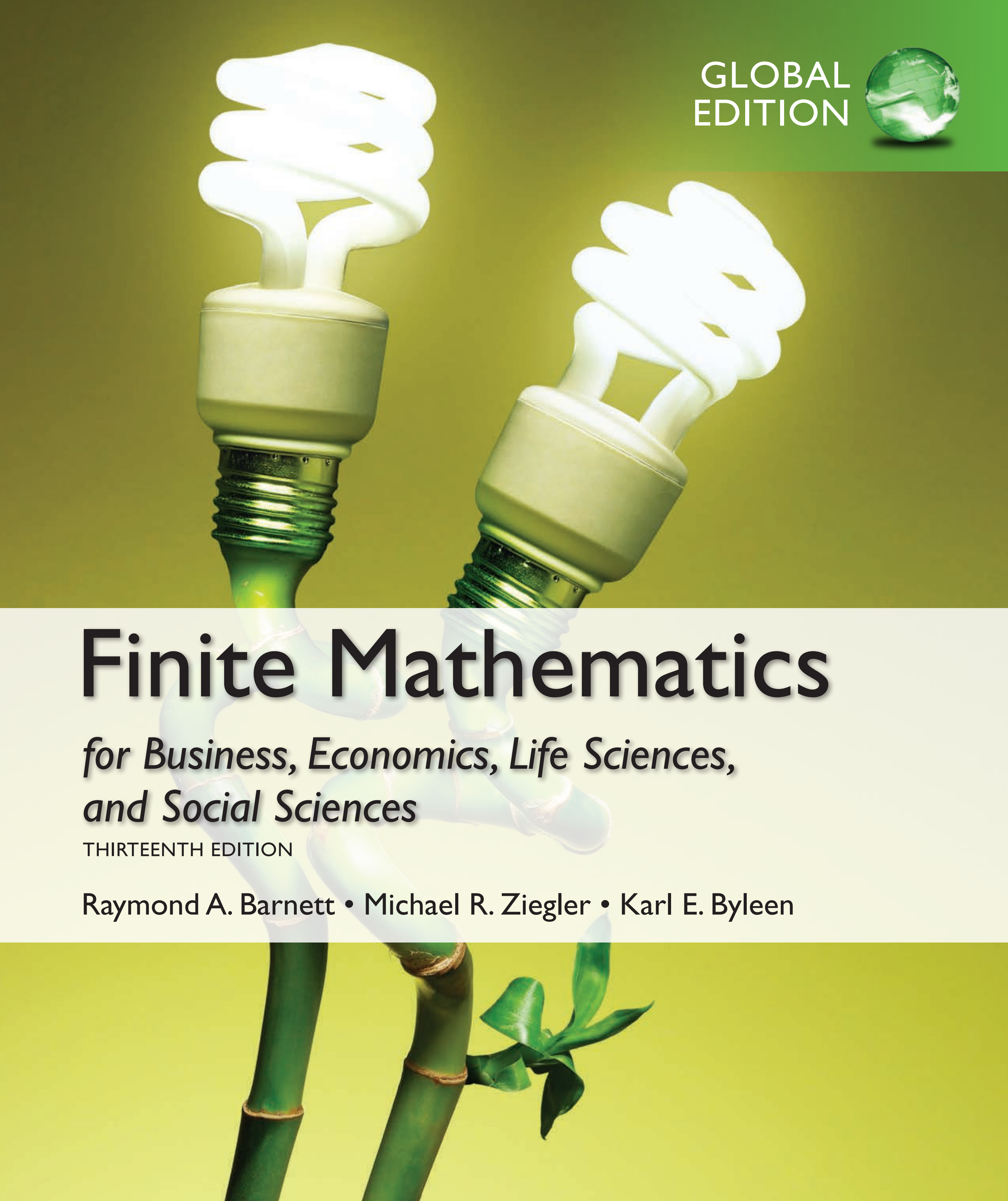

GLOBAL EDITION

Finite Mathematics

for Business, Economics, Life Sciences, and Social Sciences

THIRTEENTH EDITION

Raymond A. Barnett • Michael R. Ziegler • Karl E. Byleen

ALWAYS LEARNING

PEARSON

FINITE MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

Thirteenth Edition

Global Edition

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9. List the elements on the principal diagonal of A.

10. List the elements on the principal diagonal of B.

11. For matrix B, list the elements b_{31}, b_{22}, b_{13}.

12. For matrix A, list the elements a_{21}, a_{12}.

13. For matrix C, find $c_{11} + c_{12} + c_{13}$.

14. For matrix D, find $d_{11} + d_{21}$.

In Problems 15–18, write the coefficient matrix and the augmented matrix of the given system of linear equations.

15. $\begin{aligned} 3x_1 + 5x_2 &= 8 \\ 2x_1 - 4x_2 &= -7 \end{aligned}$

16. $\begin{aligned} -8x_1 + 3x_2 &= 10 \\ 6x_1 + 5x_2 &= 13 \end{aligned}$

17. $\begin{aligned} x_1 + 4x_2 &= 15 \\ 6x_1 &= 18 \end{aligned}$

18. $\begin{aligned} 5x_1 - x_2 &= 10 \\ 3x_2 &= 21 \end{aligned}$

In Problems 19–22, write the system of linear equations that is represented by the given augmented matrix. Assume that the variables are x_1 and x_2.

19. $\left[\begin{array}{cc|c} 2 & 5 & 7 \\ 1 & 4 & 9 \end{array}\right]$

20. $\left[\begin{array}{cc|c} 0 & 3 & 15 \\ -8 & 2 & 25 \end{array}\right]$

21. $\left[\begin{array}{cc|c} 4 & 0 & -10 \\ 0 & 8 & 40 \end{array}\right]$

22. $\left[\begin{array}{cc|c} 1 & -2 & 12 \\ 0 & 1 & 6 \end{array}\right]$

Perform the row operations indicated in Problems 23–34 on the following matrix:

$$\left[\begin{array}{cc|c} 1 & -3 & 2 \\ 4 & -6 & -8 \end{array}\right]$$

23. $R_1 \leftrightarrow R_2$

24. $\frac{1}{2}R_2 \rightarrow R_2$

25. $-4R_1 \rightarrow R_1$

26. $-2R_1 \rightarrow R_1$

27. $2R_2 \rightarrow R_2$

28. $-1R_2 \rightarrow R_2$

29. $(-4)R_1 + R_2 \rightarrow R_2$

30. $\left(-\frac{1}{2}\right)R_2 + R_1 \rightarrow R_1$

31. $(-2)R_1 + R_2 \rightarrow R_2$

32. $(-3)R_1 + R_2 \rightarrow R_2$

33. $(-1)R_1 + R_2 \rightarrow R_2$

34. $R_1 + R_2 \rightarrow R_2$

Each of the matrices in Problems 35–42 is the result of performing a single row operation on the matrix A shown below. Identify the row operation.

$$A = \left[\begin{array}{cc|c} -1 & 2 & -3 \\ 6 & -3 & 12 \end{array}\right]$$

35. $\left[\begin{array}{cc|c} -1 & 2 & -3 \\ 2 & -1 & 4 \end{array}\right]$

36. $\left[\begin{array}{cc|c} -2 & 4 & -6 \\ 6 & -3 & 12 \end{array}\right]$

37. $\left[\begin{array}{cc|c} -1 & 2 & -3 \\ 0 & 9 & -6 \end{array}\right]$

38. $\left[\begin{array}{cc|c} 3 & 0 & 5 \\ 6 & -3 & 12 \end{array}\right]$

39. $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 6 & -3 & 12 \end{array}\right]$

40. $\left[\begin{array}{cc|c} -1 & 2 & -3 \\ 2 & 5 & 0 \end{array}\right]$

41. $\left[\begin{array}{cc|c} 6 & -3 & 12 \\ -1 & 2 & -3 \end{array}\right]$

42. $\left[\begin{array}{cc|c} -1 & 2 & -3 \\ 0 & 9 & -6 \end{array}\right]$

Solve Problems 43–46 using augmented matrix methods. Graph each solution set. Discuss the differences between the graph of an equation in the system and the graph of the system's solution set.

43. $\begin{aligned} 3x_1 - 2x_2 &= 6 \\ 4x_1 - 3x_2 &= 6 \end{aligned}$

44. $\begin{aligned} x_1 - 2x_2 &= 5 \\ -2x_1 + 4x_2 &= -10 \end{aligned}$

45. $\begin{aligned} 3x_1 - 2x_2 &= -3 \\ -6x_1 + 4x_2 &= 6 \end{aligned}$

46. $\begin{aligned} x_1 - 2x_2 &= 1 \\ -2x_1 + 5x_2 &= 2 \end{aligned}$

Solve Problems 47 and 48 using augmented matrix methods. Write the linear system represented by each augmented matrix in your solution, and solve each of these systems graphically. Discuss the relationships among the solutions of these systems.

47. $\begin{aligned} x_1 + x_2 &= 5 \\ x_1 - x_2 &= 1 \end{aligned}$

48. $\begin{aligned} x_1 - x_2 &= 2 \\ x_1 + x_2 &= 6 \end{aligned}$

Each of the matrices in Problems 49–54 is the final matrix form for a system of two linear equations in the variables x_1 and x_2. Write the solution of the system.

49. $\left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 6 \end{array}\right]$

50. $\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -5 \end{array}\right]$

51. $\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 0 & 4 \end{array}\right]$

52. $\left[\begin{array}{cc|c} 1 & -2 & 7 \\ 0 & 0 & -9 \end{array}\right]$

53. $\left[\begin{array}{cc|c} 1 & -2 & 15 \\ 0 & 0 & 0 \end{array}\right]$

54. $\left[\begin{array}{cc|c} 1 & 5 & 10 \\ 0 & 0 & 0 \end{array}\right]$

Solve Problems 55–74 using augmented matrix methods.

55. $\begin{aligned} x_1 - 2x_2 &= 1 \\ 2x_1 - x_2 &= 5 \end{aligned}$

56. $\begin{aligned} x_1 + 3x_2 &= 1 \\ 3x_1 - 2x_2 &= 14 \end{aligned}$

57. $\begin{aligned} x_1 - 4x_2 &= -2 \\ -2x_1 + x_2 &= -3 \end{aligned}$

58. $\begin{aligned} x_1 - 3x_2 &= -5 \\ -3x_1 - x_2 &= 5 \end{aligned}$

59. $\begin{aligned} 3x_1 - x_2 &= 2 \\ x_1 + 2x_2 &= 10 \end{aligned}$

60. $\begin{aligned} 2x_1 + x_2 &= 0 \\ x_1 - 2x_2 &= -5 \end{aligned}$

61. $\begin{aligned} x_1 + 2x_2 &= 4 \\ 2x_1 + 4x_2 &= -8 \end{aligned}$

62. $\begin{aligned} 2x_1 - 3x_2 &= -2 \\ -4x_1 + 6x_2 &= 7 \end{aligned}$

63. $\begin{aligned} 2x_1 + x_2 &= 6 \\ x_1 - x_2 &= -3 \end{aligned}$

64. $\begin{aligned} 3x_1 - x_2 &= -5 \\ x_1 + 3x_2 &= 5 \end{aligned}$

65. $\begin{aligned} 3x_1 - 6x_2 &= -9 \\ -2x_1 + 4x_2 &= 6 \end{aligned}$

66. $\begin{aligned} 2x_1 - 4x_2 &= -2 \\ -3x_1 + 6x_2 &= 3 \end{aligned}$

67. $\begin{aligned} 4x_1 - 2x_2 &= 2 \\ -6x_1 + 3x_2 &= -3 \end{aligned}$

68. $\begin{aligned} -6x_1 + 2x_2 &= 4 \\ 3x_1 - x_2 &= -2 \end{aligned}$

69. $\begin{aligned} 2x_1 + x_2 &= 1 \\ 4x_1 - x_2 &= -7 \end{aligned}$

70. $\begin{aligned} 2x_1 - x_2 &= -8 \\ 2x_1 + x_2 &= 8 \end{aligned}$

71. $\begin{aligned} 4x_1 - 6x_2 &= 8 \\ -6x_1 + 9x_2 &= -10 \end{aligned}$

72. $\begin{aligned} 2x_1 - 4x_2 &= -4 \\ -3x_1 + 6x_2 &= 4 \end{aligned}$

73. $\begin{aligned} -4x_1 + 6x_2 &= -8 \\ 6x_1 - 9x_2 &= 12 \end{aligned}$

74. $\begin{aligned} -2x_1 + 4x_2 &= 4 \\ 3x_1 - 6x_2 &= -6 \end{aligned}$

Solve Problems 75–80 using augmented matrix methods.

75. $\begin{aligned} 3x_1 - x_2 &= 7 \\ 2x_1 + 3x_2 &= 1 \end{aligned}$

76. $\begin{aligned} 2x_1 - 3x_2 &= -8 \\ 5x_1 + 3x_2 &= 1 \end{aligned}$

77. $\begin{aligned} 3x_1 + 2x_2 &= 4 \\ 2x_1 - x_2 &= 5 \end{aligned}$

78. $\begin{aligned} 4x_1 + 3x_2 &= 26 \\ 3x_1 - 11x_2 &= -7 \end{aligned}$

79. $\begin{aligned} 0.2x_1 - 0.5x_2 &= 0.07 \\ 0.8x_1 - 0.3x_2 &= 0.79 \end{aligned}$

80. $\begin{aligned} 0.3x_1 - 0.6x_2 &= 0.18 \\ 0.5x_1 - 0.2x_2 &= 0.54 \end{aligned}$

Solve Problems 81–84 using augmented matrix methods. Use a graphing calculator to perform the row operations.

81. $\begin{aligned} 0.8x_1 + 2.88x_2 &= 4 \\ 1.25x_1 + 4.34x_2 &= 5 \end{aligned}$

82. $\begin{aligned} 2.7x_1 - 15.12x_2 &= 27 \\ 3.25x_1 - 18.52x_2 &= 33 \end{aligned}$

83. $\begin{aligned} 4.8x_1 - 40.32x_2 &= 295.2 \\ -3.75x_1 + 28.7x_2 &= -211.2 \end{aligned}$

84. $\begin{aligned} 5.7x_1 - 8.55x_2 &= -35.91 \\ 4.5x_1 + 5.73x_2 &= 76.17 \end{aligned}$

Answers to Matched Problems

1. $x_1 = -2, x_2 = 3$

2. $x_1 = 2, x_2 = -\frac{1}{2}$

3. The system is dependent. For t any real number, a solution is $x_1 = 3t - 3, x_2 = t$.

4. Inconsistent—no solution

4.3 Gauss–Jordan Elimination

- Reduced Matrices
- Solving Systems by Gauss–Jordan Elimination
- Application

Now that you have had some experience with row operations on simple augmented matrices, we consider systems involving more than two variables. We will not require a system to have the same number of equations as variables. Just as for systems of two linear equations in two variables, any linear system, regardless of the number of equations or number of variables, has either

1. Exactly one solution (consistent and independent), or
2. Infinitely many solutions (consistent and dependent), or
3. No solution (inconsistent).

Reduced Matrices

In the preceding section we used row operations to transform the augmented matrix for a system of two equations in two variables,

$$\left[\begin{array}{cc|c} a_{11} & a_{12} & k_1 \\ a_{21} & a_{22} & k_2 \end{array}\right] \qquad \begin{aligned} a_{11}x_1 + a_{12}x_2 &= k_1 \\ a_{21}x_1 + a_{22}x_2 &= k_2 \end{aligned}$$

into one of the following simplified forms:

$$\underset{\text{Form 1}}{\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array}\right]} \quad \underset{\text{Form 2}}{\left[\begin{array}{cc|c} 1 & m & n \\ 0 & 0 & 0 \end{array}\right]} \quad \underset{\text{Form 3}}{\left[\begin{array}{cc|c} 1 & m & n \\ 0 & 0 & p \end{array}\right]} \qquad (1)$$

where m, n, and p are real numbers, $p \neq 0$. Each of these reduced forms represents a system that has a different type of solution set, and no two of these forms are row equivalent.

For large linear systems, it is not practical to list all such simplified forms; there are too many of them. Instead, we give a general definition of a simplified form called a **reduced matrix**, which can be applied to all matrices and systems, regardless of size.

DEFINITION Reduced Form

A matrix is said to be in **reduced row echelon form**, or, more simply, in **reduced form**, if

1. Each row consisting entirely of zeros is below any row having at least one nonzero element.
2. The leftmost nonzero element in each row is 1.
3. All other elements in the column containing the leftmost 1 of a given row are zeros.
4. The leftmost 1 in any row is to the right of the leftmost 1 in the row above.

The following matrices are in reduced form. Check each one carefully to convince yourself that the conditions in the definition are met.

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

EXAMPLE 1 Reduced Forms The following matrices are not in reduced form. Indicate which condition in the definition is violated for each matrix. State the row operation(s) required to transform the matrix into reduced form, and find the reduced form.

(A) $\left[\begin{array}{cc|c} 0 & 1 & -2 \\ 1 & 0 & 3 \end{array}\right]$ (B) $\left[\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$

(C) $\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 1 & -2 \end{array}\right]$ (D) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right]$

SOLUTION

(A) Condition 4 is violated: The leftmost 1 in row 2 is not to the right of the leftmost 1 in row 1. Perform the row operation $R_1 \leftrightarrow R_2$ to obtain

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right]$$

(B) Condition 3 is violated: The column containing the leftmost 1 in row 2 has a nonzero element above the 1. Perform the row operation $2R_2 + R_1 \rightarrow R_1$ to obtain

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

(C) Condition 1 is violated: The second row contains all zeros and is not below any row having at least one nonzero element. Perform the row operation $R_2 \leftrightarrow R_3$ to obtain

$$\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right]$$

(D) Condition 2 is violated: The leftmost nonzero element in row 2 is not a 1. Perform the row operation $\frac{1}{2}R_2 \rightarrow R_2$ to obtain

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & -5 \end{array}\right]$$

Matched Problem 1 The matrices below are not in reduced form. Indicate which condition in the definition is violated for each matrix. State the row operation(s) required to transform the matrix into reduced form, and find the reduced form.

(A) $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 3 & -6 \end{array}\right]$

(B) $\left[\begin{array}{ccc|c} 1 & 5 & 4 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(C) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$

(D) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right]$

Solving Systems by Gauss–Jordan Elimination

We are now ready to outline the Gauss–Jordan method for solving systems of linear equations. The method systematically transforms an augmented matrix into a reduced form. The system corresponding to a reduced augmented matrix is called a **reduced system**. As we shall see, reduced systems are easy to solve.

The Gauss–Jordan elimination method is named after the German mathematician Carl Friedrich Gauss (1777–1885) and the German geodesist Wilhelm Jordan (1842–1899). Gauss, one of the greatest mathematicians of all time, used a method of solving systems of equations in his astronomical work that was later generalized by Jordan to solve problems in large-scale surveying.

EXAMPLE 2 Solving a System Using Gauss–Jordan Elimination Solve by Gauss–Jordan elimination:

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= 3 \\ 3x_1 + x_2 - x_3 &= 7 \\ x_1 - 3x_2 + 2x_3 &= 0 \end{aligned}$$

SOLUTION Write the augmented matrix and follow the steps indicated at the right.

Need a 1 here.

$$\left[\begin{array}{rrr|r} 2 & -2 & 1 & 3 \\ 3 & 1 & -1 & 7 \\ 1 & -3 & 2 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

Step 1 Choose the leftmost nonzero column and get a 1 at the top.

Need 0's here.

$$\sim \left[\begin{array}{rrr|r} 1 & -3 & 2 & 0 \\ 3 & 1 & -1 & 7 \\ 2 & -2 & 1 & 3 \end{array}\right] \quad \begin{array}{l} (-3)R_1 + R_2 \to R_2 \\ (-2)R_1 + R_3 \to R_3 \end{array}$$

Step 2 Use multiples of the row containing the 1 from step 1 to get zeros in all remaining places in the column containing this 1.

Need a 1 here.

$$\sim \left[\begin{array}{rrr|r} 1 & -3 & 2 & 0 \\ 0 & 10 & -7 & 7 \\ 0 & 4 & -3 & 3 \end{array}\right] \quad 0.1R_2 \to R_2$$

Step 3 Repeat step 1 with the *submatrix* formed by (mentally) deleting the top row.

Need 0's here.

$$\sim \left[\begin{array}{rrr|r} 1 & -3 & 2 & 0 \\ 0 & 1 & -0.7 & 0.7 \\ 0 & 4 & -3 & 3 \end{array}\right] \quad \begin{array}{l} 3R_2 + R_1 \to R_1 \\ (-4)R_2 + R_3 \to R_3 \end{array}$$

Step 4 Repeat step 2 with the *entire matrix.*

Need a 1 here.

$$\sim \left[\begin{array}{rrr|r} 1 & 0 & -0.1 & 2.1 \\ 0 & 1 & -0.7 & 0.7 \\ 0 & 0 & -0.2 & 0.2 \end{array}\right] \quad (-5)R_3 \to R_3$$

Step 3 Repeat step 1 with the *submatrix* formed by (mentally) deleting the top rows.

Need 0's here.

$$\sim \left[\begin{array}{rrr|r} 1 & 0 & -0.1 & 2.1 \\ 0 & 1 & -0.7 & 0.7 \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \begin{array}{l} 0.1R_3 + R_1 \to R_1 \\ 0.7R_3 + R_2 \to R_2 \end{array}$$

Step 4 Repeat step 2 with the *entire matrix.*

$$\sim \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

The matrix is now in reduced form, and we can solve the corresponding reduced system.

$$\begin{aligned} x_1 \qquad\quad &= 2 \\ x_2 \quad\; &= 0 \\ x_3 &= -1 \end{aligned}$$

The solution to this system is $x_1 = 2, x_2 = 0, x_3 = -1$. You should check this solution in the original system.

Matched Problem 2 Solve by Gauss–Jordan elimination:

$$\begin{aligned} 3x_1 + x_2 - 2x_3 &= 2 \\ x_1 - 2x_2 + x_3 &= 3 \\ 2x_1 - x_2 - 3x_3 &= 3 \end{aligned}$$

PROCEDURE Gauss–Jordan Elimination

Step 1 Choose the leftmost nonzero column and use appropriate row operations to get a 1 at the top.

Step 2 Use multiples of the row containing the 1 from step 1 to get zeros in all remaining places in the column containing this 1.

(Continued)

Step 3 Repeat step 1 with the **submatrix** formed by (mentally) deleting the row used in step 2 and all rows above this row.

Step 4 Repeat step 2 with the **entire matrix,** including the rows deleted mentally. Continue this process until the entire matrix is in reduced form.

Note: If at any point in this process we obtain a row with all zeros to the left of the vertical line and a nonzero number to the right, we can stop before we find the reduced form since we will have a contradiction: $0 = n$, $n \neq 0$. We can then conclude that the system has no solution.

Remarks

1. Even though each matrix has a unique reduced form, the sequence of steps presented here for transforming a matrix into a reduced form is not unique. For example, it is possible to use row operations in such a way that computations involving fractions are minimized. But we emphasize again that we are not interested in the most efficient hand methods for transforming small matrices into reduced forms. Our main interest is in giving you a little experience with a method that is suitable for solving large-scale systems on a graphing calculator or computer.
2. Most graphing calculators have the ability to find reduced forms. Figure 1 illustrates the solution of Example 2 on a TI-86 graphing calculator using the rref command (rref is an acronym for reduced row echelon form). Notice that in row 2 and column 4 of the reduced form the graphing calculator has displayed the very small number –3.5E – 13, instead of the exact value 0. This is a common occurrence on a graphing calculator and causes no problems. Just replace any very small numbers displayed in scientific notation with 0.

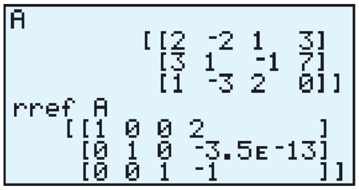

Figure 1 **Gauss–Jordan elimination on a graphing calculator**

EXAMPLE 3 Solving a System Using Gauss–Jordan Elimination Solve by Gauss–Jordan elimination:

$$\begin{aligned} 2x_1 - 4x_2 + x_3 &= -4 \\ 4x_1 - 8x_2 + 7x_3 &= 2 \\ -2x_1 + 4x_2 - 3x_3 &= 5 \end{aligned}$$

SOLUTION

$$\left[\begin{array}{ccc|c} 2 & -4 & 1 & -4 \\ 4 & -8 & 7 & 2 \\ -2 & 4 & -3 & 5 \end{array}\right] \quad 0.5R_1 \to R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 0.5 & -2 \\ 4 & -8 & 7 & 2 \\ -2 & 4 & -3 & 5 \end{array}\right] \quad \begin{array}{l} (-4)R_1 + R_2 \to R_2 \\ 2R_1 + R_3 \to R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 0.5 & -2 \\ 0 & 0 & 5 & 10 \\ 0 & 0 & -2 & 1 \end{array}\right] \quad 0.2R_2 \to R_2$$

Note that column 3 is the leftmost nonzero column in this submatrix.

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 0.5 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & 1 \end{array}\right] \quad \begin{array}{l} (-0.5)R_2 + R_1 \to R_1 \\ 2R_2 + R_3 \to R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

We stop the Gauss–Jordan elimination, even though the matrix is not in reduced form, since the last row produces a contradiction.

The system has no solution.