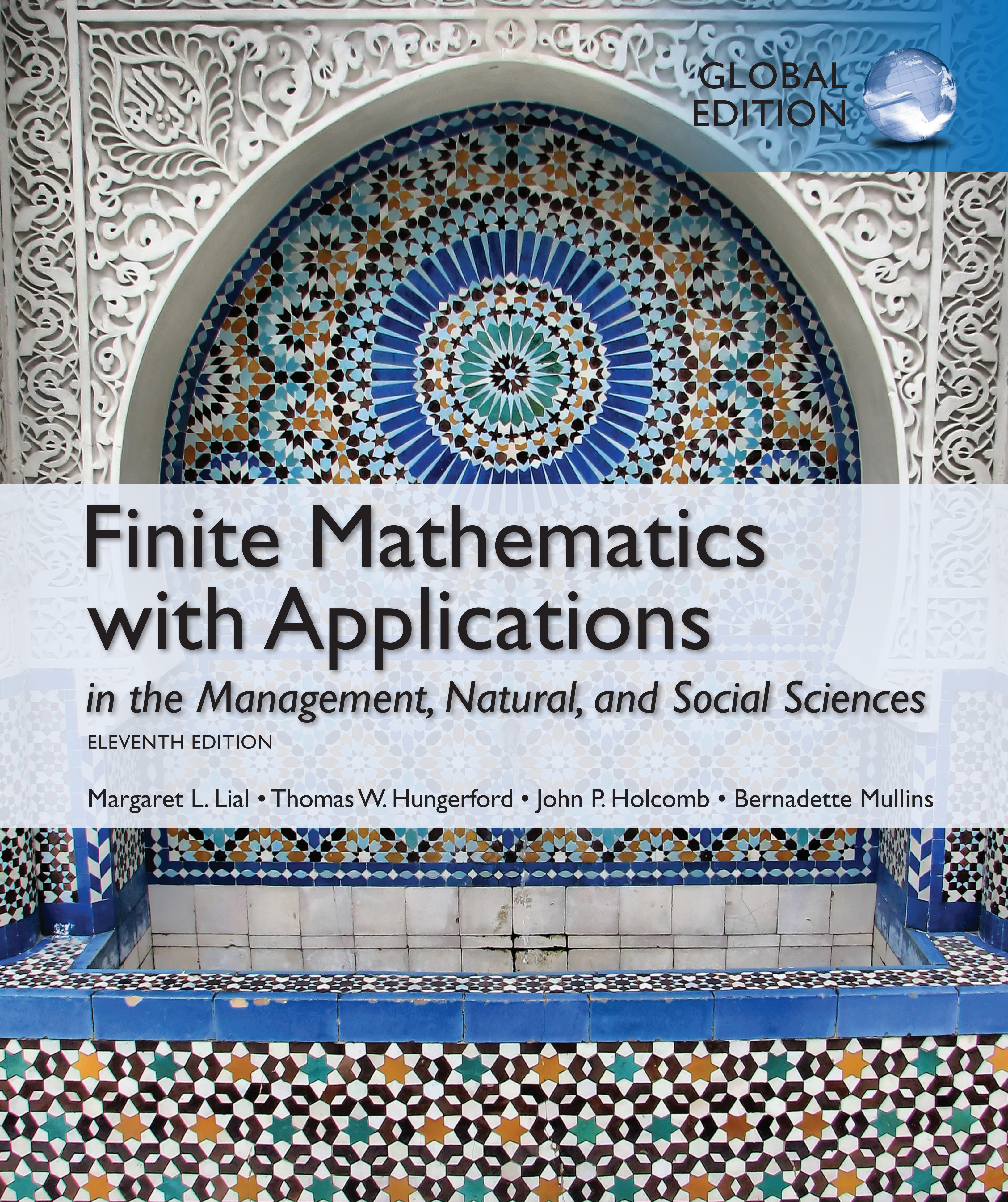

GLOBAL EDITION

Finite Mathematics with Applications

in the Management, Natural, and Social Sciences

ELEVENTH EDITION

Margaret L. Lial • Thomas W. Hungerford • John P. Holcomb • Bernadette Mullins

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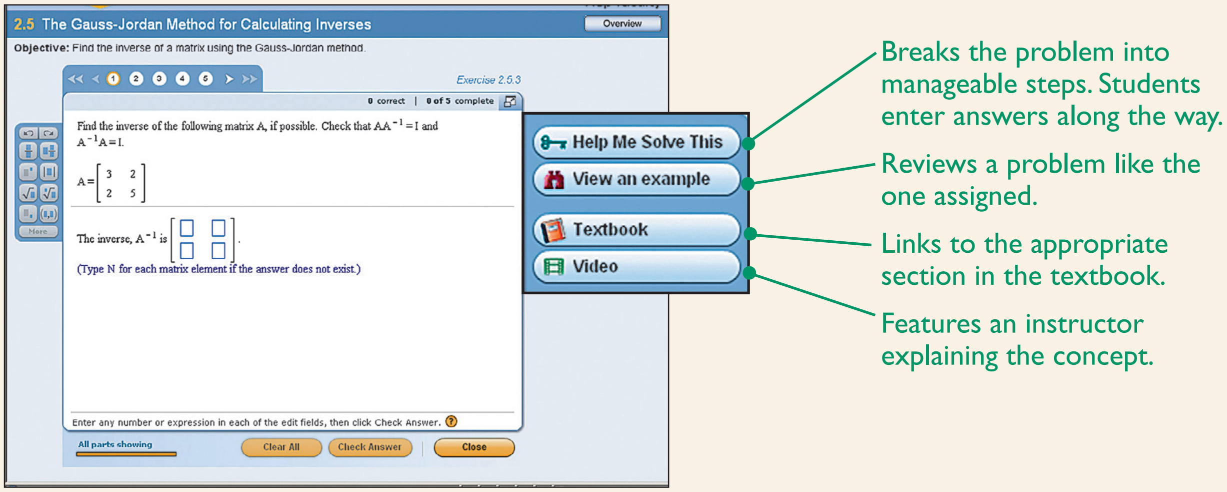

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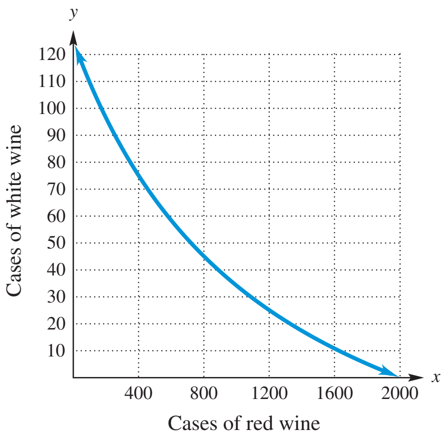


Figure 3.54

The maximum value of y occurs when $x = 0$, so the maximum amount of white wine that can be produced is 125 cases, as given by the y-intercept. The x-intercept gives the maximum amount of red wine that can be produced: 2000 cases. ✓5

Checkpoint 5

Rework Example 5 with the product-exchange function

$$y = \frac{70{,}000 - 10x}{70 + x}$$

to find the maximum amount of each wine that can be produced.

Example 6 **Business** A retailer buys 2500 specialty lightbulbs from a distributor each year. In addition to the cost of each bulb, there is a fee for each order, so she wants to order as few times as possible. However, storage costs are higher when there are fewer orders (and hence more bulbs per order to store). Past experience shows that the total annual cost (for the bulbs, ordering fees, and storage costs) is given by the rational function.

$$C(x) = \frac{.98x^2 + 1200x + 22{,}000}{x},$$

where x is the number of bulbs ordered each time. How many bulbs should be ordered each time in order to have the smallest possible cost?

Solution Graph the cost function $C(x)$ in a window with $0 \le x \le 2500$ (because the retailer cannot order a negative number of bulbs and needs only 2500 for the year).

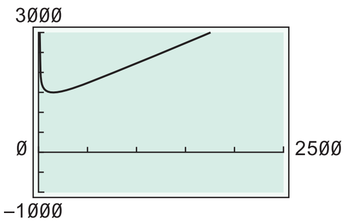


Figure 3.55

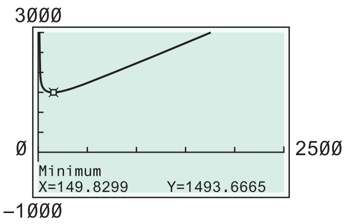


Figure 3.56

For each point on the graph in Figure 3.55,

the x-coordinate is the number of bulbs ordered each time;
the y-coordinate is the annual cost when x bulbs are ordered each time.

Use the minimum finder on a graphing calculator to find the point with the smallest y-coordinate, which is approximately (149.83, 1493.67), as shown in Figure 3.56. Since the retailer cannot order part of a lightbulb, she should order 150 bulbs each time, for an approximate annual cost of \$1494.

3.6 Exercises

Graph each function. Give the equations of the vertical and horizontal asymptotes. (See Examples 1–3.)

1. $f(x) = \frac{1}{x + 5}$

2. $g(x) = \frac{-7}{x - 6}$

3. $f(x) = \frac{-3}{2x + 5}$

4. $h(x) = \frac{-4}{2 - x}$

5. $f(x) = \frac{3x}{x - 1}$

6. $g(x) = \frac{x - 2}{x}$

7. $f(x) = \frac{x + 1}{x - 4}$

8. $f(x) = \frac{x - 3}{x + 5}$

9. $f(x) = \frac{2 - x}{x - 3}$

10. $g(x) = \frac{3x - 2}{x + 3}$

11. $f(x) = \frac{3x + 2}{2x + 4}$

12. $f(x) = \frac{4x - 8}{8x + 1}$

13. $h(x) = \frac{x + 1}{x^2 + 2x - 8}$

14. $g(x) = \frac{1}{x(x + 2)^2}$

15. $f(x) = \frac{x^2 + 4}{x^2 - 4}$

16. $f(x) = \frac{x - 1}{x^2 - 2x - 3}$

Find the equations of the vertical asymptotes of each of the given rational functions.

17. $f(x) = \frac{x - 3}{x^2 + x - 2}$

18. $g(x) = \frac{x + 2}{x^2 - 1}$

19. $g(x) = \frac{x^2 + 2x}{x^2 - 4x - 5}$

20. $f(x) = \frac{x^2 - 2x - 4}{x^3 - 2x^2 + x}$

Work these problems. (See Example 4.)

21. Natural Science Suppose a cost–benefit model is given by

$$f(x) = \frac{4.3x}{100 - x},$$

where $f(x)$ is the cost, in thousands of dollars, of removing x percent of a given pollutant. Find the cost of removing each of the given percentages of pollutants.

(a) 50% **(b)** 70% **(c)** 80%
(d) 90% **(e)** 95% **(f)** 98%
(g) 99%
(h) Is it possible, according to this model, to remove *all* the pollutant?
(i) Graph the function.

22. Business Suppose a cost–benefit model is given by

$$f(x) = \frac{6.2x}{112 - x},$$

where $f(x)$ is the cost, in thousands of dollars, of removing x percent of a certain pollutant. Find the cost of removing the given percentages of pollutants.

(a) 0% **(b)** 50%
(c) 80% **(d)** 90%
(e) 95% **(f)** 99%
(g) 100% **(h)** Graph the function.

23. Natural Science The function

$$f(x) = \frac{\lambda x}{1 + (ax)^b}$$

is used in population models to give the size of the next generation $f(x)$ in terms of the current generation x. (See J. Maynard Smith, *Models in Ecology* [Cambridge University Press, 1974].)

(a) What is a reasonable domain for this function, considering what x represents?
(b) Graph the function for $\lambda = a = b = 1$ and $x \geq 0$.
(c) Graph the function for $\lambda = a = 1$ and $b = 2$ and $x \geq 0$.
(d) What is the effect of making b larger?

24. Natural Science The function

$$f(x) = \frac{Kx}{A + x}$$

is used in biology to give the growth rate of a population in the presence of a quantity x of food. This concept is called Michaelis–Menten kinetics. (See Leah Edelstein-Keshet, *Mathematical Models in Biology* [Random House, 1988].)

(a) What is a reasonable domain for this function, considering what x represents?
(b) Graph the function for $K = 5$, $A = 2$, and $x \geq 0$.
(c) Show that $y = K$ is a horizontal asymptote.
(d) What do you think K represents?
(e) Show that A represents the quantity of food for which the growth rate is half of its maximum.

25. Social Science The average waiting time in a line (or queue) before getting served is given by

$$W = \frac{S(S - A)}{A},$$

where A is the average rate at which people arrive at the line and S is the average service time. At a certain fast-food restaurant, the average service time is 3 minutes. Find W for each of the given average arrival times.

(a) 1 minute
(b) 2 minutes
(c) 2.5 minutes
(d) What is the vertical asymptote?
(e) Graph the equation on the interval (0, 3).
(f) What happens to W when $A > 3$? What does this mean?

Business *Sketch the portion of the graph in Quadrant I of each of the functions defined in Exercises 26 and 27, and then estimate the maximum quantities of each product that can be produced. (See Example 5.)*

26. The product-exchange function for gasoline x and heating oil y, in hundreds of gallons per day, is

$$y = \frac{125{,}000 - 25x}{125 + 2x}.$$

27. A drug factory found that the product-exchange function for a red tranquilizer x and a blue tranquilizer y is

$$y = \frac{900{,}000{,}000 - 30{,}000x}{x + 90{,}000}.$$

28. **Physical Science** The failure of several O-rings in field joints was the cause of the fatal crash of the *Challenger* space shuttle in 1986. NASA data from 24 successful launches prior to *Challenger* suggested that O-ring failure was related to launch temperature by a function similar to

$$N(t) = \frac{600 - 7t}{4t - 100} \quad (50 \le t \le 85),$$

where t is the temperature (in °F) at launch and N is the approximate number of O-rings that fail. Assume that this function accurately models the number of O-ring failures that would occur at lower launch temperatures (an assumption NASA did not make).

- **(a)** Does $N(t)$ have a vertical asymptote? At what value of t does it occur?
- **(b)** Without actually graphing the function, what would you conjecture that the graph would look like just to the right of the vertical asymptote? What does this suggest about the number of O-ring failures that might be expected near that temperature? (The temperature at the *Challenger* launching was 31°.)
- **(c)** Confirm your conjecture by graphing $N(t)$ between the vertical asymptote and $t = 85$.

29. **Business** A company has fixed costs of $40,000 and a marginal cost of $2.60 per unit.

- **(a)** Find the linear cost function.
- **(b)** Find the average cost function. (Average cost was defined in Section 3.3.)
- **(c)** Find the horizontal asymptote of the graph of the average cost function. Explain what the asymptote means in this situation. (How low can the average cost be?)

Use a graphing calculator to do Exercises 30–33. (See Example 6.)

30. **Finance** Another model of a Laffer curve (see Exercise 27 of Section 3.5) is given by

$$f(x) = \frac{300x - 3x^2}{10x + 200},$$

where $f(x)$ is government revenue (in billions of dollars) from a tax rate of x percent. Find the revenue from the given tax rates.

- **(a)** 16%
- **(b)** 25%
- **(c)** 40%
- **(d)** 55%
- **(e)** Graph $f(x)$.
- **(f)** What tax rate produces maximum revenue? What is the maximum revenue?

31. **Business** When no more than 110 units are produced, the cost of producing x units is given by

$$C(x) = .2x^3 - 25x^2 + 1531x + 25{,}000.$$

How many units should be produced in order to have the lowest possible average cost?

32. **(a)** Graph $f(x) = \dfrac{x^3 + 3x^2 + x + 1}{x^2 + 2x + 1}$.

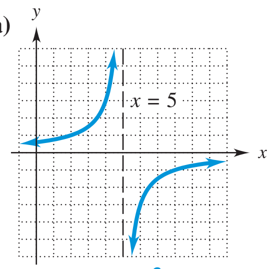
- **(b)** Does the graph appear to have a horizontal asymptote? Does the graph appear to have some nonhorizontal straight line as an asymptote?
- **(c)** Graph $f(x)$ and the line $y = x + 1$ on the same screen. Does this line appear to be an asymptote of the graph of $f(x)$?

33. **(a)** Graph $g(x) = \dfrac{x^3 - 2}{x - 1}$ in the window with $-5 \le x \le 5$ and $-6 \le y \le 12$.

- **(b)** Graph $g(x)$ and the parabola $y = x^2 + x + 1$ on the same screen. How do the two graphs compare when $|x| \ge 2$?

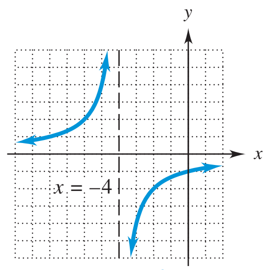
Checkpoint Answers

1. (a)



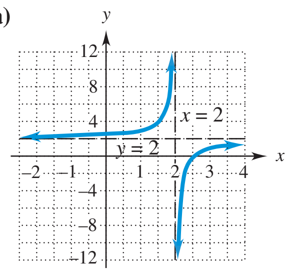
$f(x) = \dfrac{3}{5 - x}$

(b)

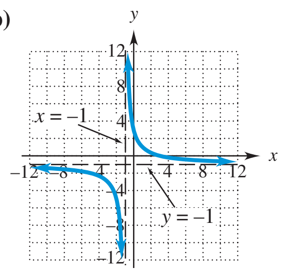


$f(x) = \dfrac{-4}{x + 4}$

2. (a)



(b)



3. **(a)** Vertical, $x = -5$; horizontal, $y = 3$

(b) Vertical, $x = -2$ and $x = 2$; horizontal, $y = -1$

4. **(a)** $35,000

(b) About $73,000

(c) About $221,000

5. 7000 cases of red, 1000 cases of white

CHAPTER 3 Summary and Review

Key Terms and Symbols

3.1 function
domain
range
functional notation
piecewise-defined function
graph reading
vertical-line test

3.2 graph
linear function
piecewise linear function
absolute-value function
greatest-integer function
step function

3.3 fixed costs
variable cost
average cost
linear depreciation
rate of change
marginal cost
linear cost function
linear revenue function
break-even point
supply and demand curves
equilibrium point
equilibrium price
equilibrium quantity

3.4 quadratic function
parabola
vertex
axis
quadratic model

3.5 polynomial function
graph of $f(x) = ax^n$
properties of polynomial graphs
polynomial models

3.6 rational function
linear rational function
vertical asymptote
horizontal asymptote

Chapter 3 Key Concepts

Functions

A **function** consists of a set of inputs called the **domain,** a set of outputs called the **range,** and a rule by which each input determines exactly one output.

If a vertical line intersects a graph in more than one point, the graph is not that of a function.

Linear Functions

A **linear cost function** has equation $C(x) = mx + b$, where m is the **marginal cost** (the cost of producing one more item) and b is the **fixed cost.**

If $p = f(q)$ gives the price per unit when q units can be supplied and $p = g(q)$ gives the price per unit when q units are demanded, then the **equilibrium price** and **equilibrium quantity** occur at the q-value such that $f(q) = g(q)$.

Quadratic Functions

The **quadratic function** defined by $f(x) = a(x - h)^2 + k$ has a graph that is a **parabola** with vertex (h, k) and axis of symmetry $x = h$. The parabola opens upward if $a > 0$ and downward if $a < 0$.

If the equation is in the form $f(x) = ax^2 + bx + c$, the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Polynomial Functions

When $|x|$ is large, the graph of a **polynomial function** resembles the graph of its highest-degree term ax^n. The graph of $f(x) = ax^n$ is described on page 180.

On the graph of a polynomial function of degree n,

the total number of peaks and valleys is at most $n - 1$;

the total number of x-intercepts is at most n.

Rational Functions

If a number c makes the denominator of a **rational function** 0, but the numerator nonzero, then the line $x = c$ is a **vertical asymptote** of the graph.

Whenever the values of y approach, but do not equal, some number k as $|x|$ gets larger and larger, the line $y = k$ is a **horizontal asymptote** of the graph.

If the numerator of a rational function is of *smaller* degree than the denominator, then the x-axis is the horizontal asymptote of the graph.

If the numerator and denominator of a rational function are of the *same* degree, say, $f(x) = \frac{ax^n + \cdots}{cx^n + \cdots}$, then the line $y = \frac{a}{c}$ is the horizontal asymptote of the graph.

Chapter 3 Review Exercises

In Exercises 1–6, state whether the given rule defines a function or not.

1.

x	3	2	1	0	1	2
y	8	5	2	0	−2	−5

2.

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

3. $y = \sqrt{x}$

4. $y = x^{\frac{1}{3}}$

5. $x = y^2 + 1$

6. $y = 5x - 2$

For the functions in Exercises 7–10, find

(a) $f(6)$; (b) $f(-2)$; (c) $f(p)$; (d) $f(r + 1)$.

7. $f(x) = 4x - 1$ **8.** $f(x) = 3 - 4x$

9. $f(x) = -x^2 + 2x - 4$ **10.** $f(x) = 8 - x - x^2$

11. Let $f(x) = 5x - 3$ and $g(x) = -x^2 + 4x$. Find each of the following:

(a) $f(-2)$ **(b)** $g(3)$ **(c)** $g(-k)$

(d) $g(3m)$ **(e)** $g(k - 5)$ **(f)** $f(3 - p)$

12. Let $f(x) = 3x^2 + 4x + 8$. Find each of the following:

(a) $f(2)$ **(b)** $f(-1)$ **(c)** $f(4)$

(d) Based on your answers in parts (a)–(c), is it true that $f(a + b) = f(a) + f(b)$ for all real number a and b?

Graph the functions in Exercises 13–24.

13. $f(x) = |x| - 3$ **14.** $f(x) = -|x| - 2$

15. $f(x) = -|x + 1| + 3$ **16.** $f(x) = 2|x - 3| - 4$

17. $f(x) = [x - 3]$ **18.** $f(x) = \left[\frac{1}{2}x - 2\right]$

19. $f(x) = \begin{cases} -4x + 2 & \text{if } x \le 1 \\ 3x - 5 & \text{If } x > 1 \end{cases}$

20. $f(x) = \begin{cases} 3x + 1 & \text{if } x < 2 \\ -x + 4 & \text{if } x \ge 2 \end{cases}$

21. $f(x) = \begin{cases} |x| & \text{if } x < 3 \\ 6 - x & \text{if } x \ge 3 \end{cases}$

22. $f(x) = \sqrt{x^2}$

23. $g(x) = x^2/8 - 3$

24. $h(x) = \sqrt{x} + 2$

25. Business Let f be a function that gives the cost to rent a power washer for x hours. The cost is a flat $45 for renting the washer, plus $20 per day or fraction of a day for using the washer.

(a) Graph f.

(b) Give the domain and range of f.

(c) John McDonough wants to rent the washer, but he can spend no more than $90. What is the maximum number of days he can use it?

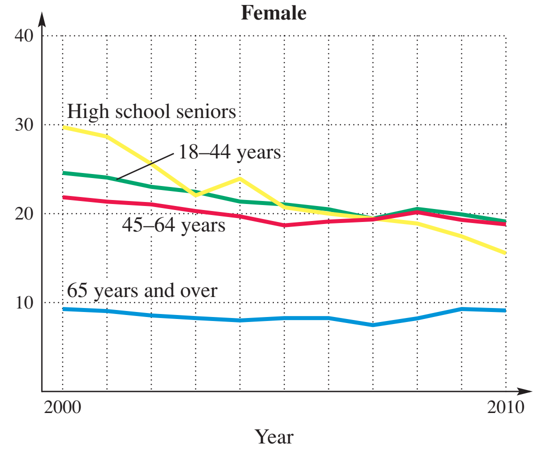
26. Health A city hospital charges $20 per hour for hospitalization.The consultant charges are $100 per day. If a patient is hospitalized for x days, then

(a) Define a function $Y = f(x)$.

(b) Graph the function.

(c) If $x = 10$ days, then find the cost to the patient.

27. Health The following graph, which tracks the percentage of the female population of the United States who smoke cigarettes, appeared in a report from the Centers for Disease Control and Prevention.

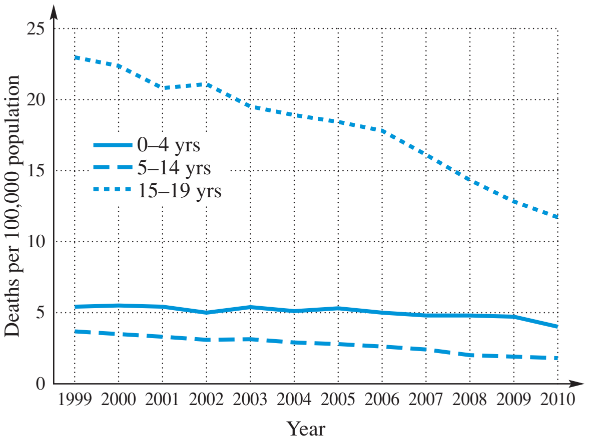


(a) What is the general trend for female high school seniors? What is the general trend for females 65 years and over?

(b) Let $x = 0$ correspond to the year 2000. Approximate a linear function for female high school seniors using the points (0, 30) and (10, 16).

(c) If the trend continues, estimate the percentage of female high school seniors who smoke cigarettes in 2012.

28. Health The following graph, which tracks the rate of traumatic brain injury-related deaths per 100,000 people, appeared on a website for the Centers for Disease Control and Prevention.



(a) What is the general trend for youth age 15–19 years?

(b) Let $x = 9$ correspond to the year 1999. Approximate a linear function to fit the data using 22 as the number of deaths in 2000 and 11 for 2010.

(c) If the trend continues, estimate the deaths per 100,000 people in the year 2012.

Business *In Exercises 29–32, find the following:*

(a) *the linear cost function;*

(b) *the marginal cost;*

(c) *the average cost per unit to produce 100 units.*

29. Fifteen unit cost $625; ten unit cost $425

30. Fixed cost is $2000; 36 units cost $8480.

31. Twelve units cost $445; 50 units cost $1585.

32. Ten unit cost $345; eighteen unit cost $545.

33. **Business** The cost of producing x ink cartridges for a printer is given by $C(x) = 24x + 18{,}000$. Each cartridge can be sold for \$28.

(a) What are the fixed costs?

(b) Find the revenue function.

(c) Find the break-even point.

(d) If the company sells exactly the number of cartridges needed to break even, what is its revenue?

34. **Business** The cost of producing x laser printers is given by $C(x) = 325x + 2{,}300{,}000$. Each printer can be sold for \$450.

(a) What are the fixed costs?

(b) Find the revenue function.

(c) Find the break-even point.

(d) If the company sells exactly the number of printers needed to break even, what is its revenue?

35. **Business** A manufacturer of batteries estimates that when x batteries are manufactured each month, the total cost will be $C(x) = \frac{1}{10}x^2 + 3x + 20$. All batteries can be sold at a price of $p(x) = 70 - x$ dollars per unit. What are the fixed costs? Also determine the break-even point.

36. **Business** Suppose the supply and price for prescription-strength Tylenol are related by $p = .0015q + 1$, where p is the price (in dollars) of a 30-day prescription. If the demand is related to price by $p = -.0025q + 64.36$, what are the equilibrium quantity and price?

Without graphing, determine whether each of the following parabolas opens upward or downward, and find its vertex.

37. $f(x) = 3(x - 2)^2 + 6$

38. $f(x) = \sqrt{x + 2} - 2$

39. $g(x) = -4(x + 1)^2 + 8$

40. $g(x) = -5(x - 4)^2 - 6$

Graph each of the following quadratic functions, and label its vertex.

41. $f(x) = x^2 - 9$

42. $f(x) = 5 - 2x^2$

43. $f(x) = x^2 + 2x - 6$

44. $g(x) = -x^2 - x + \frac{15}{4}$

45. $f(x) = -x^2 - 6x + 5$

46. $f(x) = 5x^2 + 20x - 2$

47. $f(x) = 2x^2 - 12x + 10$

48. $f(x) = -3x^2 - 12x - 2$

Determine whether the functions in Exercises 49–52 have a minimum or a maximum value, and find that value.

49. $f(x) = \frac{1}{3}x^3 + x^2 + 4$

50. $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - 2x^2 + 4x + 6$

51. $g(x) = -4x^2 + 8x + 3$

52. $f(x) = x^3 - 3x^2 + 3x - 1$

Solve each problem.

53. **Business** The weekly future price for a barrel of oil can be approximated by the function $f(x) = -.002x^2 + .644x + 45.32$, where x is the number of weeks since the beginning of the year 2009. How many weeks from the beginning of the year 2009 until the peak oil futures price occurs? What is the peak oil futures price with this model? (Data from: U.S. Energy Information Administration.)

54. **Social Science** The percent of children living below the poverty line can be approximated by the function $f(x) = .096x^2 - 2.40x + 31.7$, where $x = 6$ corresponds to the year 1996. What year saw the lowest percent living in poverty? What was the lowest percent?

55. **Social Science** The declining birth rate in China has demographers believing that the population of China will soon peak. A model for the population (in millions) for China is $g(x) = -.173x^2 + 12.86x + 1152$, where $x = 0$ corresponds to the year 1990. According to this model, what year will China reach its peak population? What is the estimate of the peak population? (Data from: U.S. Census Bureau.)

56. **Business** The amount of energy (in kilowatt hours) a new refrigerator typically used for a year of operation can be approximated by the function $h(x) = -1.44x^2 + 220x - 6953$, where $x = 50$ corresponds to the year 1950. According to this model, what year did peak electrical use occur? What was that amount? (Data from: U.S. Energy Information Administration.)

57. **Business** The following table shows the average cost of tuition and fees at private colleges in various years. (Data from: U.S. Center for Educational Statistics.)

Year	Cost
1975	3672
1980	5470
1985	8885
1990	12,910
1995	17,208
2000	21,373
2005	26,908
2010	32,026

(a) Let $x = 5$ correspond to the year 1975. Find a quadratic function $f(x) = a(x - h)^2 + k$ that models these data using (5, 3672) as the vertex and the data for 2005.

(b) Estimate the average tuition and fees at private colleges in 2015.

58. **Social Science** According the U.S. Department of Justice, the following table gives the number (in thousands) of violent crimes committed in the given years.

Year	Crimes
2004	1360
2005	1391
2006	1418
2007	1408
2008	1393
2009	1326
2010	1246

(a) Let $x = 4$ correspond to the year 2004. Find a quadratic function $f(x) = a(x - h)^2 + k$ that models these data using (6, 1418) as the vertex and the data for 2009.

(b) Estimate the number of violent crimes in 2012.