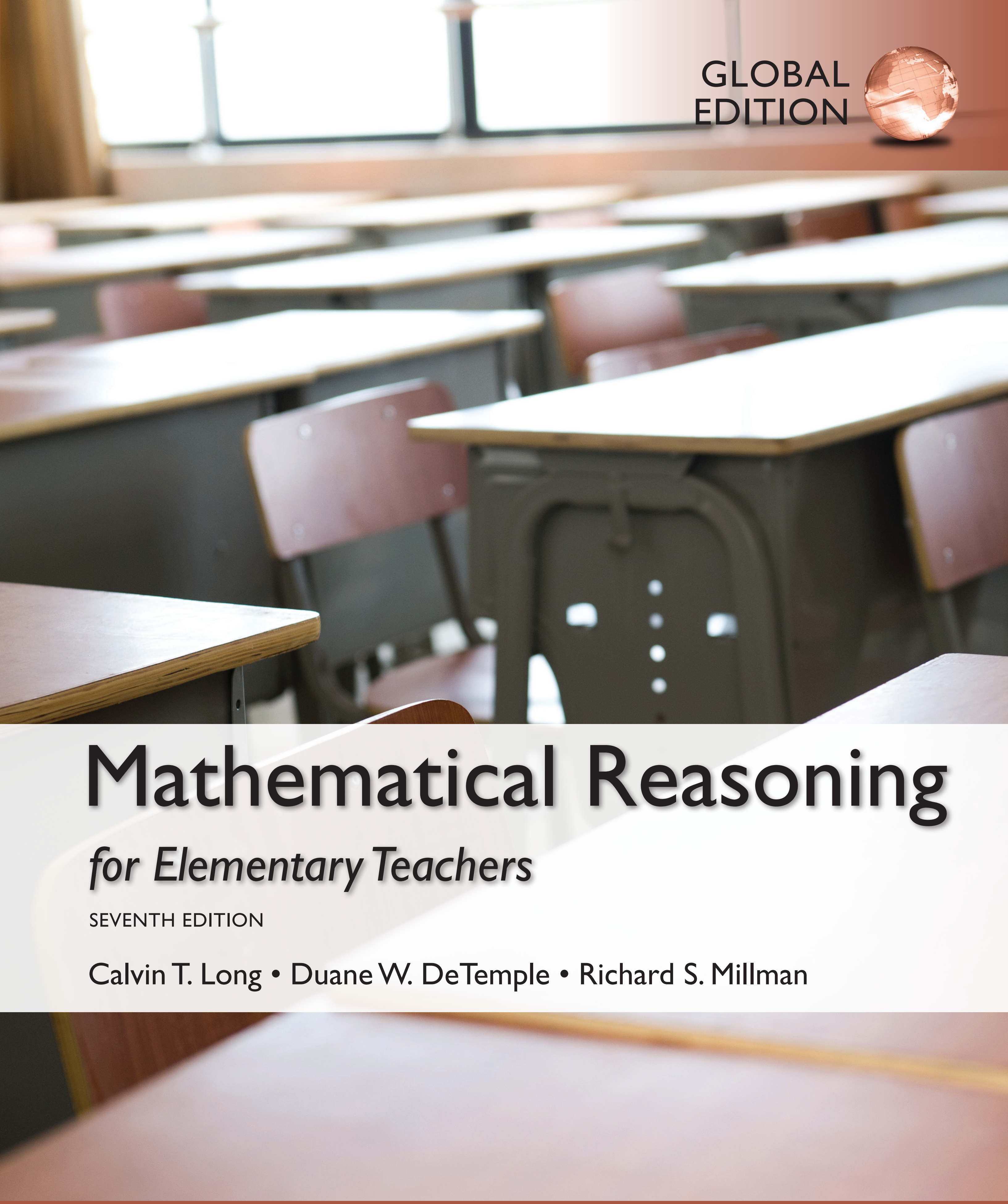

GLOBAL EDITION

Mathematical Reasoning

for Elementary Teachers

SEVENTH EDITION

Calvin T. Long • Duane W. DeTemple • Richard S. Millman

ALWAYS LEARNING

PEARSON

Mathematical Reasoning

FOR

ELEMENTARY TEACHERS

This book presents the mathematical knowledge needed for teaching, with an emphasis on why future teachers are learning the content, as well as when and how they will use it in the classroom. The Seventh Edition teaches the ***content in context*** to prepare today's students for tomorrow's classroom.

The **Common Core State Standards for Mathematics** include **8 Standards for Mathematical Practice (SMP)**, which have been integrated throughout this text. It's important for future teachers to know what will be expected of them when they are in the classroom, and these SMP references ensure that future teachers be both familiar and comfortable with these mathematical practices. Instances where an SMP applies are called out with an icon and highlighted text.

Continuing from the Common Core, the following eight Standards for Mathematical Practice are designed to teach students to

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice elaborate and reinforce the importance of Pólya's four principles of problems solving. In particular, special attention is given to the fourth principle, to

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quantities and their operation including differing units, such as cm, cm^2, cm^3, Fahrenheit versus Celsius temperature, and so on. Computations with different units can cause a real change in a problem. Unfortunately, you will see an example of a disaster in the paragraph immediately after SMP 2.

"Mathematically proficient students make sense of quantities and their relationships in problem situations . . . Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them. . . ."

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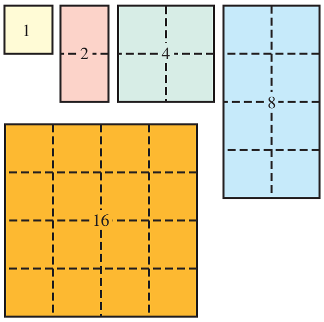
COOPERATIVE INVESTIGATION

Numbers from Rectangles

Material Needed

1. One rectangle of each of these shapes for each student:



2. One record sheet like this for each student:

	16	8	4	2	1
0	0	0	0	0	0
1	0	0	0	0	1
18	1	0	0	1	0
19	1	0	0	1	1

	16	8	4	2	1
20	1	0	1	0	0
21					
38					
39					

Directions

Step 1. Use the rectangles to determine whether or not there are representations of each of the numbers 0, 1, 2, . . . , 39 as a sum of the numbers 1, 2, 4, 8, or 16, with each of the latter group of numbers used at most once.

Step 2. For each representation determined in step 1, record the numbers (rectangles) used by placing a 0 or a 1 in the appropriate columns of the record sheet. The rows for 0, 1, 18, 19, and 20 have been done for you.

(a) Do all the numbers from 0 through 39 have such a representation?

(b) What additional numbers could be represented if you had a 32 rectangle?

(c) Describe any interesting patterns you see on your record sheet.

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COOPERATIVE INVESTIGATIONS begin each chapter, offering content-related games and puzzles that motivate the chapter. These can be easily adapted for use in the elementary classroom.

Bringing something *to* you is adding.

Addition of Integers by Using Mail-Time Stories

A second useful approach to the addition of integers is by means of mail-time stories.

At mail time, suppose you receive a check for \$13 and another check for \$6. Are you richer or poorer, and by how much? Answer: Richer by \$19. This story illustrates that $13 + 6 = 19$.

EXAMPLE 5.10

Adding Integers by Using Mail-Time Stories

Write the addition equation illustrated by each of these stories:

(a) At mail time, you receive a bill for \$2 and another bill for \$4. Are you richer or poorer, and by how much?

(b) At mail time, you receive a bill for \$3 and a check for \$5. Are you richer or poorer, and by how much?

(c) At mail time, you receive a check for \$5 and a bill for \$7. Are you richer or poorer, and by how much?

(d) At mail time, you receive a check for \$4 and a bill for \$4. Are you richer or poorer, and by how much?

Solution

(a) Receiving a bill for \$2 and another bill for \$4 makes you \$6 poorer. The story illustrates that $(-2) + (-4) = -6$.

(b) Receiving a bill for \$3 and a check for \$5 makes you \$2 richer. The story illustrates that $(-3) + 5 = 2$.

(c) Receiving a check for \$5 makes you richer by \$5, but receiving a bill for \$7 makes you \$7 poorer. The net effect is that you are \$2 poorer. The story illustrates that $5 + (-7) = -2$.

(d) Receiving a \$4 check and a \$4 bill exactly balances out, and you are neither richer nor poorer. The story illustrates that $4 + (-4) = 0$.

Note that these results are exactly the results of Example 5.8. Moreover, the arguments hold in general, and we are again led to the same definition of addition of integers.

INTO THE CLASSROOM

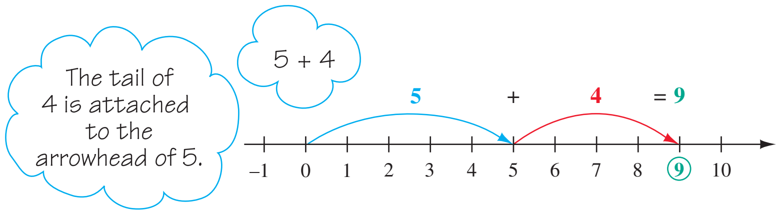
Kristin Hanley Using Contextual Narratives to Build Understanding of Integers

Similar to mail-time stories, I use contextual narratives to build understanding in my classroom. Let's say I had \$3 but wanted to buy a book at the book fair that cost \$5. My friend loaned me exactly how much I needed in order to buy the book. How much would I owe my friend? Most students can quickly determine I would owe \$2. Then I applaud them for their knowledge of negative numbers! They always look surprised. We introduce a way to write the equation they solved $3 + (-5) = -2$. I use the colored counters to represent the amounts of money spent and money owed in the story. Then the students are given sets of counters to complete new stories. They love to create their own stories where they gain and lose points while playing video games. These are similar to the mail-time stories where points are gained and lost continuously and present a high level of engagement for my students.

Addition of Integers by Using a Number Line

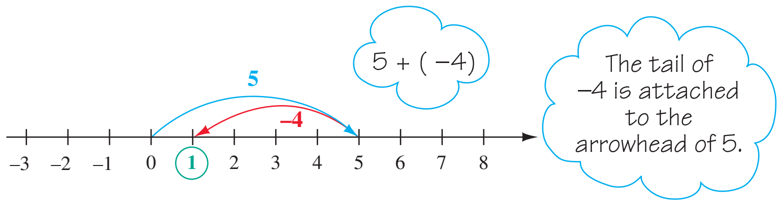
Suppose we want to illustrate $5 + 4$ on a number line. This addition can be thought of as starting at 0 and then combining a jump of 5 with a jump of 4. Figure 5.11 shows that we arrive at the point 9 on the number-line. Thus, $5 + 4 = 9$.

Figure 5.11
Illustrating $5 + 4 = 9$ on a number line



The integer -4 is represented by a left-pointing arrow of length 4. Thus, the addition $5 + (-4)$ is depicted on the number line as in Figure 5.12, where the right-pointing 5 arrow is combined with the left-pointing -4 arrow to show that $5 + (-4) = 1$.

Figure 5.12
Illustrating $5 + (-4) = 1$ on a number line

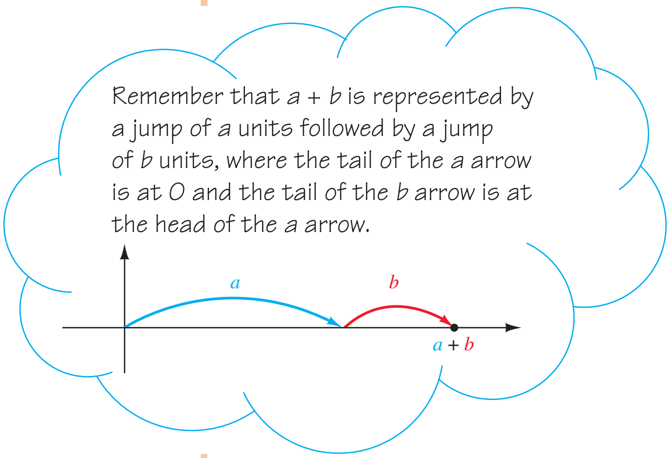


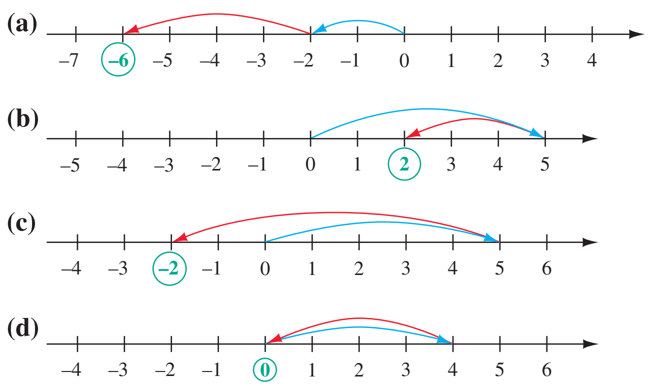
If we had first made a jump -4 to the left and then a jump 5 to the right, we would still arrive at 1. That is, $(-4) + 5 = 1$.

EXAMPLE 5.11

Adding Integers on a Number Line

What addition fact is illustrated by each of these diagrams?





Solution

Since counting in the direction indicated by an arrow means adding, these diagrams represent the following sums:

(a) $(-2) + (-4) = -6$ **(b)** $5 + (-3) = 2$
(c) $5 + (-7) = -2$ **(d)** $4 + (-4) = 0$

The results are the same as in Example 5.8.

EXAMPLE 5.12

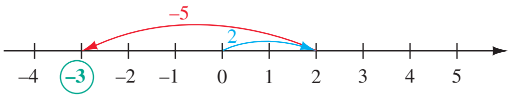
Drawing Number-Line Addition Diagrams

Draw diagrams to illustrate the following integer additions:

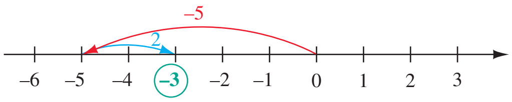
(a) $2 + (-5)$ **(b)** $(-5) + 2$

Solution

(a) A jump of 2 units to the right is followed by a jump to the left of 5 units, arriving at the point -3 on the number line. Therefore, $2 + (-5) = -3$.



(b) The arrows are now drawn in the reverse order, with a jump to the left of 5 units followed by a jump to the right by 2 units. This shows that $(-5) + 2 = -3$.



The diagrams confirm an instance of the commutative property of addition:

$$2 + (-5) = (-5) + 2.$$

Subtraction of Integers

Our conceptual models of subtraction—take away, missing addend, number line—will continue to be our guide as we extend subtraction to the integers. As with addition, we make subtraction meaningful by interpreting the operation with colored counters, mail-time stories, and jumps on the number line.

Subtraction of Integers with Colored Counters

Consider the subtraction

$$7 - (-3).$$

Modeling 7 with counters, we must "take away" a representation of -3. Since any representation of -3 must have at least three red counters, we must use a representation of 7 with at least three red counters. The simplest representation of this subtraction is shown in Figure 5.13.

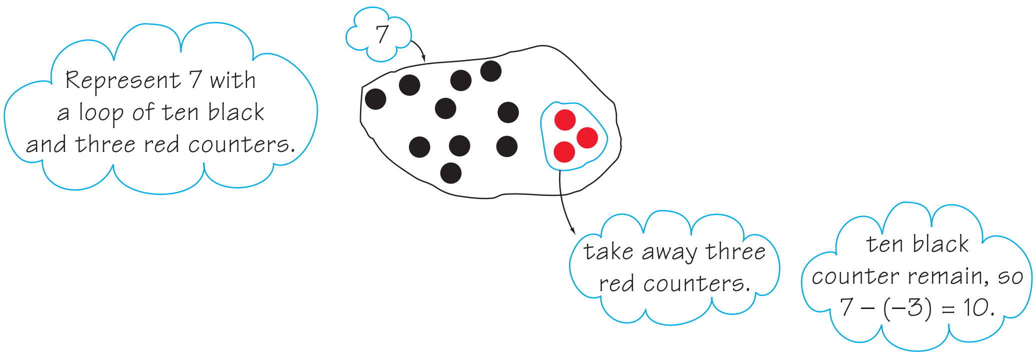


Figure 5.13 Colored-counter representation of $7 - (-3) = 10$

Thus, $7 - (-3) = 10$. The result does not change if we use another representation of 7 with at least three red counters. A second representation of $7 - (-3) = 10$ is shown in Figure 5.14, where 7 is represented with a loop containing 14 black and 7 red counters.

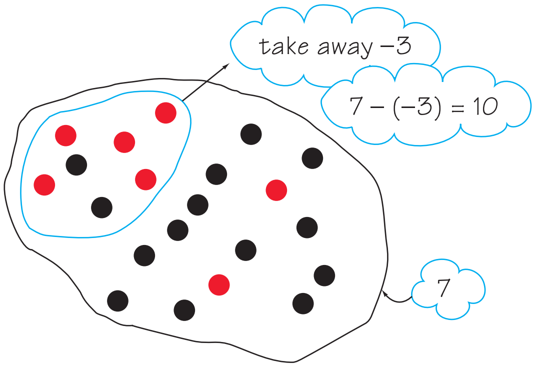
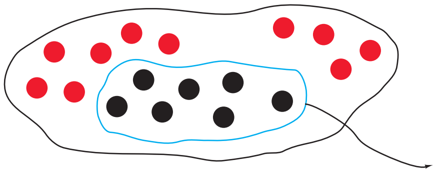


Figure 5.14
A second colored-counter representation of $7 - (-3) = 10$

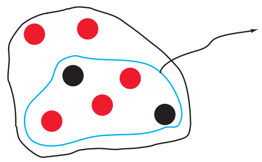
EXAMPLE 5.13 Subtracting by Using Colored Counters

Write out the subtraction equations illustrated by these diagrams:

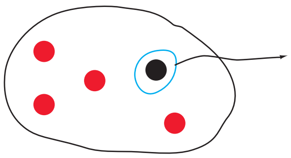
(a)



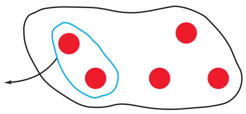
(b)



(c)



(d)



Solution

(a) Since the large loop contains ten red counters and seven black counters, it represents -3. The small loop contains seven black counters and thus represents 7. If we remove the small loop of counters as indicated, we are left with ten red counters, representing -10. Thus, this diagram illustrates the subtraction $(-3) - 7 = -10$.
(b) This diagram represents the subtraction $(-3) - (-1) = -2$.
(c) This diagram represents the subtraction $(-3) - 1 = -4$.
(d) This diagram represents the subtraction $(-5) - (-2) = -3$.

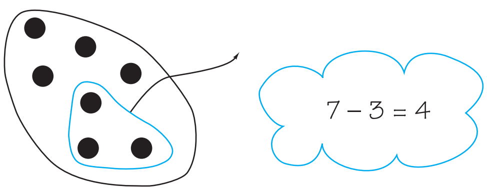
EXAMPLE 5.14 Drawing Diagrams for Given Subtraction Problems

Draw a diagram of colored counters to illustrate each of these subtractions, and determine the result in each case:

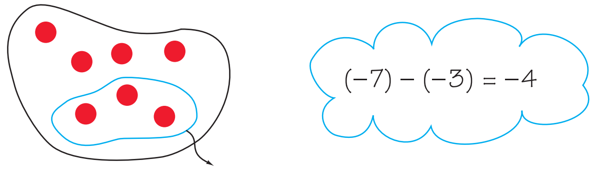
(a) $7 - 3$ **(b)** $(-7) - (-3)$ **(c)** $7 - (-3)$ **(d)** $(-7) - 3$

Solution

(a) Many different diagrams could be drawn, but the simplest is shown here:

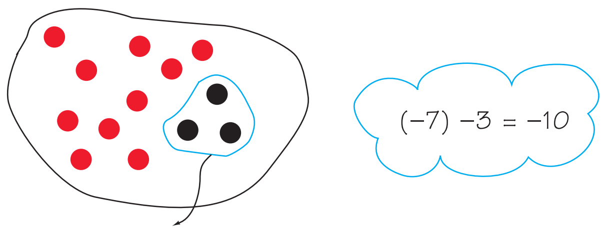


(b) As in part (a), we can use a diagram with counters of only one color:



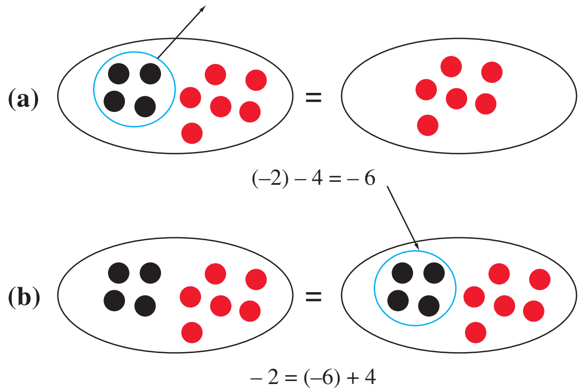
(c) This subtraction is illustrated in Figures 5.13 and 5.14.

(d) Here, in order to remove three black counters (that is, subtract 3), the representation for -7 must have at least three black counters. The simplest diagram is the following:



The colored-counter model illustrates why subtraction is the inverse of addition. For example, Figure 5.15a shows that $(-2) - 4 = -6$, since removing four black counters from the loop 4B6R that represents -2 leaves a loop with six red counters. However, if the procedure is done in reverse order, as in Figure 5.15b, we see that a loop representing -2 is obtained by adding four black counters to a loop of six red counters. Thus, $-2 = (-6) + 4$.

Figure 5.15
Showing the subtraction $(-2) - 4 = -6$ is equivalent to the addition fact $-2 = (-6) + 4$



Since the addition of integers has already been defined, subtraction is defined as the inverse of addition.

DEFINITION Subtraction of Integers

If a, b, and c are integers, then

$$a - b = c \text{ if and only if } a = c + b.$$

By the commutative property of addition, $b + c = c + b$, we obtain the fact family

$$a - b = c, a = c + b, a = b + c, a - c = b.$$

If one of these equations is true, then all the equations are true.

The Equivalence of Subtraction with Addition of the Opposite

We know that $5 - 2 = 3$, since removing two black counters from a loop of five black counters leaves three black counters in the loop. However, there is a second way to remove two black counters from the loop: Annihilate them by adding two red counters. This still leaves two black counters, so the subtraction $5 - 2$ can be replaced by the equivalent addition $5 + (-2)$. The addition and subtraction diagrams in Figure 5.16 show that $5 - 2 = 5 + (-2) = 3$.

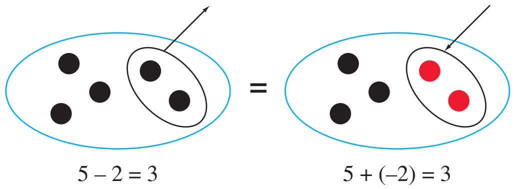


Figure 5.16
Showing subtraction by 2 is equivalent to addition by -2

Since removing counters of one color is equivalent to adding counters of the opposite color, we obtain the following theorem.

To subtract, add the negative.

THEOREM Subtracting by Adding the Opposite
Let a and b be any integers. Then

$$a - b = a + (-b).$$

Since the set of integers is closed under addition, a consequence of this theorem is that *the set of integers is closed under subtraction.*

THEOREM Closure Property for the Subtraction of Integers
The set of integers is closed under subtraction.

For example, in the set of whole numbers, there is no answer to $3 - 5$. But in the set of integers, $3 - 5 = -2$.

EXAMPLE 5.15 Subtracting by Adding the Opposite

Perform each of these subtractions as additions:

(a) $7 - 3$ **(b)** $(-7) - (-3)$ **(c)** $7 - (-3)$ **(d)** $(-7) - 3$

Solution

We make use of the theorem stating that $a - b = a + (-b)$ for any integers a and b.

(a) $7 - 3 = 7 + (-3) = 4$
(b) $(-7) - (-3) = (-7) + 3 = -4$
(c) $7 - (-3) = 7 + 3 = 10$
(d) $(-7) - 3 = -7 + (-3) = -10$

Subtraction of Integers by Using Mail-Time Stories

For this model of subtraction to work, we must imagine a situation where checks and bills are immediately credited or debited to your account as soon as they are delivered, whether they are really intended for you or not. If an error has been made by the mail carrier, he or she must return and reclaim delivered mail and take it to the intended recipient. Thus,

bringing a check adds a positive number,
bringing a bill adds a negative number,
taking away a check subtracts a positive number, and
taking away a bill subtracts a negative number.