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Amortisation and depreciation

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AMORTISATION

This chapter introduces methods of amortising loans or leases and compares the result with depreciation methods for writing off equipment.

Amortisation is the reduction of the value of an asset by prorating its initial cost over a number of periods. You can calculate the payment required under an annuity and then you need to split each payment into capital and interest. Due to the compounding nature of the time value of money, more capital is outstanding in the earlier periods and therefore more interest is payable on the outstanding balance. In effect, you pay for the capital outstanding. As the loan progresses, less interest is payable on the notional balance outstanding and more capital can be repaid in each period. Typical applications include house mortgages, bank loans or finance leases. To solve these problems, you need to construct a cash flow grid with the period number, rental payment, interest paid, capital repayment and balance outstanding.

The example below uses a capital value of 100,000 lent at a nominal rate of 8 per cent over three years with quarterly rentals. One rental payment is due on signing. Since there is no final rental, this results in a rental of 9,270.55 per quarter using the `PMT` function.

The amount lent is 100,000 less the first rental, which is 90,729.45. To calculate the interest for the first period, you multiply the capital outstanding by the periodic interest rate. This is 8 per cent/four rentals per annum. The first calculation is:

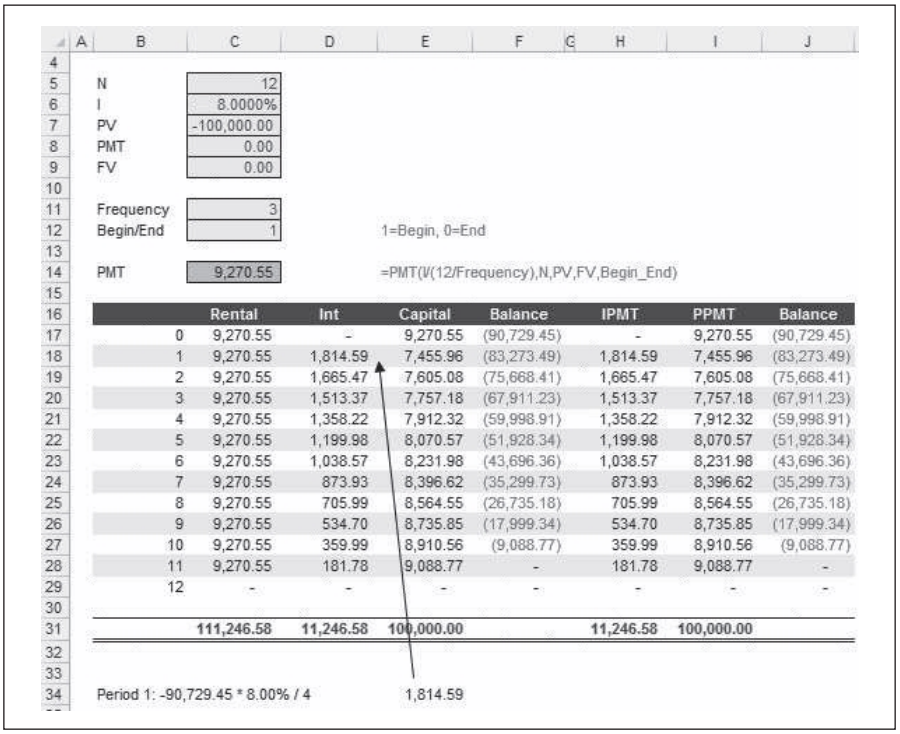
*90,729.45 * 2 per cent = 1,814.59*

The capital repayment is then the difference between 9,270.55 and the interest payable. The capital of 7,455.96 is added to the capital to form the capital outstanding at the end of period 1. The process is then repeated with the period 2 interest calculated from the capital outstanding at the end of period 1, and so on. Progressing through the 12 periods means that the interest per period declines while the capital repayable grows. In the final period the capital outstanding reduces to zero.

The checks are that the total interest equals the total charges and the capital repayments add up to the original capital. As seen in Figure 7.1 the interest is 11,246.58 and the capital is 100,000.

There are also functions in Excel for calculating directly the interest or the capital for any period. `IPMT` will calculate the amortised interest and `PPMT` the capital. The functions take the same arguments as the `PMT` and `PV` procedures with the addition of the current period number. Figure 7.2 shows the inputs for the `IPMT` function in the first period.

Figure 7.1 **Amortisation**



N	12	
I	8.0000%	
PV	-100,000.00	
PMT	0.00	
FV	0.00	
Frequency	3	
Begin/End	1	1=Begin, 0=End
PMT	9,270.55	=PMT(I/(12/Frequency),N,PV,FV,Begin_End)

	Rental	Int	Capital	Balance	IPMT	PPMT	Balance
0	9,270.55	-	9,270.55	(90,729.45)	-	9,270.55	(90,729.45)
1	9,270.55	1,814.59	7,455.96	(83,273.49)	1,814.59	7,455.96	(83,273.49)
2	9,270.55	1,665.47	7,605.08	(75,668.41)	1,665.47	7,605.08	(75,668.41)
3	9,270.55	1,513.37	7,757.18	(67,911.23)	1,513.37	7,757.18	(67,911.23)
4	9,270.55	1,358.22	7,912.32	(59,998.91)	1,358.22	7,912.32	(59,998.91)
5	9,270.55	1,199.98	8,070.57	(51,928.34)	1,199.98	8,070.57	(51,928.34)
6	9,270.55	1,038.57	8,231.98	(43,696.36)	1,038.57	8,231.98	(43,696.36)
7	9,270.55	873.93	8,396.62	(35,299.73)	873.93	8,396.62	(35,299.73)
8	9,270.55	705.99	8,564.55	(26,735.18)	705.99	8,564.55	(26,735.18)
9	9,270.55	534.70	8,735.85	(17,999.34)	534.70	8,735.85	(17,999.34)
10	9,270.55	359.99	8,910.56	(9,088.77)	359.99	8,910.56	(9,088.77)
11	9,270.55	181.78	9,088.77	-	181.78	9,088.77	-
12	-	-	-	-	-	-	-
	111,246.58	11,246.58	100,000.00		11,246.58	100,000.00	

Period 1: -90,729.45 * 8.00% / 4 1,814.59

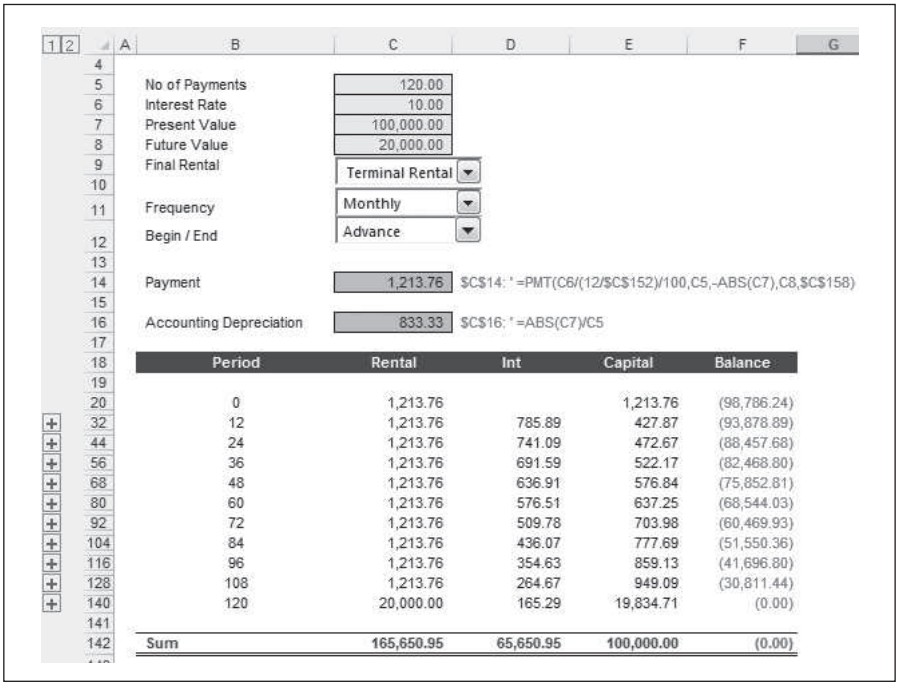
Figure 7.2 **IPMT/PPMT function**

FULL AMORTISATION

It is possible to scale up the initial calculations. As an example, Figure 7.3 shows the amortisation of a 10-year loan with monthly rentals in advance at a nominal rate of 10 per cent. There is a final payment or residual value of 20,000. The same calculations are used to multiply out the outstanding capital by the periodic interest rate. Since there is a rental due on signing, the first capital value is the net advance of 98,786.24.

Ten-year amortisation

Figure 7.3



5	No of Payments	120.00				
6	Interest Rate	10.00				
7	Present Value	100,000.00				
8	Future Value	20,000.00				
9	Final Rental	Terminal Rental				
11	Frequency	Monthly				
12	Begin / End	Advance				
14	Payment	1,213.76	$C$14: ' =PMT(C6/(12/$C$152)/100,C5,-ABS(C7),C8,$C$158)			
16	Accounting Depreciation	833.33	$C$16: ' =ABS(C7)/C5			
18	Period	Rental	Int	Capital	Balance	
20	0	1,213.76		1,213.76	(98,786.24)	
32	12	1,213.76	785.89	427.87	(93,878.89)	
44	24	1,213.76	741.09	472.67	(88,457.68)	
56	36	1,213.76	691.59	522.17	(82,468.80)	
68	48	1,213.76	636.91	576.84	(75,852.81)	
80	60	1,213.76	576.51	637.25	(68,544.03)	
92	72	1,213.76	509.78	703.98	(60,469.93)	
104	84	1,213.76	436.07	777.69	(51,550.36)	
116	96	1,213.76	354.63	859.13	(41,696.80)	
128	108	1,213.76	264.67	949.09	(30,811.44)	
140	120	20,000.00	165.29	19,834.71	(0.00)	
142	Sum	165,650.95	65,650.95	100,000.00	(0.00)	

Again the check is the totals row where the total payable less the charges equals the capital value. The cash flow above contains 121 payments and is grouped in years in order to show the summary.

DELAYED PAYMENTS

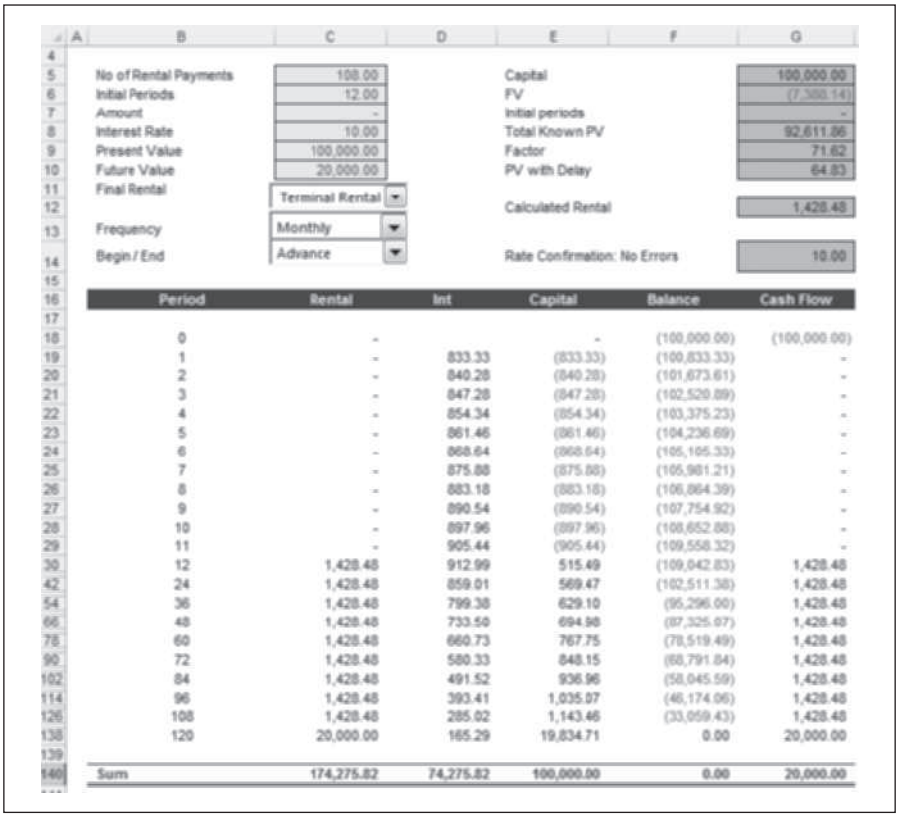
The amortisation period of 120 payments can be structured to accept periods of lower rentals and can still be structured to split the subsequent rentals into interest and capital. If no rental is being paid then the capital outstanding rises due to the continuing cost of funding the balance outstanding.

Instead of the capital reducing, it rises due to the lack of repayments, since the outstanding balance has to be notionally financed.

The rental cannot be calculated directly and needs to be computed using the factors or $1 method. While you could use Solver or Goal Seek, this procedure will compute a rental or payment directly. The method is as follows:

- Derive the original capital value with cash out as negative and cash in as positive.
- Present value and payments during the period together with the final value.
- Add these two values to form the known present value (A).
- Present-value factors using one instead of the rental. In this case the model will need to present-value 11 cash flows of zero (since one is due on signing) followed by 108 payments of one at the period discount rate of 10 per cent/12. This is (B).
- The periodic rental is calculated by dividing (A) by (B).

Figure 7.4 **Delayed rentals amortisation**



No of Rental Payments	108.00		Capital		100,000.00
Initial Periods	12.00		FV		(7,388.14)
Amount	-		Initial periods		-
Interest Rate	10.00		Total Known PV		92,611.86
Present Value	100,000.00		Factor		71.62
Future Value	20,000.00		PV with Delay		64.83
Final Rental	Terminal Rental		Calculated Rental		1,428.48
Frequency	Monthly				
Begin / End	Advance		Rate Confirmation: No Errors		10.00

Period	Rental	Int	Capital	Balance	Cash Flow
0	-		-	(100,000.00)	(100,000.00)
1	-	833.33	(833.33)	(100,833.33)	-
2	-	840.28	(840.28)	(101,673.61)	-
3	-	847.28	(847.28)	(102,520.89)	-
4	-	854.34	(854.34)	(103,375.23)	-
5	-	861.46	(861.46)	(104,236.69)	-
6	-	868.64	(868.64)	(105,105.33)	-
7	-	875.88	(875.88)	(105,981.21)	-
8	-	883.18	(883.18)	(106,864.39)	-
9	-	890.54	(890.54)	(107,754.92)	-
10	-	897.96	(897.96)	(108,652.88)	-
11	-	905.44	(905.44)	(109,558.32)	-
12	1,428.48	912.99	515.49	(109,042.83)	1,428.48
24	1,428.48	859.01	569.47	(102,511.38)	1,428.48
36	1,428.48	799.38	629.10	(95,296.00)	1,428.48
48	1,428.48	733.59	694.98	(87,325.97)	1,428.48
60	1,428.48	660.73	767.75	(78,519.49)	1,428.48
72	1,428.48	580.33	848.15	(68,791.84)	1,428.48
84	1,428.48	491.52	936.96	(58,045.59)	1,428.48
96	1,428.48	393.41	1,035.07	(46,174.06)	1,428.48
108	1,428.48	285.02	1,143.46	(33,059.43)	1,428.48
120	20,000.00	165.29	19,834.71	0.00	20,000.00
Sum	174,275.82	74,275.82	100,000.00	0.00	20,000.00